Lezione 20 22/05/2024

mercoledì 22 maggio 2024 13:57

Lezione #20

22/5/2024

Eq^ne stato gas perfetti

$L = \int p\, dV \rightarrow$ area

$PV = nRT$

$Q_V = n C_V \Delta T$

$Q_P = n C_P \Delta T$

$\Delta E_{INT} = Q - L$

$\Delta S = \dfrac{Q}{T}$

in un ciclo:

$\Delta E_{INT} = 0 \qquad \Delta S = 0$

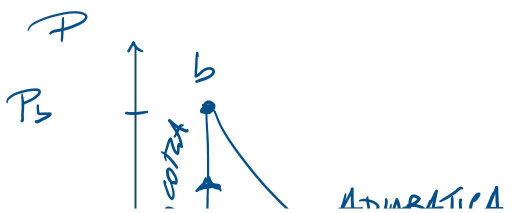
$Q = L$

Riprendiamo esercizio precedente:

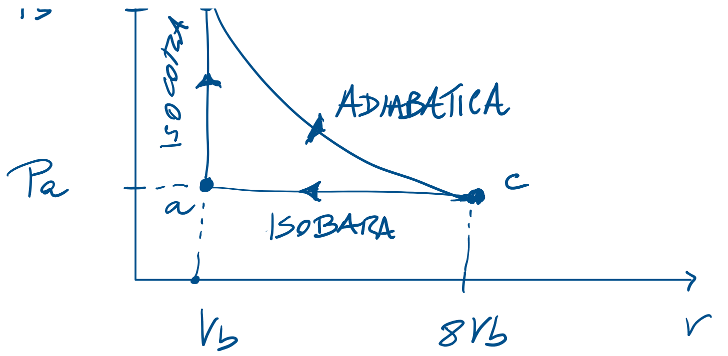
Esercizio #25 (CAP. $\underline{XX}$) Halliday-Resnick

A una mole di un gas ideale monoatomico ($C_V = \frac{3}{2}R$; $C_P = \frac{5}{2}R$)

viene fatto percorrere il ciclo:



$$\begin{cases} R = 8{,}31 \ J/mol\,K \\ P_a = 0{,}3165 \ bar \\ P_b = 10{,}1 \ bar \end{cases}$$



$$\begin{cases} P_b = 10{,}1 \text{ bar} \\ V_b = 10^{-3}\, m^3 \\ 1 \text{ bar} = 10^5 \text{ Pa} \end{cases}$$

Calcolare:
1) Calore assorbito dal gas
2) " ceduto dal gas
3) Lavoro totale compiuto dal gas
4) rendimento η

1) Calore viene assorbito solo nella prima trasformazione isocora ($V =$ cost mentre $P \nearrow$)

$$Q = n c_v \Delta T = n c_v (T_b - T_a)$$

Dal momento che $pV = nRT$

$$T = \frac{pV}{nR} \Rightarrow T_a = \frac{P_a V_A}{nR}$$

$$T_b = \frac{P_b V_B}{nR}$$

$$Q = \cancel{n} \underbrace{\frac{3}{2} \cancel{R}}_{c_v} \left(\frac{P_b V_B}{\cancel{nR}} - \frac{P_a V_A}{\cancel{nR}} \right)$$

$$\begin{cases} V_A = V_b \\ V_B = V_b \end{cases}$$

$$= \frac{3}{2}\left(P_B V_B - P_a V_A\right) = \frac{3}{2}\left(P_b V_b - P_a V_b\right)$$

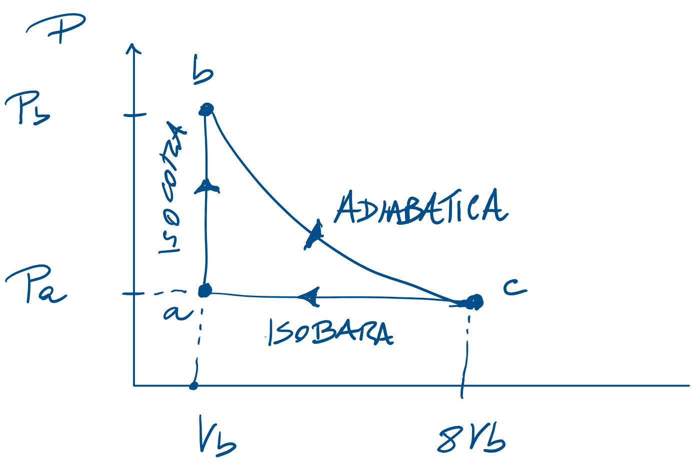
$$= \frac{3}{2} V_b \left(P_b - P_a\right)$$

$$Q = \frac{3}{2} \cancel{10^{-3}} \left(10{,}1 - 0{,}3165\right) \cdot \underbrace{\cancel{10^{5}}\, 10^{2}}_{\text{Pascal}} =$$

