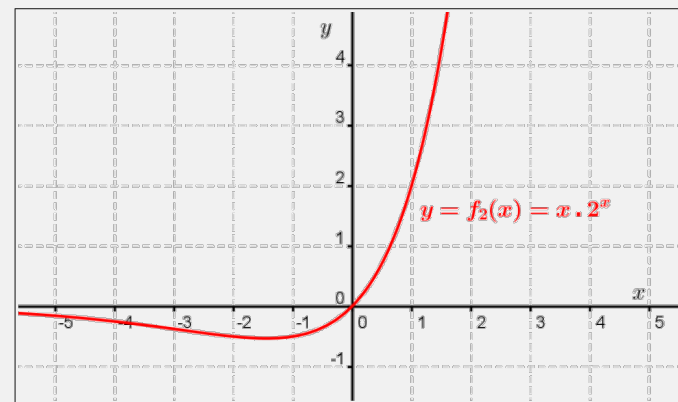
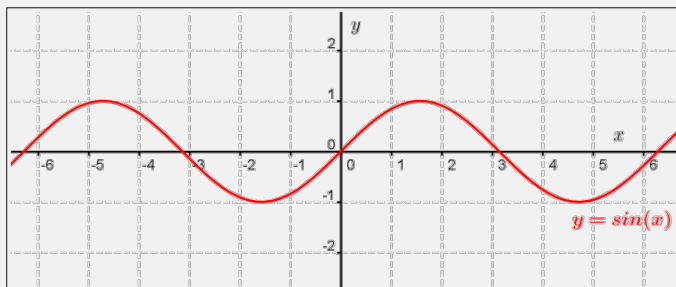


- **DA FUNZIONI ELEMENTARI A FUNZIONI SEMI-ELEMENTARI**
(trasformazioni elementari)

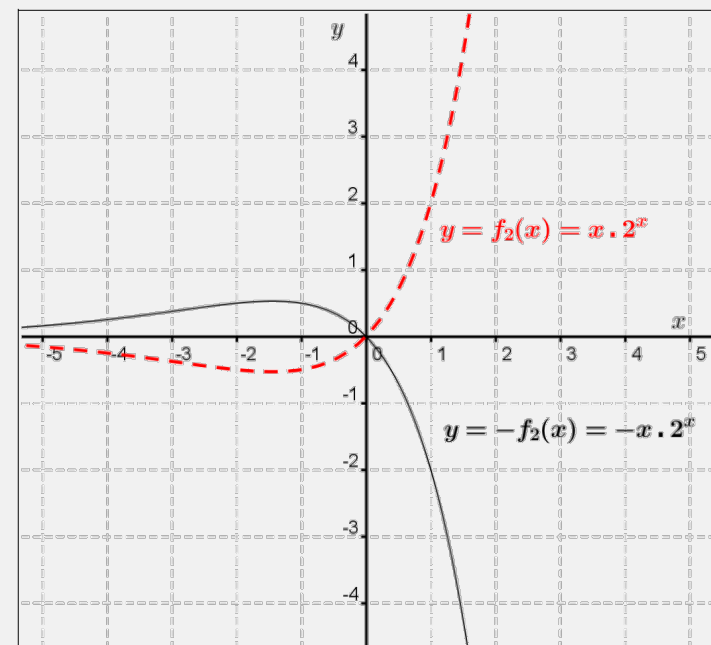
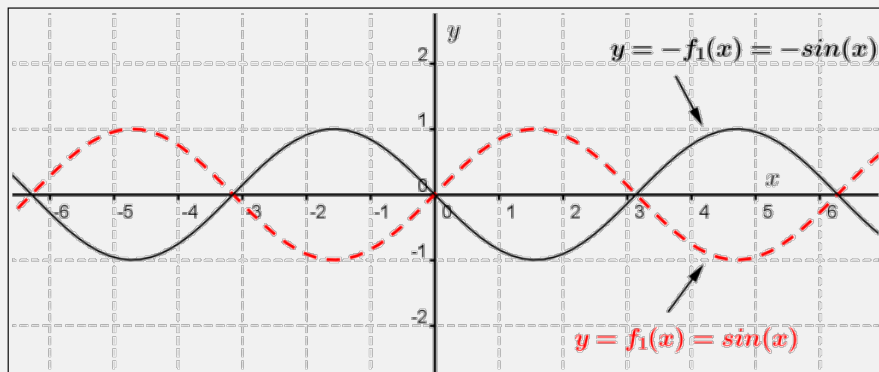
Funzione opposta \rightarrow ribaltamento verticale $g(x) = -f(x)$

$$y = f_1(x) = \sin x$$



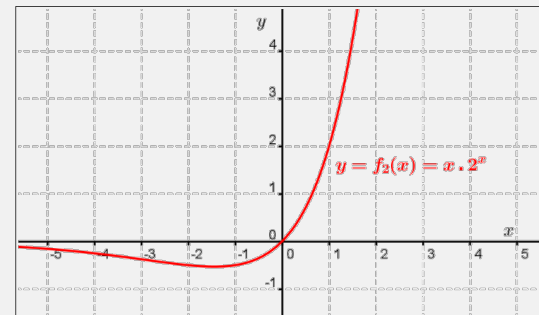
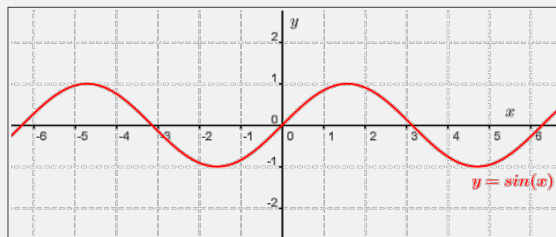
$$y = f_2(x) = x \cdot 2^x$$

Il grafico ottenuto è formato da punti che hanno le ordinate di segno opposto:



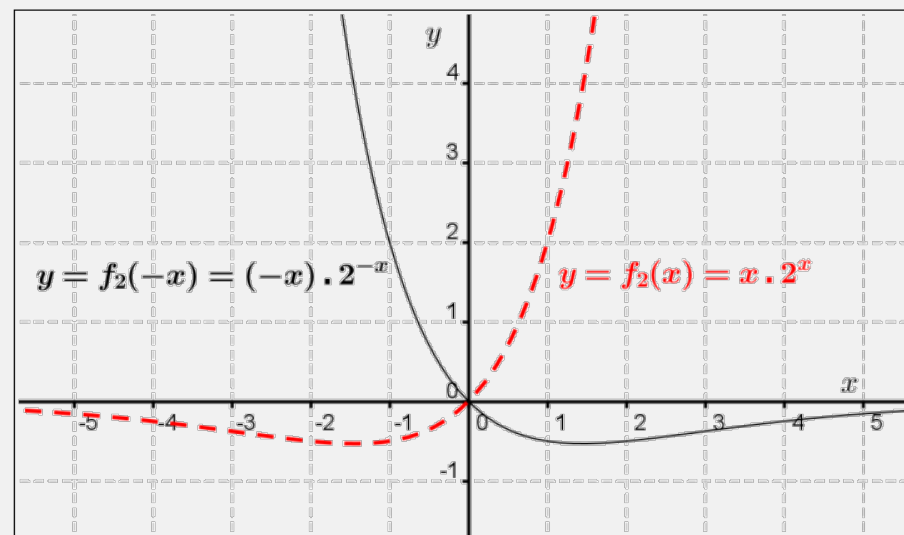
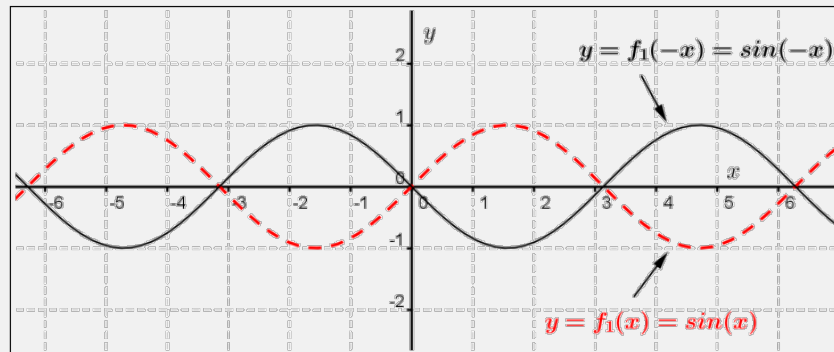
Ribaltamento orizzontale $g(x) = f(-x)$

$$y = f_1(x) = \sin x$$



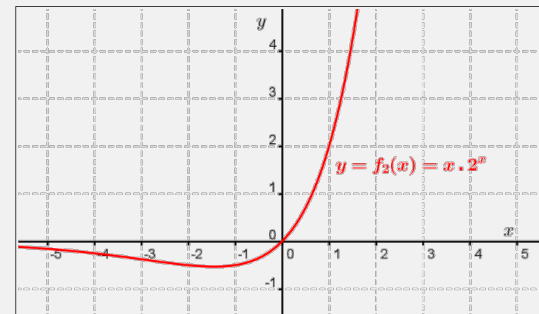
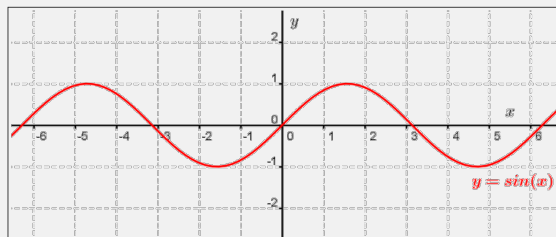
$$y = f_2(x) = x \cdot 2^x$$

Il grafico ottenuto è formato da punti che hanno le ascisse di segno opposto:



Traslazione verticale $g(x) = f(x) + k$

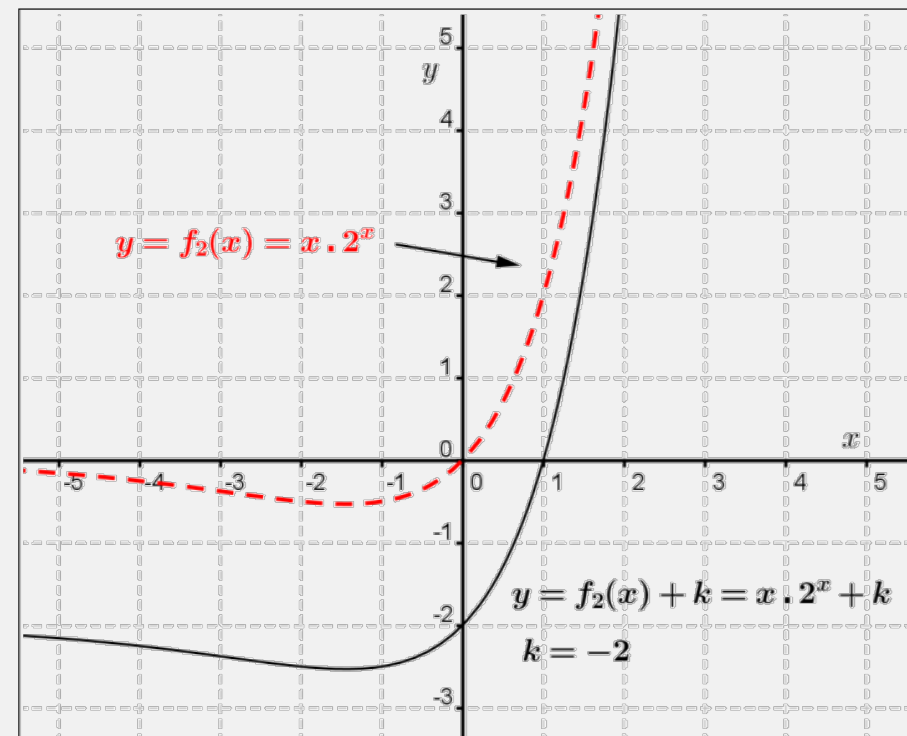
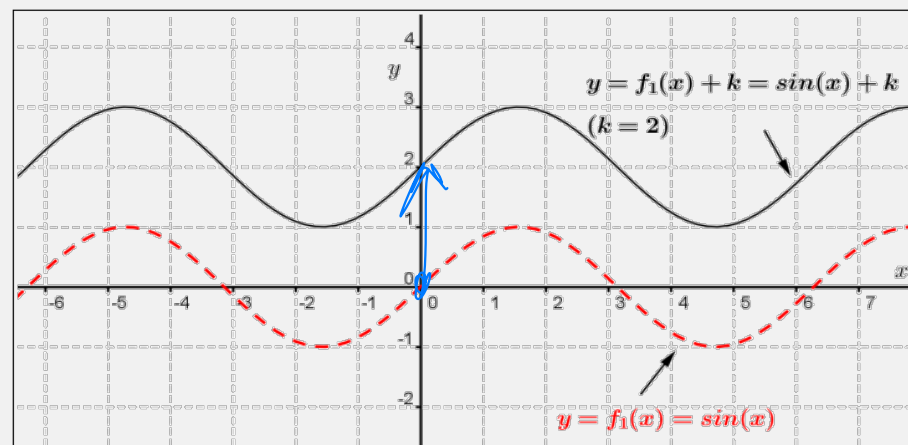
$$y = f_1(x) = \sin x$$



$$y = f_2(x) = x \cdot 2^x$$

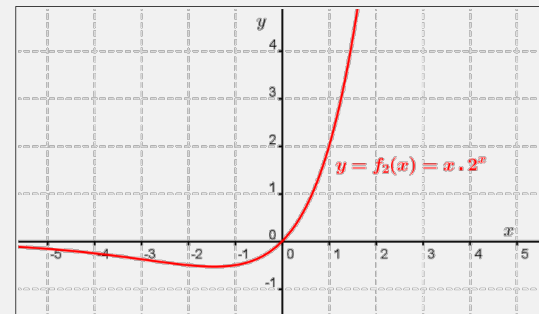
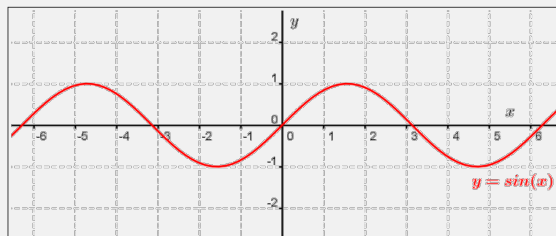
Il grafico ottenuto è formato da punti che hanno le ordinate spostate di k .

- Se $k > 0$ la traslazione è verso l'alto
- Se $k < 0$ la traslazione è verso il basso



Traslazione orizzontale $g(x) = f(x + k)$

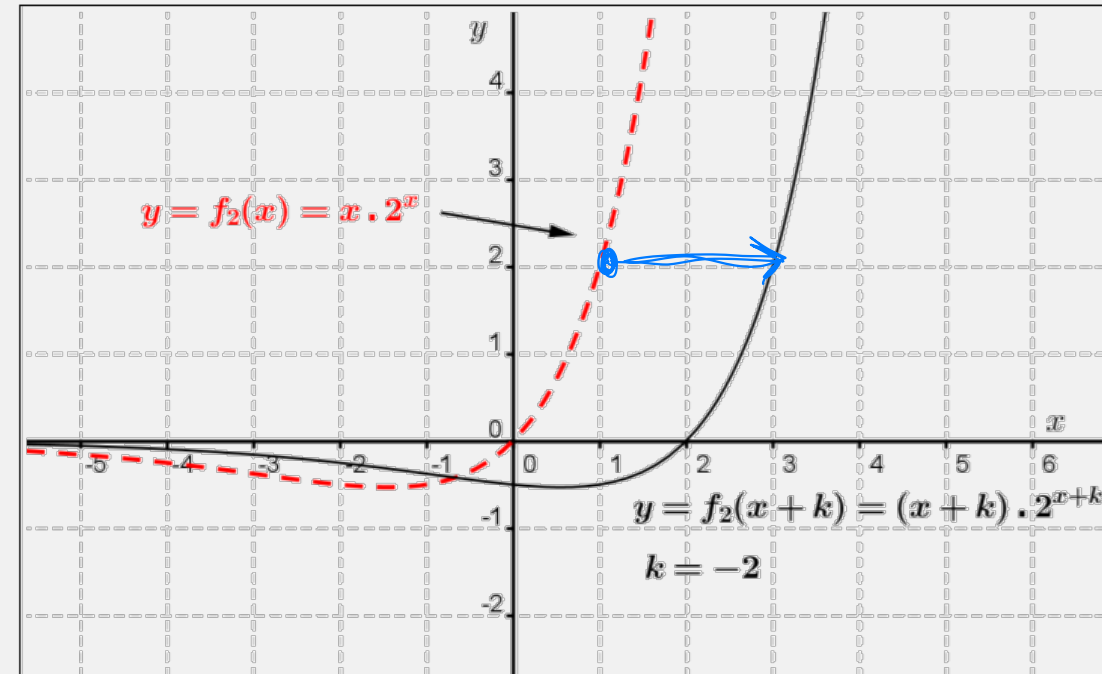
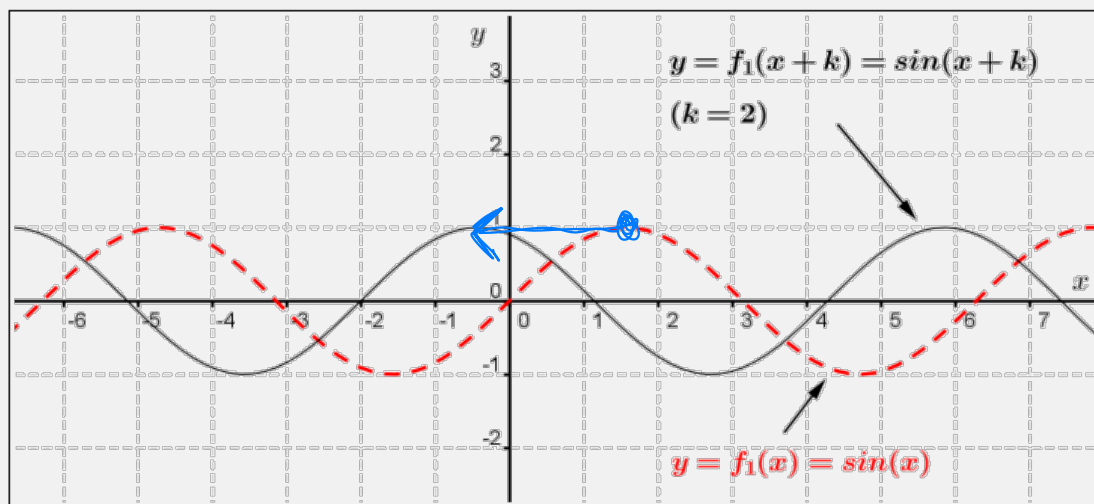
$$y = f_1(x) = \sin x$$



$$y = f_2(x) = x \cdot 2^x$$

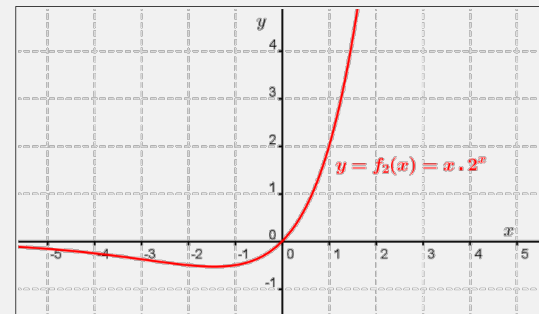
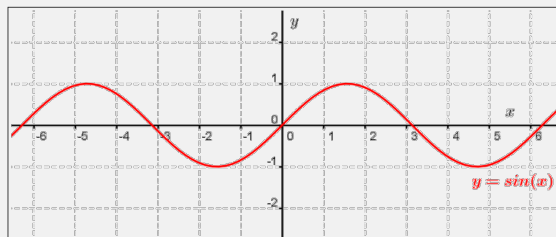
Il grafico ottenuto è formato da punti che hanno le ascisse spostate di k .

