

## Focus: limiti

$$\rightarrow +\infty^3 + \infty^2 + 2 = +\infty + \infty = +\infty$$

$$\rightarrow \lim_{x \rightarrow +\infty} (x^3 + x^2 + 2) = +\infty$$

$$\frac{(-1)^2 - 1}{-1 + 1} = \frac{+1 - 1}{-1 + 1} = \frac{0}{0}$$

$$\rightarrow \lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = \frac{0}{0} \rightarrow \text{forma indeterminata}$$

$$\text{Scomponiamo il limite: } \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{x+1} = \lim_{x \rightarrow -1} (x-1) = -2 \neq \frac{0}{0}$$

$$\rightarrow \lim_{x \rightarrow 7} e^{\frac{1}{x}} = e^{\lim_{x \rightarrow 7} \frac{1}{x}} = e^{1/7}$$

$$\rightarrow \lim_{x \rightarrow +\infty} \frac{x^3 + x^2 - 1}{2x^2 + 2} = \frac{+\infty}{+\infty} \rightarrow \text{forma indeterminata}$$

$$\text{Scomponiamo il limite: } \lim_{x \rightarrow +\infty} \frac{x^3 \left(1 + \frac{1}{x} - \frac{1}{x^3}\right)}{x^3 \left(\frac{2}{x} + \frac{2}{x^3}\right)} = \frac{+1 + 0 - 0}{0 + 0} = \frac{+1}{0} = +\infty$$

Esercizio esemplificativo sul limite notevole:  $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e$

➤  $\lim_{x \rightarrow +\infty} \left(1 + \frac{a}{x}\right)^x, a \in \mathbb{R}$

DIMOSTRAZIONE  
↓

Il limite notevole originale veniva da  $1^\infty = e$ . Anche questa variazione viene da  $1^\infty$ , ma avendo una  $a$  non posso usare la forma del limite notevole: devo ricondurlo a tale forma:

$$y = \frac{x}{a} \rightarrow x = ya$$

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{\frac{x}{a}}\right)^x = \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{y}\right)^x, y = \frac{x}{a} \Rightarrow x \rightarrow +\infty : y = +\frac{\infty}{a} = +\infty$$

y

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{y}\right)^{ya} \Leftrightarrow a > 0, a \in \mathbb{R}^+$$

$$\lim_{x \rightarrow +\infty} \left[\left(1 + \frac{1}{y}\right)^y\right]^a = e^a \quad \text{la stessa cosa vale per } a < 0, a \in \mathbb{R}^-$$

↓  
 $e^a, a = \frac{1}{3} \rightarrow e^{1/3}$

**Esercizio esemplificativo sul limite notevole:**  $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e$

➤  $\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{3x}\right)^x$

Altra variazione dello stesso limite notevole. Chiamo  $3x = y$ :

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{3x}\right)^x = \lim_{y \rightarrow +\infty} \left[ \left(1 + \frac{1}{y}\right)^y \right]^{\frac{1}{3}} = e^{\frac{1}{3}}$$

Lo si può capire anche scrivendo:  $\lim_{x \rightarrow +\infty} \left(1 + \frac{1/3}{x}\right)^x$

Se  $x \rightarrow +\infty$ , anche  $3x \rightarrow +\infty$

*In questo modo, 1/3 sarebbe la  $a$ , che va come esponente di  $e$*

# Focus: limiti

$$\lim_{x \rightarrow +\infty} \frac{x+2+\cos x}{3-x} = -1$$

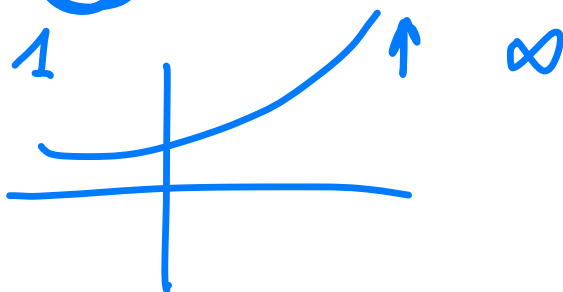


$$\lim_{x \rightarrow +\infty} \frac{x+2+\cos x}{3-x} = \lim_{x \rightarrow +\infty} \frac{x \left( 1 + \underbrace{\frac{2}{x}}_{\downarrow 0} + \underbrace{\frac{\cos x}{x}}_{\downarrow 0} \right)}{x \left( \underbrace{\frac{3}{x}}_{\downarrow 0} - 1 \right)} = -1$$