Pascal $[1 \text{ bar} = 10^{5} \text{ Pascal}]$

$$\boxed{Q_{ASS} = 1{,}4675 \cdot 10^{3} \text{ J}}$$

2) Calore ceduto



Q_{CEDUTO}

$\downarrow$

ISOBARA

$$Q_P = n\,C_P\,(T_f - T_i)$$

$$T_f = \frac{P_a V_b}{nR}$$

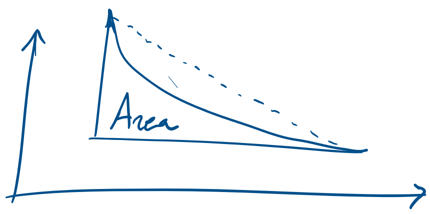
$$T_i = \frac{P_a\, 8 V_b}{nR}$$

$$= \cancel{n}\,\frac{5}{2}\,\cancel{R}\left(\frac{P_a V_b}{\cancel{nR}} - \frac{P_a V_b\, 8}{\cancel{nR}}\right)$$

$$= \frac{5}{2} P_a V_b (1 - 8) = \frac{5}{2}(0{,}3165\,\cancel{10^5})(\cancel{10^{-3}})(-7)\,10^2$$

$$\boxed{Q_{CEDUTO} = -553{,}87\ J}$$

3) $L_{SVOLTO} = ?$



Area = ? Non ~~so~~ come calcolarla ...

Possiamo invece sfruttare il fatto che sia un ciclo

$$\Delta E_{INT} = 0 \Rightarrow 0 = Q - L \Rightarrow L = Q\ !$$

$$L = |Q_{ASS}| - |Q_{CEDUTO}| = (1{,}4675\,10^3 - 553{,}87) =$$

$$\boxed{L = 913{,}12\ J}$$

4) $\eta = ?$

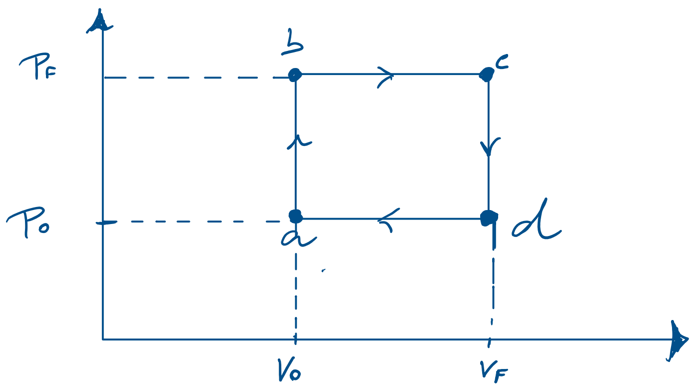
$$\eta = \frac{L}{|Q_{ASS}|} = \frac{|Q_{ASS}| - |Q_{CEDUTO}|}{|Q_{ASS}|}$$

$$= 1 - \frac{|Q_{CEDUTO}|}{|Q_{ASS}|}$$

$$\eta = 1 - \frac{553{,}84}{(1{,}4675 \cdot 10^3)} = 0{,}62 = 62\%$$

Es. 26 (CAP XX) Hallyday-Resnick

$n = 1$
gas monoatomico



$P_F = 2P_0$

$V_F = 2V_0$

$P_0 = 1{,}01 \cdot 10^5\ Pa$

$V_0 = 0{,}0225\ m^3$

$\left[c_v = \frac{3}{2} R ; \quad c_p = \frac{5}{2} R \right]$

a) L

b) $Q_{FORNITO}$ durante abc

c) η

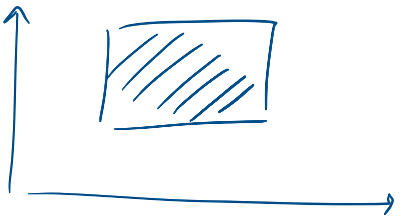
d) η_{CARNOT} se la macc. lavora tra

$V_0 = 0{,}0225\ m$

c) η

d) η_{CARNOT} se la macc. lavora tra la T più bassa e alta del ciclo

a) $\mathcal{L} = \int_i^f p\, dV$ = area racchiusa dal ciclo



$$\text{Area} = (V_F - V_0)(P_F - P_0) = (2V_0 - V_0)(2P_0 - P_0) = P_0 V_0$$

$$= (1{,}01\ 10^5 \cdot 0{,}0225) = 2272{,}5\ J$$

$$\mathcal{L}_{TOT} = \underset{\substack{\| \\ 0 \\ \text{(ISOCORA)}}}{\mathcal{L}_{ab}} + \mathcal{L}_{bc} + \underset{\substack{\| \\ 0 \\ \text{(ISOCORA)}}}{\mathcal{L}_{cd}} + \mathcal{L}_{da} = \mathcal{L}_{bc} + \mathcal{L}_{da}$$