- Se $k > 0$ la traslazione è verso sinistra
- Se $k < 0$ la traslazione è verso destra



Dilatazione/Contrazione verticale $g(x) = k \cdot f(x), k > 0$

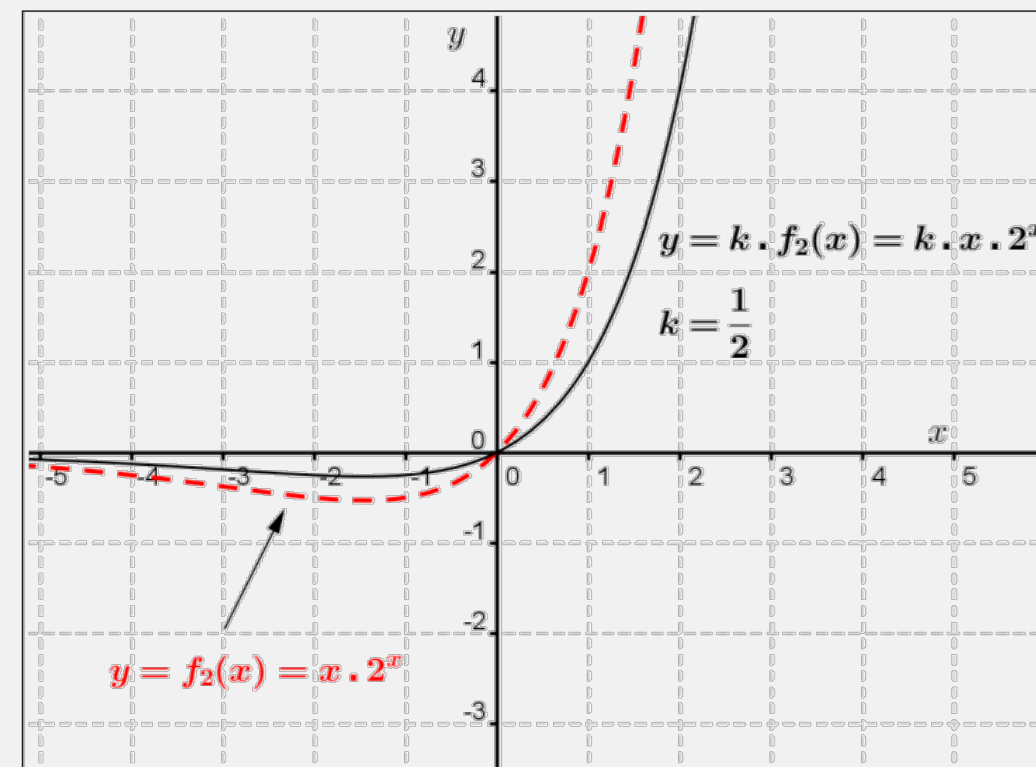
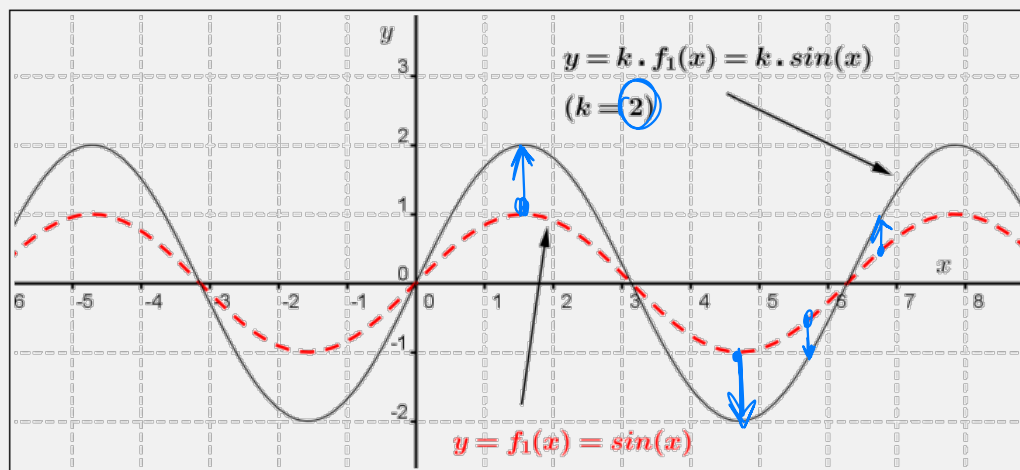
$$y = f_1(x) = \sin x$$



$$y = f_2(x) = x \cdot 2^x$$

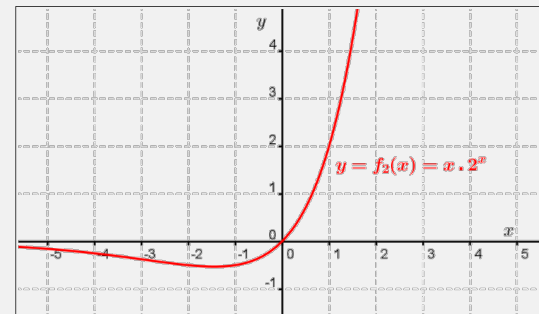
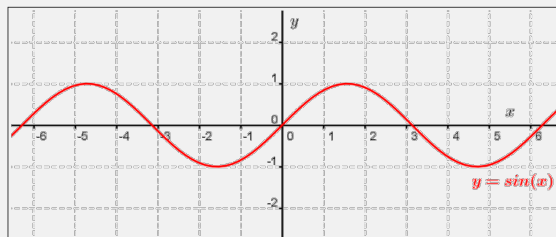
Il grafico ottenuto è formato da punti che hanno le ordinate moltiplicate per k .

- Se $k > 1$ si ha una dilatazione
- Se $0 < k < 1$ si ha una contrazione



Dilatazione/Contrazione orizzontale $g(x) = f(x \cdot k), k > 0$

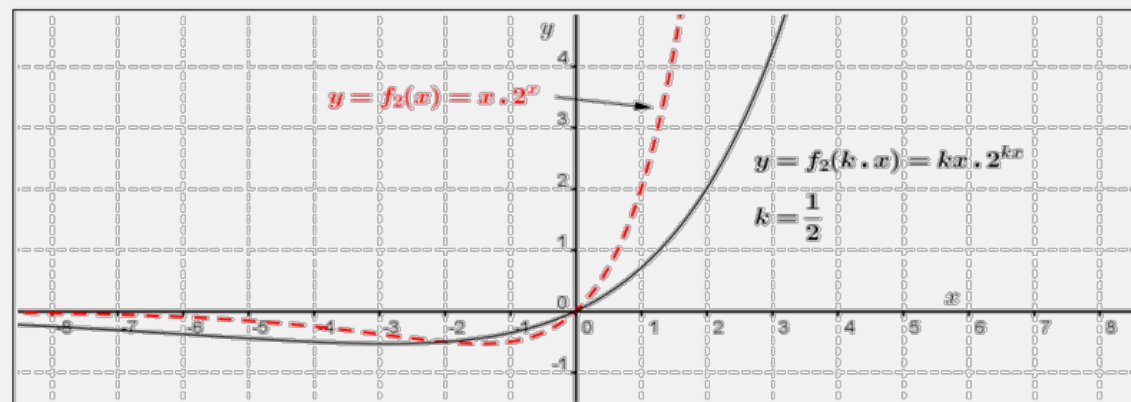
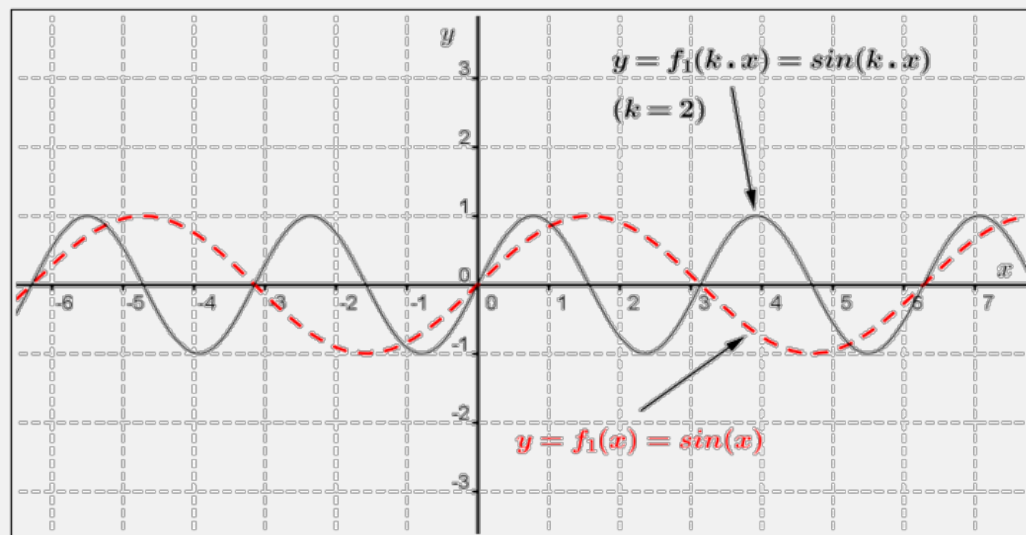
$$y = f_1(x) = \sin x$$



$$y = f_2(x) = x \cdot 2^x$$

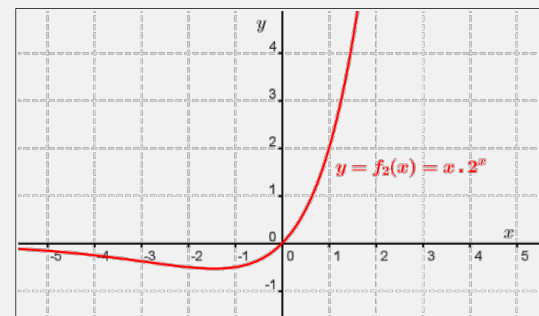
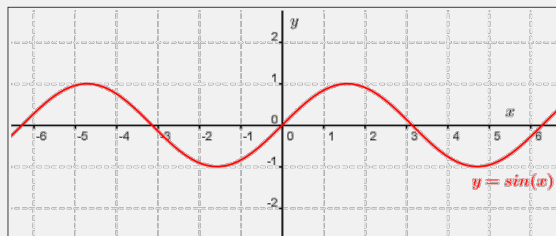
Il grafico ottenuto è formato da punti che hanno le ascisse mutate di un fattore k .