$$\lim_{x \rightarrow +\infty} e^x (\cos x - 2 \sin x) = 1$$

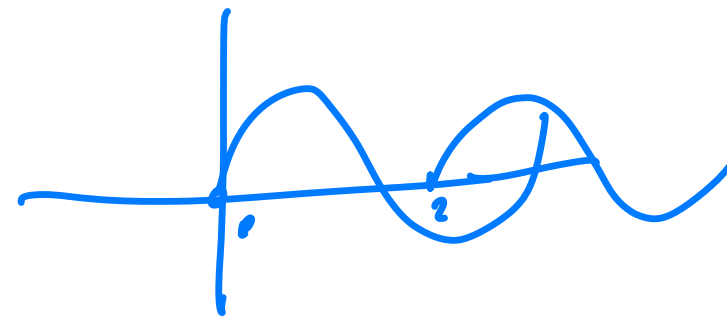
$$1(1-0) = 1$$

Prodotto di funzioni continue: il limite si valuta calcolando il valore della funzione in quel punto



$$\lim_{x \rightarrow -\infty} \frac{x^2 + \sin e^x}{2x} = -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 + \sin e^x}{2x} = \lim_{x \rightarrow -\infty} \frac{x^2 \left( 1 + \underbrace{\frac{\sin e^x}{x^2}}_{\downarrow 0} \right)}{2x} = -\infty$$



## Focus: limiti

$$\blacktriangleright \lim_{x \rightarrow +\infty} \frac{x+3\sqrt{x}}{2x-5\sqrt{x}} = \frac{1}{2}$$

$$\lim_{x \rightarrow +\infty} \frac{x+3\sqrt{x}}{2x-5\sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{x \left( 1 + \frac{3}{\sqrt{x}} \right)}{x \left( 2 - \frac{5}{\sqrt{x}} \right)} = \frac{1}{2}$$

$$\blacktriangleright \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{\left(x - \frac{\pi}{2}\right)^2} = -\frac{1}{2}$$

$$\begin{aligned} & \psi = x - \frac{\pi}{2} \\ & \uparrow \\ & \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{\left(x - \frac{\pi}{2}\right)^2} \stackrel{x - \frac{\pi}{2} = y}{=} \lim_{y \rightarrow 0} \frac{\sin\left(y + \frac{\pi}{2}\right) - 1}{y^2} \\ & = \lim_{y \rightarrow 0} \frac{\sin y \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \cos y - 1}{y^2} = \lim_{y \rightarrow 0} \frac{\cos y - 1}{y^2} = -\frac{1}{2} \end{aligned}$$

$$\blacktriangleright \lim_{x \rightarrow 0} \frac{\sin x - 2 \log(1+x)}{x + \sin x} = -\frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\sin x - 2 \log(1+x)}{x + \sin x} = \lim_{x \rightarrow 0} \frac{x \left( \frac{\sin x}{x} - 2 \frac{\log(1+x)}{x} \right)}{x \left( 1 + \frac{\sin x}{x} \right)} = -\frac{1}{2}$$