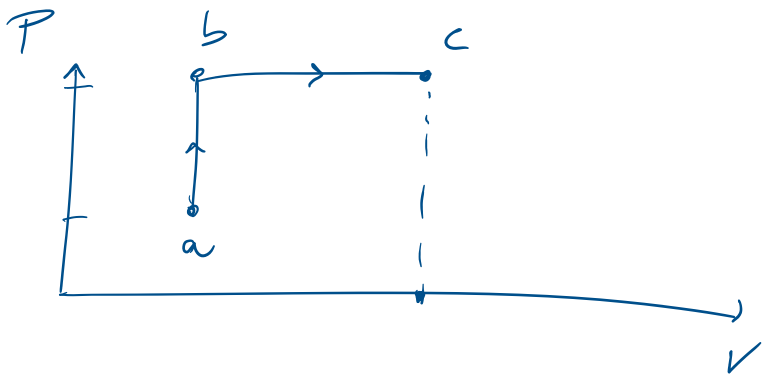
ISOBARA

$$= P_F (V_F - V_0) + P_0 (V_0 - V_F) =$$

$$= 2p_0 (V_F - V_0) - p_0 (V_F - V_0) =$$

$$= p_0 (V_F - V_0) = p_0 V_0 \quad \checkmark$$

2) $Q_{FORNITO}$ ABC



$$Q_{FORNITO} = Q_{ab} + Q_{bc} = n C_V \Delta T + n C_P \Delta T$$

$$= n C_V \left(\overset{T_b}{\frac{P_F V_0}{nR}} - \overset{T_a}{\frac{P_0 V_0}{nR}} \right) + n C_P \left(\overset{T_c}{\frac{P_F V_F}{nR}} - \overset{T_b}{\frac{P_F V_0}{nR}} \right)$$

$$= \cancel{n} \frac{3}{2} \cancel{R} \left(\frac{P_F V_0}{\cancel{nR}} - \frac{P_0 V_0}{\cancel{nR}} \right) + \cancel{n} \frac{5}{2} \cancel{R} \left(\frac{P_F V_F}{\cancel{nR}} - \frac{P_F V_0}{\cancel{nR}} \right)$$

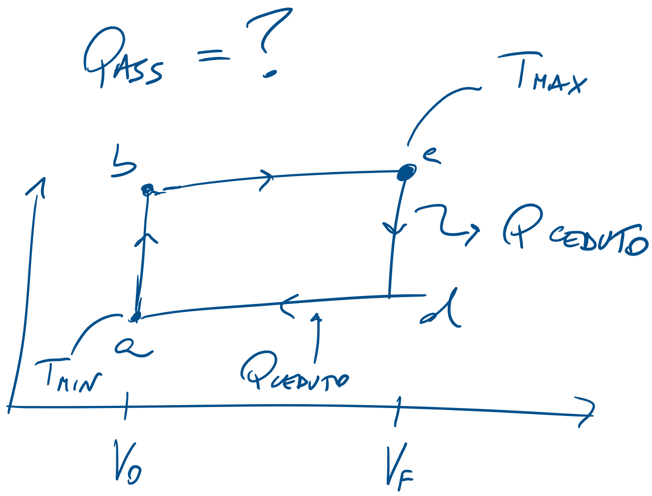
$$= \frac{3}{2} \left(2P_0 V_0 - P_0 V_0 \right) + \frac{5}{2} \left(2P_0 2V_0 - 2P_0 V_0 \right)$$

$$= \frac{3}{2} P_0 V_0 + \frac{5}{2} 2P_0 V_0 = \frac{13}{2} P_0 V_0$$

$$\boxed{Q_{abc} = \frac{13}{2} P_0 V_0 = 1{,}4771 \; 10^4 \, J}$$

3) $\eta = \frac{L}{|Q_{ASS}|}$

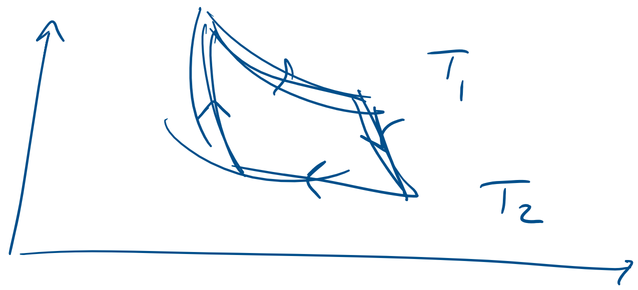
$Q_{ASS} = ?$



$|Q_{ASS}| = Q_{ab} + Q_{bc}$

$$\eta = \frac{2272,5}{14,772} \simeq 0,15 = 15\%$$

4) η_{CARNOT}



$T_1 = T_c$

$T_2 = T_c$

$$\eta_{CARNOT} = 1 - \frac{T_2}{T_1} = 1 - \frac{P_0 \cancel{V_0}}{\cancel{nR}} \Big/ \frac{2\cancel{P_0} 2\cancel{V_0}}{\cancel{nR}}$$

$$= 1 - \frac{1}{4} = 0,75 = 75\%$$