- Se $k > 1$ si ha una contrazione
- Se $0 < k < 1$ si ha una dilatazione



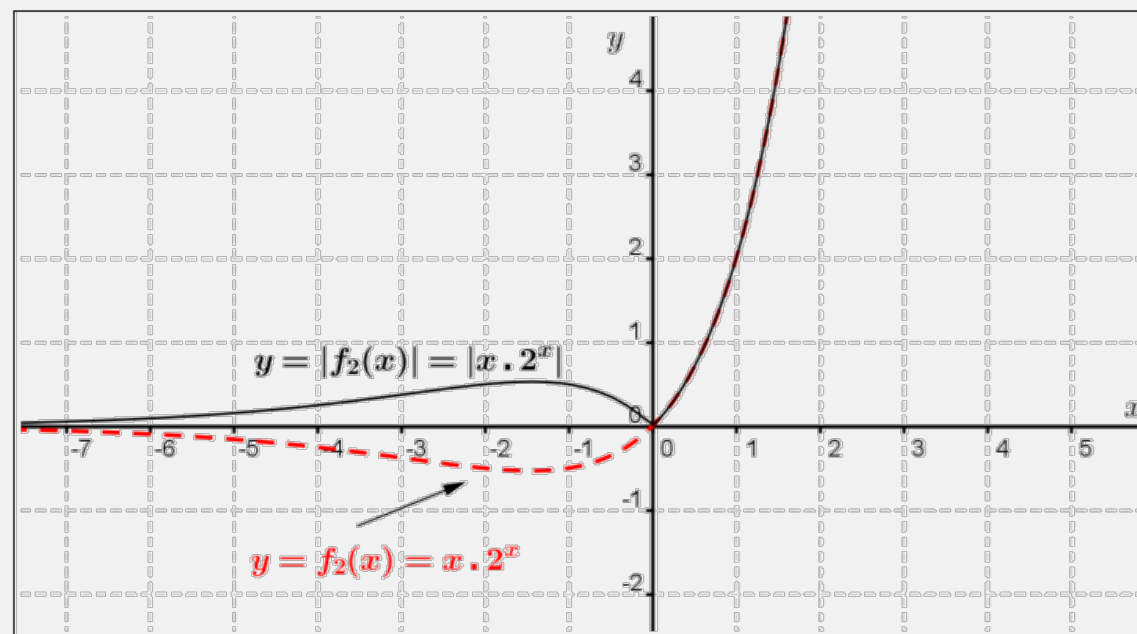
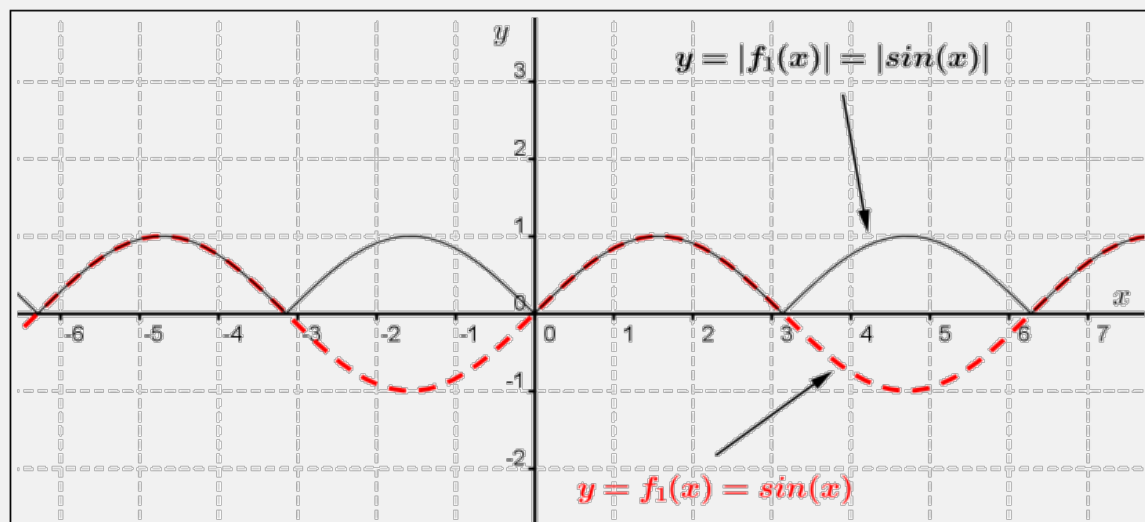
Valore assoluto della funzione $g(x) = |f(x)|$

$$y = f_1(x) = \sin x$$

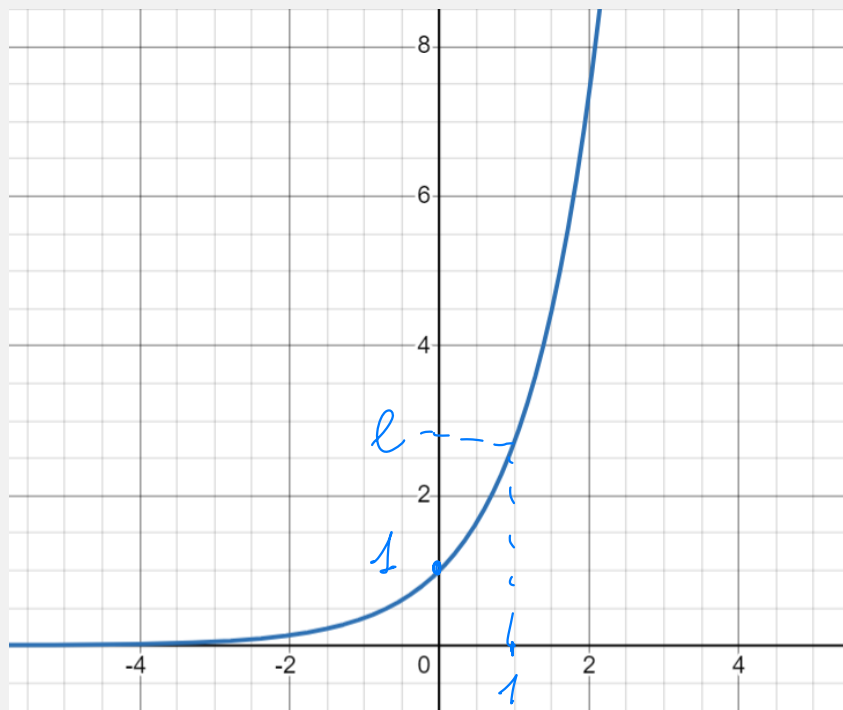


$$y = f_2(x) = x \cdot 2^x$$

Il grafico ottenuto è formato da punti le cui ordinate sono il modulo di quelle del grafico iniziale.
La parte positiva della funzione rimane invariata; quella negativa viene ribaltata nel semi-piano delle ordinate positive.

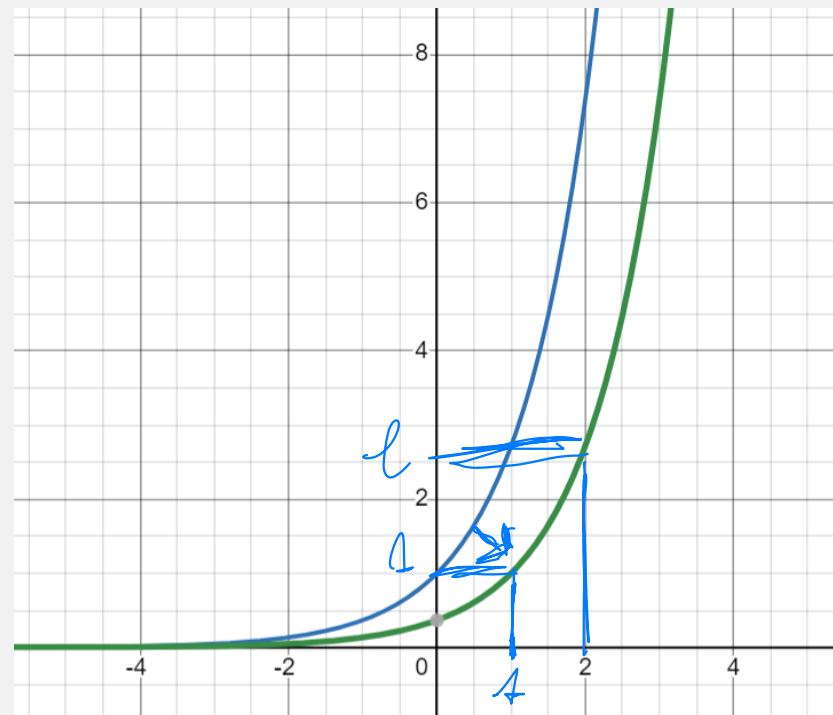


Esempio. $f(x) = |e^{x-1} - 2|$?