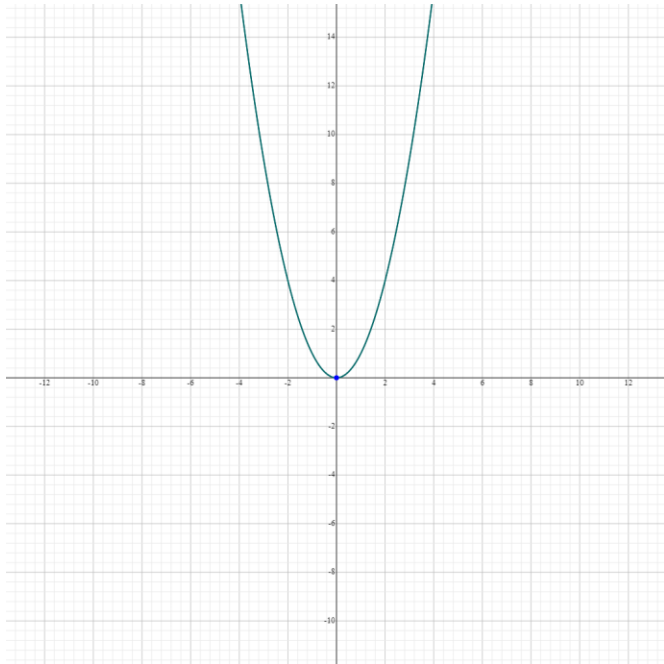
Ricorda: limiti notevoli  $\rightarrow$   $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$   $\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \log_a e$

**Esercizio.** Valutare dominio, continuità e limiti.

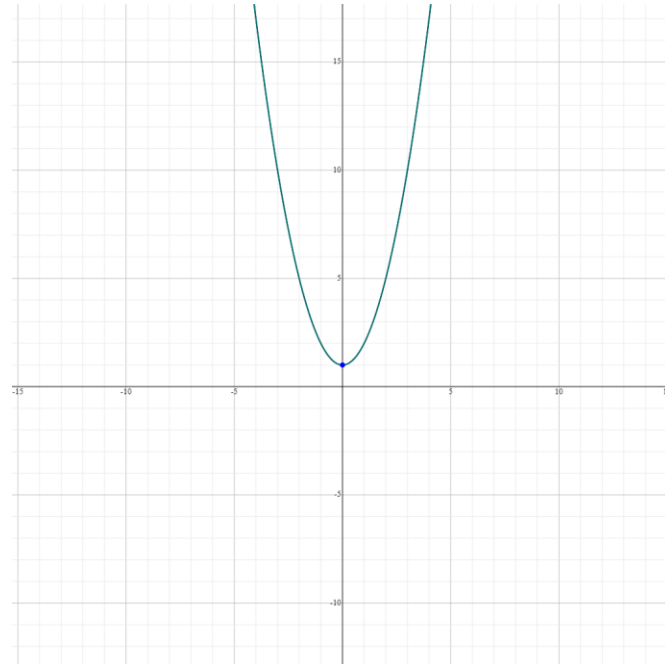
$$f(x) = \sqrt{x^2 + 1}$$

➤  $D_f = \mathbb{R}$       $x^2 + 1 \geq 0$       $x^2 \geq -1 \rightarrow \text{sempre}$

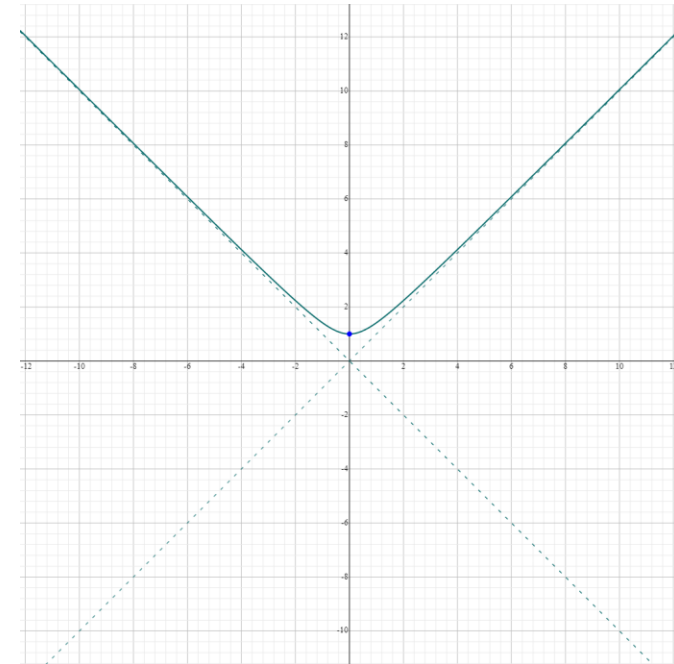
➤ la parabola è continua, la radice è continua, quindi la composizione di due funzioni continue dà una funzione continua