$$f(x) = e^x$$

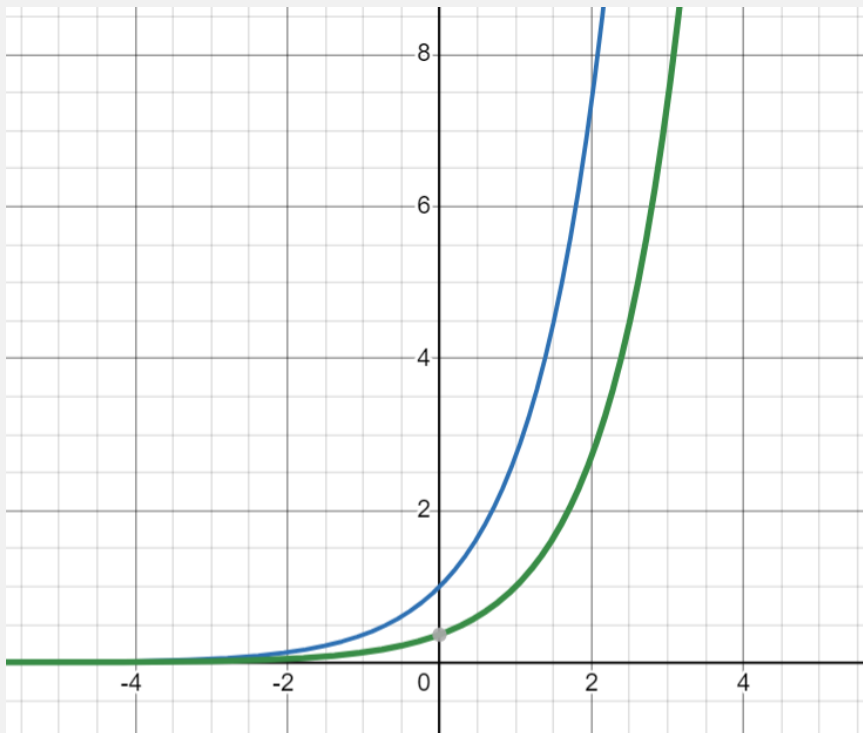
Punti importanti: $(0,1)$, $(1, e)$



$$f(x) = e^{x-1}$$

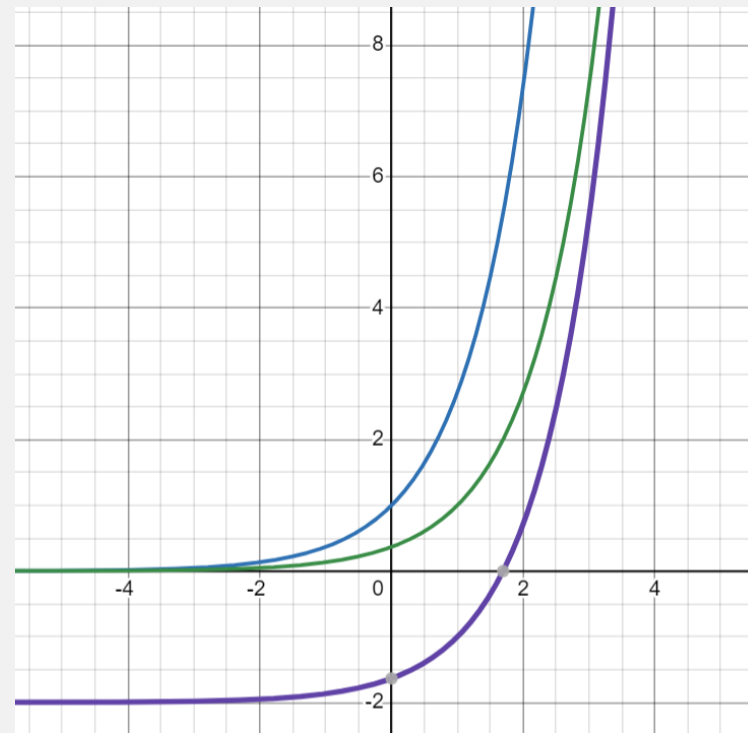
Punti importanti: $(1,1)$, $(2, e)$

Esempio. $f(x) = |e^{x-1} - 2|$?



$$f(x) = e^{x-1}$$

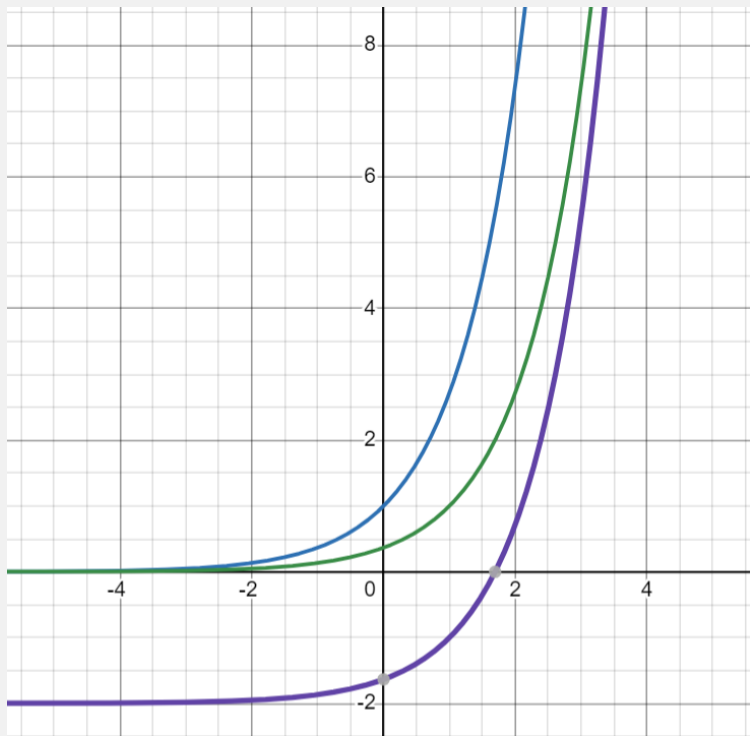
Punti importanti: (1,1), (2, e)



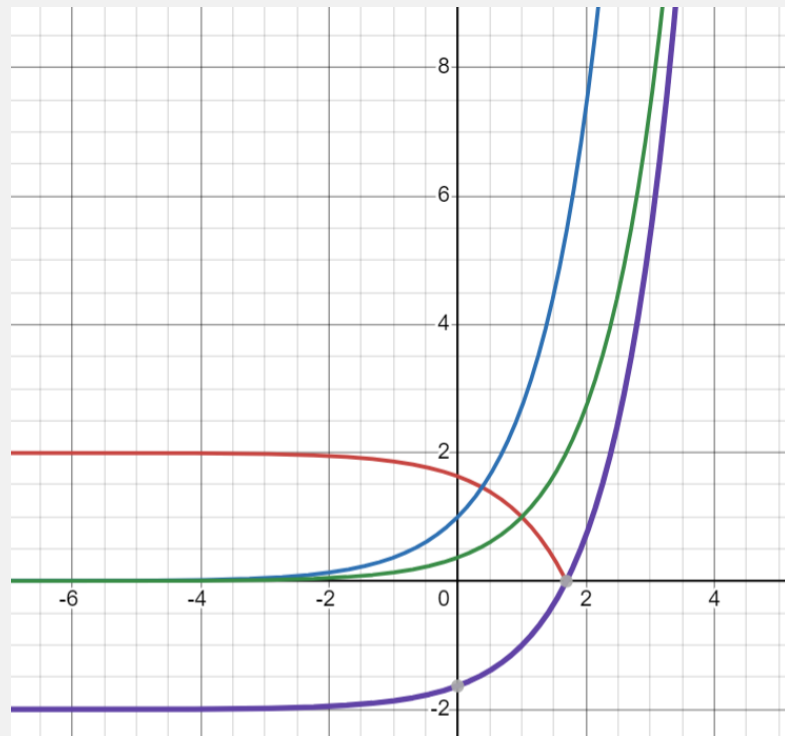
$$f(x) = e^{x-1} - 2$$

Punti importanti: (1, -1), (2,0)

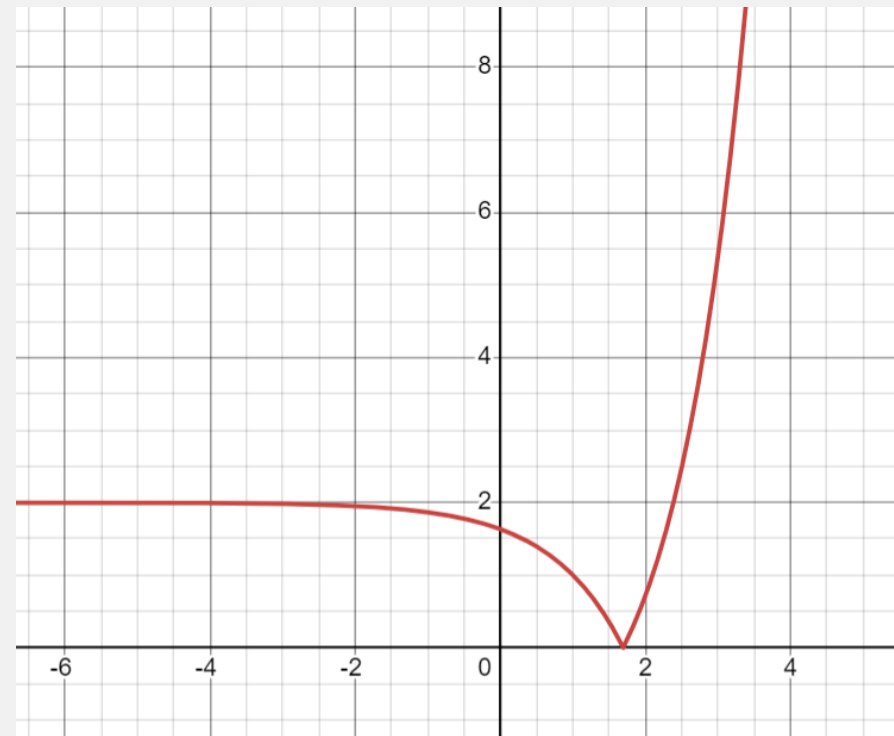
Esempio. $f(x) = |e^{x-1} - 2|$?



$f(x) = e^{x-1} - 2$
Punti importanti: (1, -1), (2, 0)

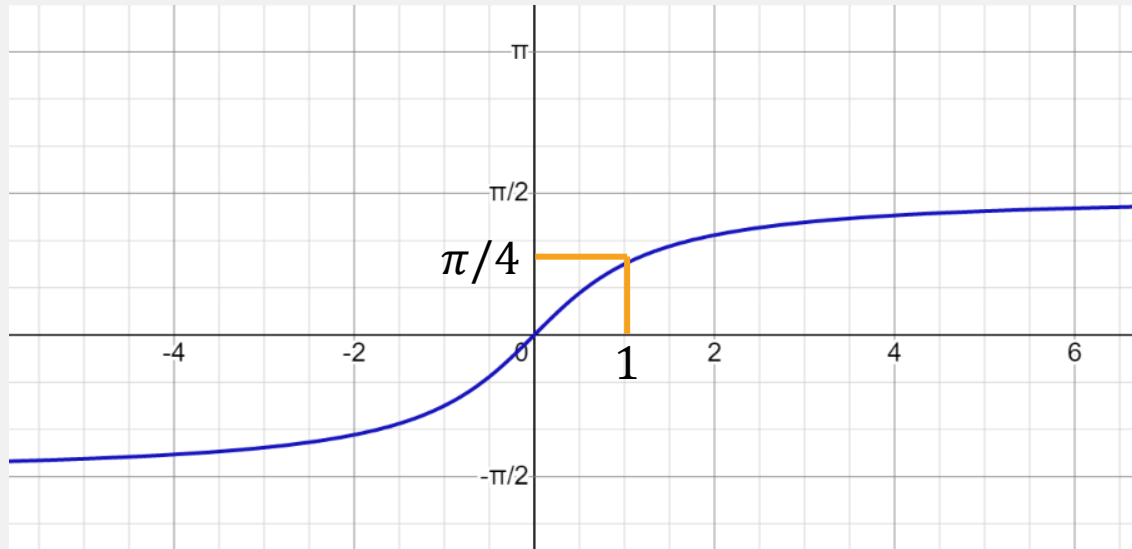


$f(x) = |e^{x-1} - 2|$
Punti importanti: (1, 1), (2, 0)

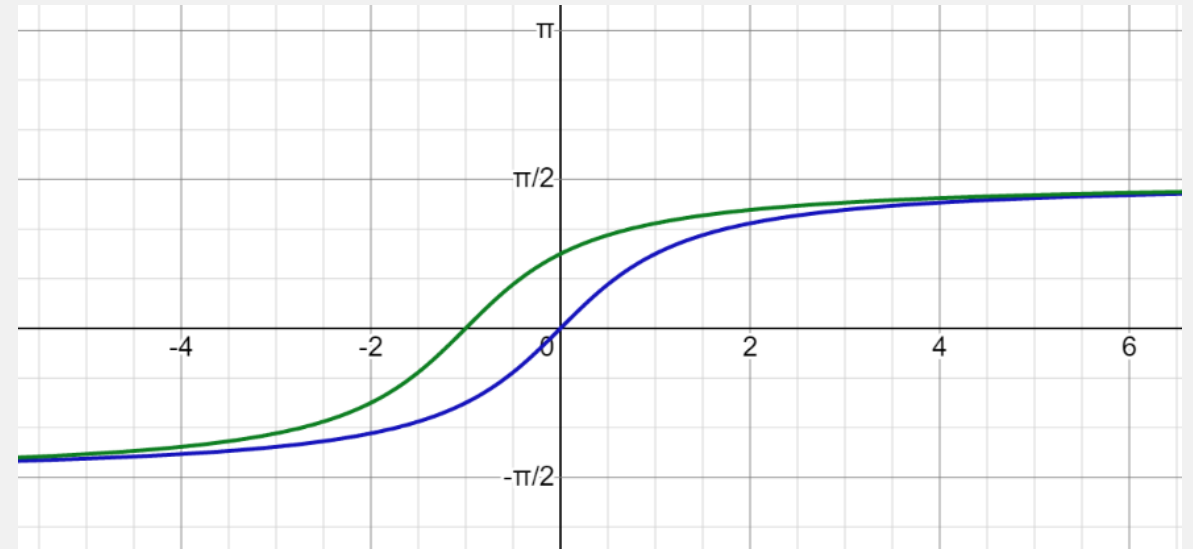


$f(x) = |e^{x-1} - 2|$

Esempio. $f(x) = |\arctg(x + 1)| + \pi$

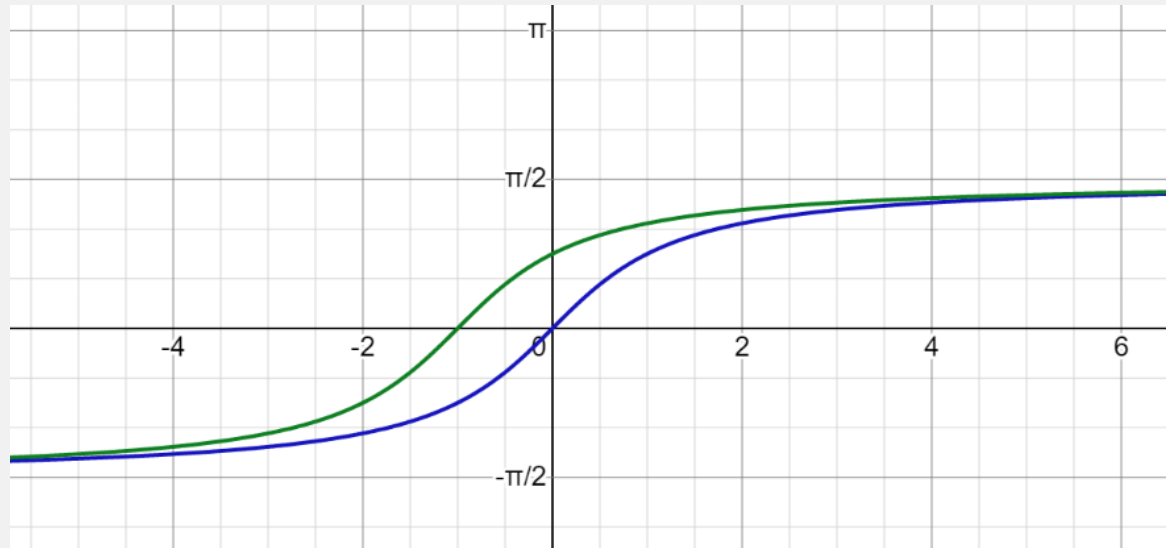


$$f(x) = \arctan x$$

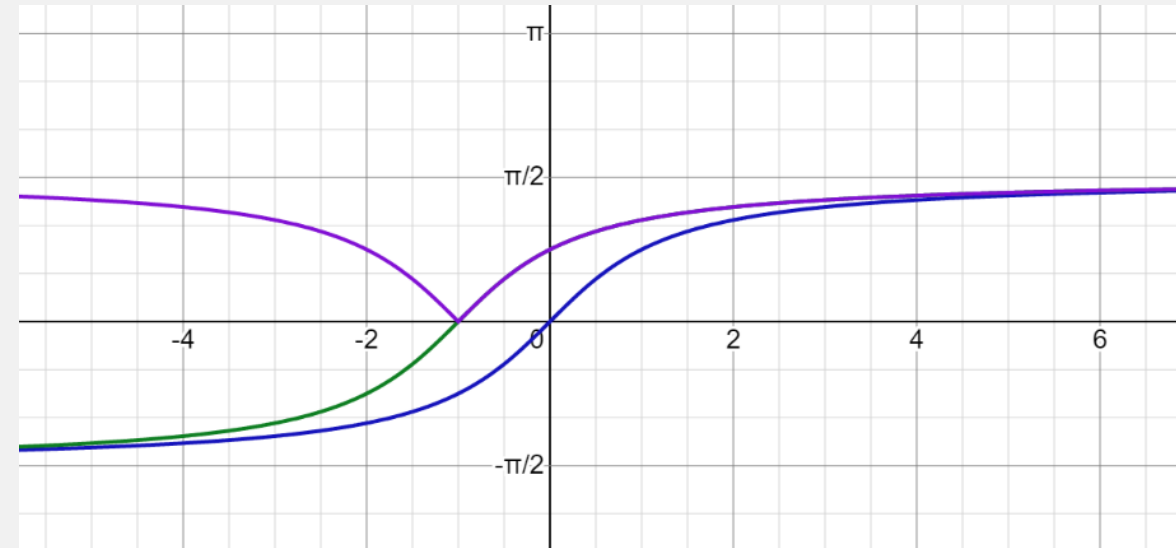


$$f(x) = \arctan(x + 1)$$

Esempio. $f(x) = |\operatorname{arctg}(x + 1)| + \pi$

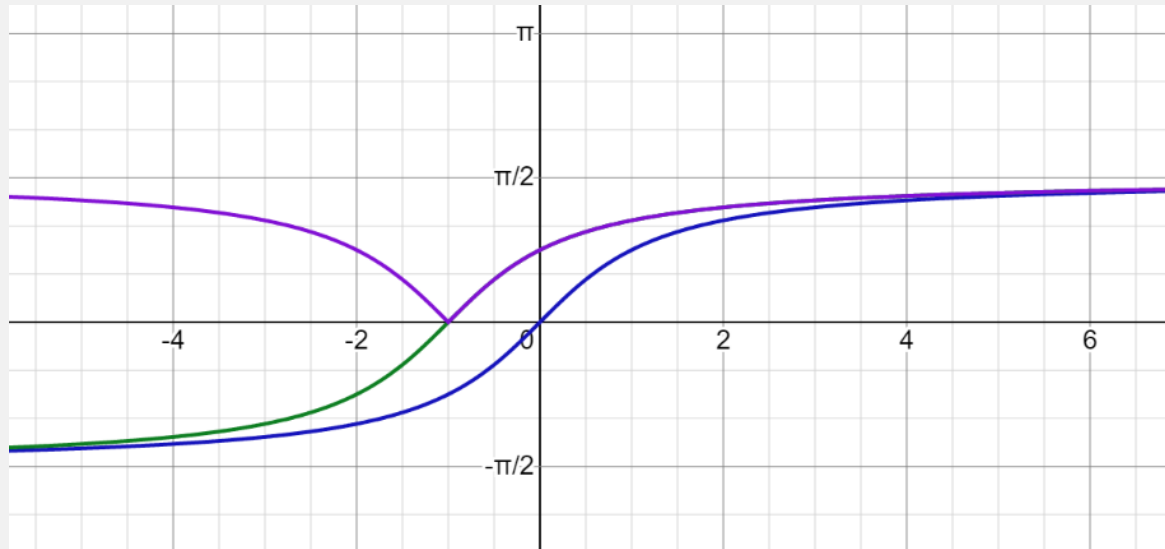


$$f(x) = \arctan(x + 1)$$

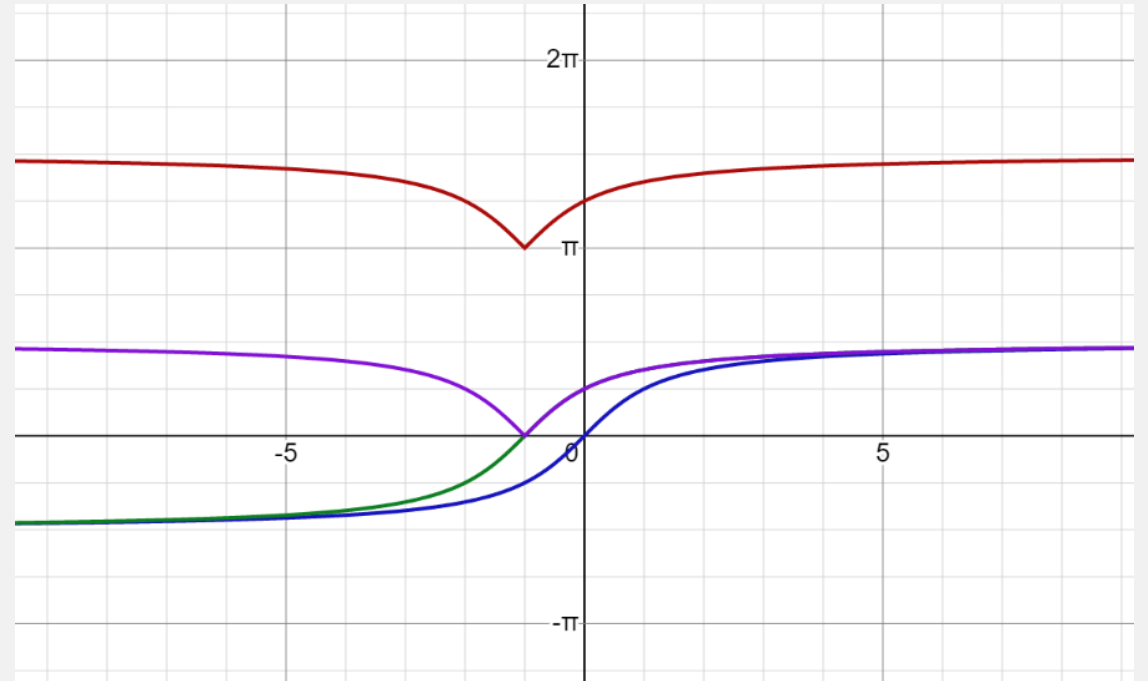


$$f(x) = |\arctan(x + 1)|$$

Esempio. $f(x) = |\arctg(x + 1)| + \pi$



$$f(x) = |\arctan(x + 1)|$$



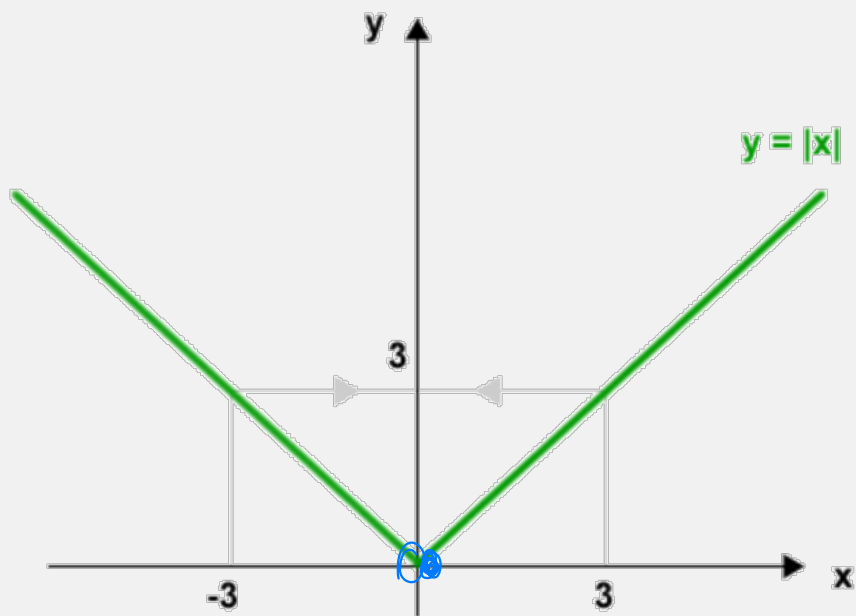
$$f(x) = |\arctan(x + 1)| + \pi$$

- **FUNZIONI DEFINITE A TRATTI**

Caso più semplice: $f(x) = |x|$

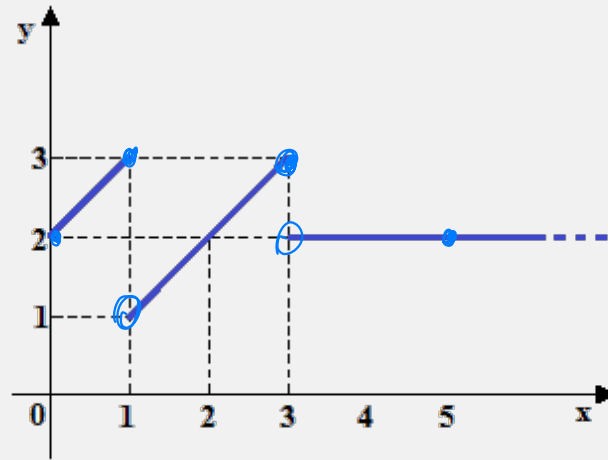
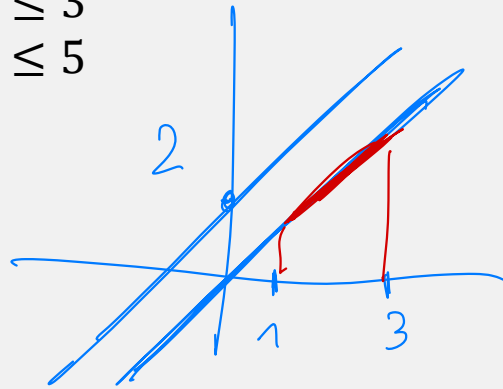