$$f(x) = x^2$$



$$f(x) = x^2 + 1$$

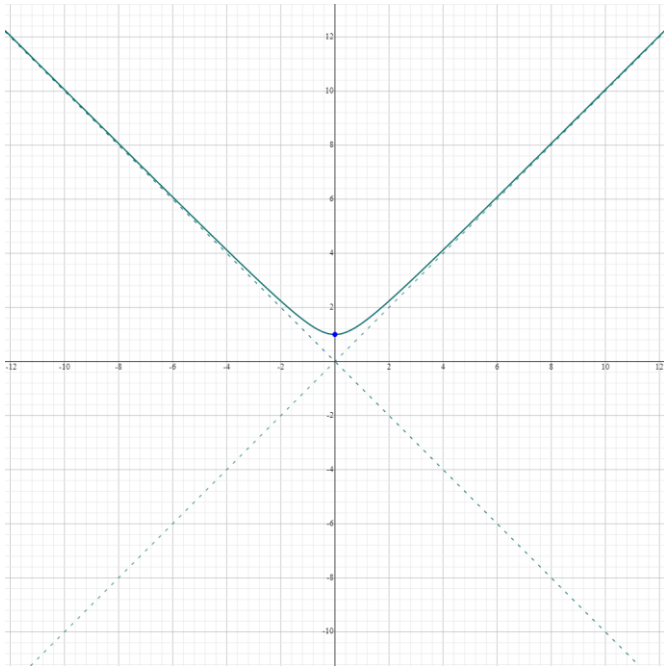


$$f(x) = (x^2 + 1)^{\frac{1}{2}} = \sqrt{x^2 + 1}$$

**Esercizio.** Valutare dominio, continuità e limiti.

$$f(x) = \sqrt{x^2 + 1}$$

- $D_f = \mathbb{R}$
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Ricordando:

- Se  $\lim_{x \rightarrow \pm\infty} f(x) = k \rightarrow$  asintoto orizzontale
- Se  $\lim_{x \rightarrow c} f(x) = \pm\infty \rightarrow$  asintoto verticale
- Se  $\lim_{x \rightarrow \pm\infty} f(x) = \pm\infty \rightarrow$  asintoto obliquo, con

- $m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x}$

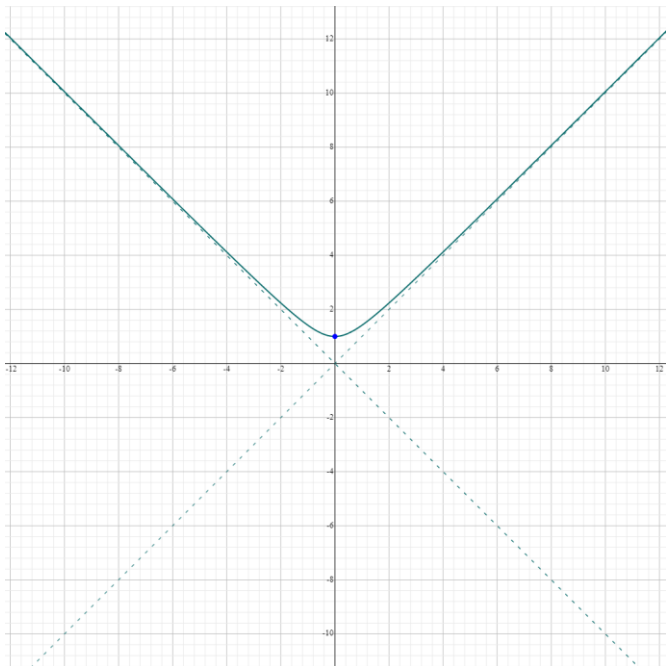
- $q = \lim_{x \rightarrow +\infty} [f(x) - mx]$

$$f(x) = \sqrt{x^2 + 1} \rightarrow \lim_{x \rightarrow \pm\infty} \sqrt{x^2 + 1} = +\infty \rightarrow \text{as. obliquo!}$$