La funzione è composta da due espressioni: la prima (x) si considera solo per i valori non negativi; la seconda ($-x$) solo per i valori negativi della x

$$y = |x| = f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

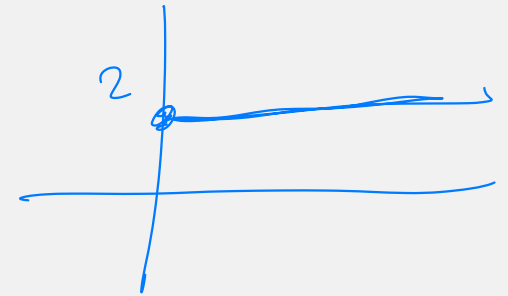


Per ogni x positiva o nulla, la funzione è crescente.
Per ogni x negativa, la funzione è decrescente.

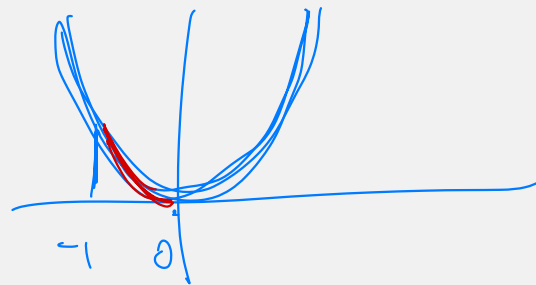
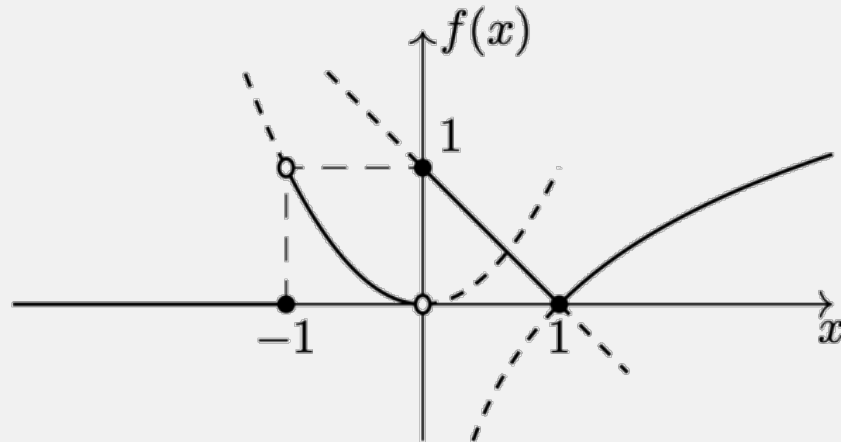
$$f(x) = \begin{cases} x + 2, & 0 \leq x \leq 1 \\ x, & 1 < x \leq 3 \\ 2, & 3 < x \leq 5 \end{cases}$$



$$D_f = \{x \in \mathbb{R} : x \geq 0\}$$

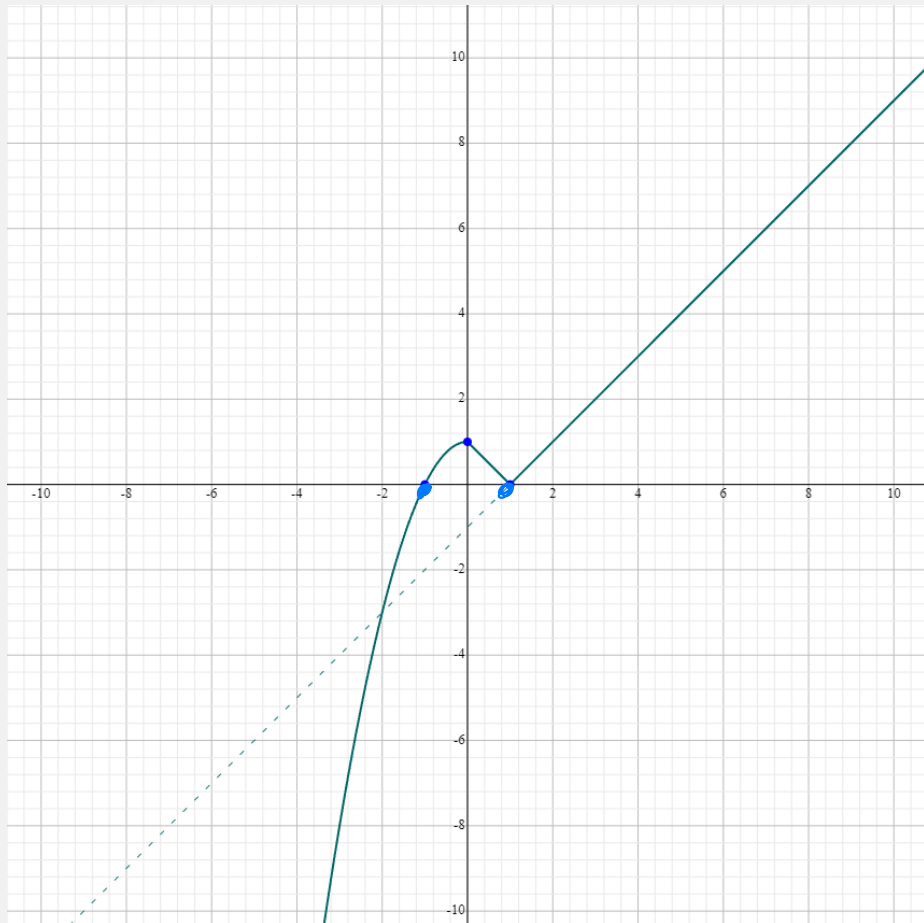
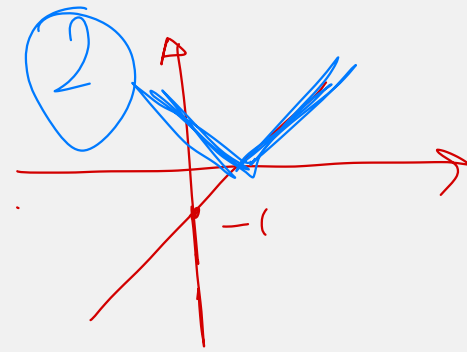
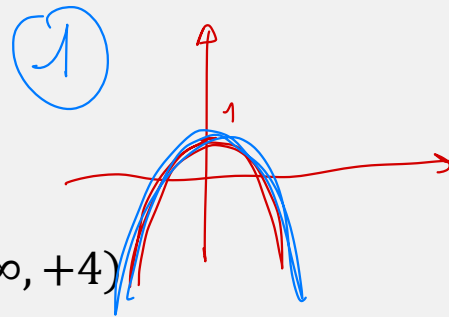


$$f(x) = \begin{cases} 0, & x \leq -1 \\ x^2, & -1 < x < 0 \\ 1 - x, & 0 \leq x < 1 \\ \ln x, & x \geq 1 \end{cases}$$



$$f(x) = \begin{cases} 1 - x^2, & x \leq 0 \\ |x - 1|, & x > 0 \end{cases}$$

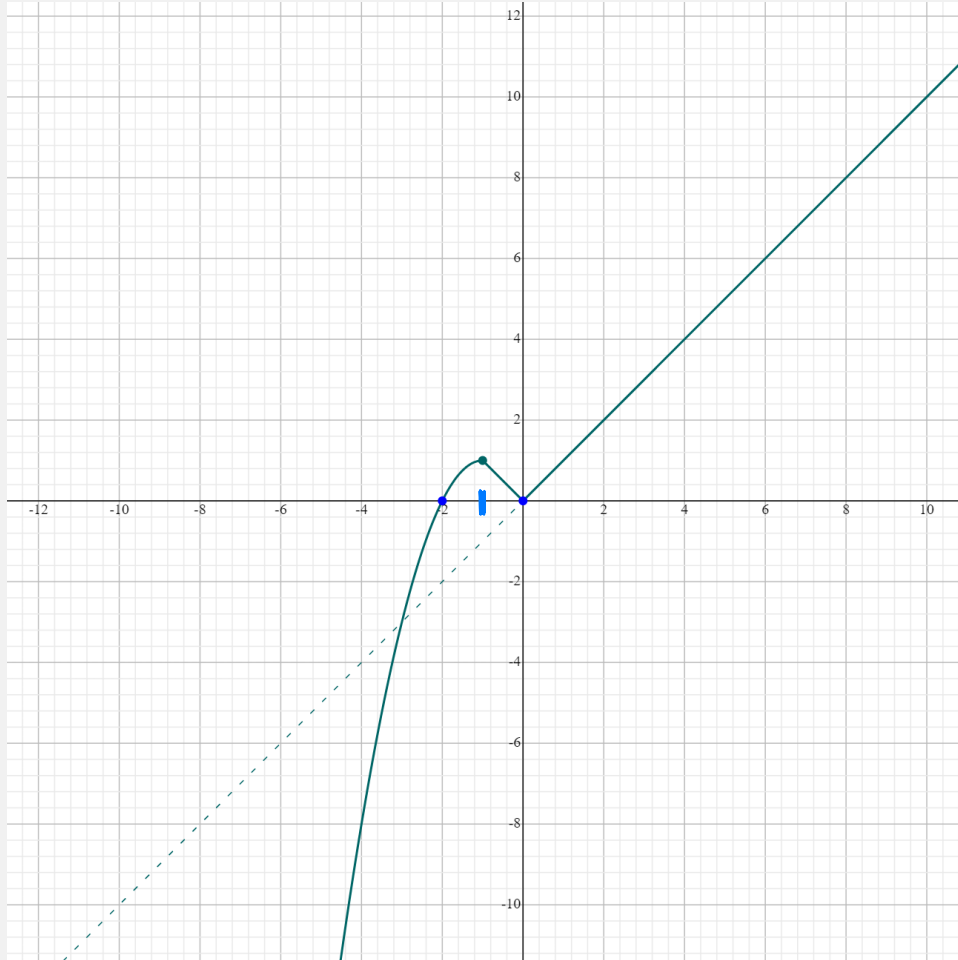
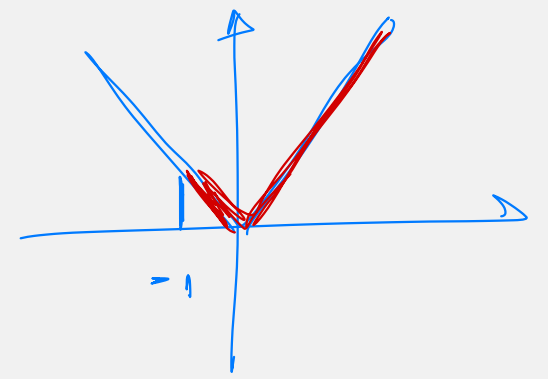
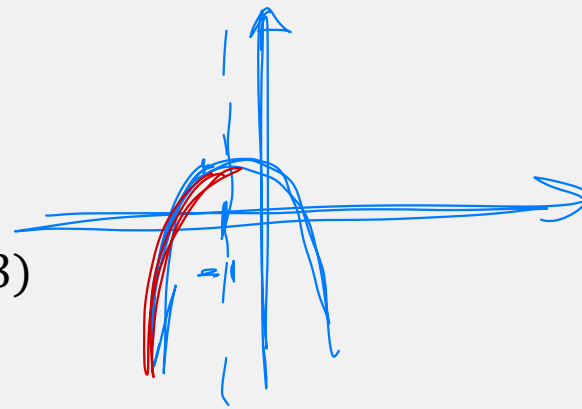
Determinare massimi/minimi assoluti/relativi per $x \in (-\infty, +4)$



- Dominio: $-\infty < x < \infty = x \in \mathbb{R} = \mathbb{R}$
- Intersezioni X: $(-1, 0), (1, 0)$; Intersezioni Y: $(0, 1)$
- Intervalli monotoni: crescente $-\infty < x < 0$; decrescente $0 < x < 1$
- Non esiste minimo assoluto
- Minimo relativo: $x = 1, y = 0$
- Massimo assoluto: $x = 4, y = 3$
- Massimo relativo: $x = 0, y = 1$

$$f(x) = \begin{cases} -x^2 - 2x, & x \leq -1 \\ |x|, & x > -1 \end{cases}$$

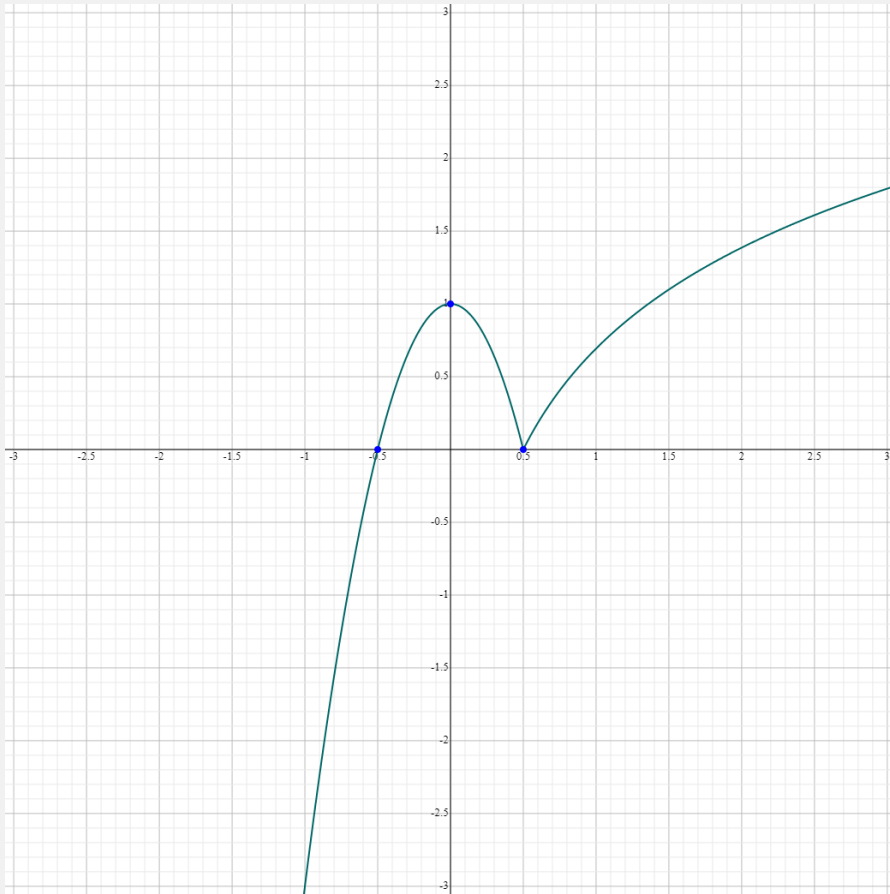
Determinare massimi/minimi assoluti/relativi per $x \in [-\infty, +3)$



- **Dominio:** $-\infty < x < \infty$
- **Intersezioni X:** $(-2, 0), (0, 0)$; **Intersezioni Y:** $(0, 0)$
- **Intervalli monotoni:** crescente $-\infty < x < -1$; decrescente $-1 < x < 0$
- **Non esiste minimo assoluto**
- **Minimo relativo:** $x = 0, y = 0$
- **Massimo assoluto:** $x = 3, y = 3$
- **Massimo relativo:** $x = -1, y = 1$

$$f(x) = \begin{cases} 1 - 4x^2, & x \leq \frac{1}{2} \\ \log_e 2x & x > \frac{1}{2} \end{cases}$$

Determinare massimi/minimi assoluti/relativi per $x \in [-1, +\infty)$



- **Dominio:** $-\infty < x < \infty$
- **Intersezioni X:** $(\frac{1}{2}, 0), (-\frac{1}{2}, 0)$; **Intersezioni Y:** $(0, 1)$
- **Intervalli monotoni:** crescente $-\infty < x < 0$; decrescente $0 < x < \frac{1}{2}$
- **Non esiste massimo assoluto**
- **Massimo relativo:** $x = 0, y = 1$
- **Minimo assoluto:** $x = -1, y = -3$
- **Minimo relativo:** $x = \frac{1}{2}, y = 0$