**Esercizio.** Valutare dominio, continuità e limiti.

$$f(x) = \sqrt{x^2 + 1}$$

- $D_f = \mathbb{R}$
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$$f(x) = \sqrt{x^2 + 1} \rightarrow \lim_{x \rightarrow \pm\infty} \sqrt{x^2 + 1} = +\infty \rightarrow \text{as. obliquo!}$$

$$m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1}}{x} = \lim_{x \rightarrow +\infty} \sqrt{\frac{x^2 + 1}{x^2}} = \lim_{x \rightarrow +\infty} \sqrt{1 + \frac{1}{x^2}} = 1$$

$$q = \lim_{x \rightarrow +\infty} [f(x) - mx] = \lim_{x \rightarrow +\infty} \sqrt{x^2 + 1} - 1x = \infty - \infty \rightarrow f.i.$$

$$= \lim_{x \rightarrow +\infty} \sqrt{x^2 + 1} - 1x \cdot \frac{\sqrt{x^2 + 1} + 1x}{\sqrt{x^2 + 1} + 1x} = \lim_{x \rightarrow +\infty} \frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + 1x} = \frac{1}{+\infty + \infty} = \frac{1}{+\infty} = 0^+$$

$$m = 1, q = 0 \rightarrow y = 1x + 0 = x$$

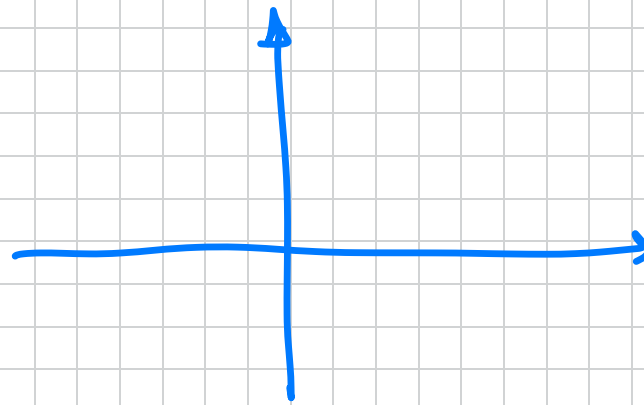
cosa accade per  $-\infty$ ?  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x} = \lim_{x \rightarrow +\infty} -\sqrt{1 + \frac{1}{x^2}} = -1 \rightarrow y = -x$

So che le  $x$  che prendo sono negative



$$f(x) = x(x^2 - 1)^2$$

$$D_f? \rightarrow \mathbb{R}$$



$$\lim_{x \rightarrow \pm \infty} x(x^2 - 1)^2 = \pm \infty \rightarrow \text{NO ORIZZ.}$$

$$\lim_{x \rightarrow \pm \infty} \frac{x(x^2 - 1)^2}{x} = +\infty \neq m \rightarrow \text{NO OBLIQUO}$$

$$f(x) = \frac{x+1}{x-1}$$

$$x-1 \neq 0 \quad x \neq 1$$

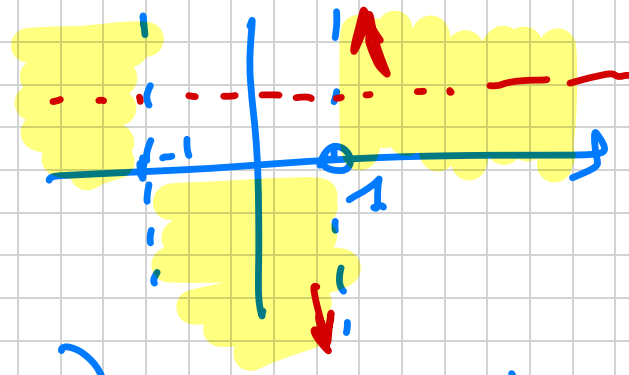
$$D_f = x \in \mathbb{R} : x \neq 1 ; \mathbb{R} - \{1\} ; (-\infty, 1) \cup (1, +\infty)$$

$$\lim_{x \rightarrow \pm \infty} \frac{x+1}{x-1} = 1 \quad \text{AS. ORIZ.}$$

$$\lim_{x \rightarrow 1^-} \frac{x+1}{x-1} = \frac{1+1}{1-1} = \frac{2}{0^-} = -\infty$$

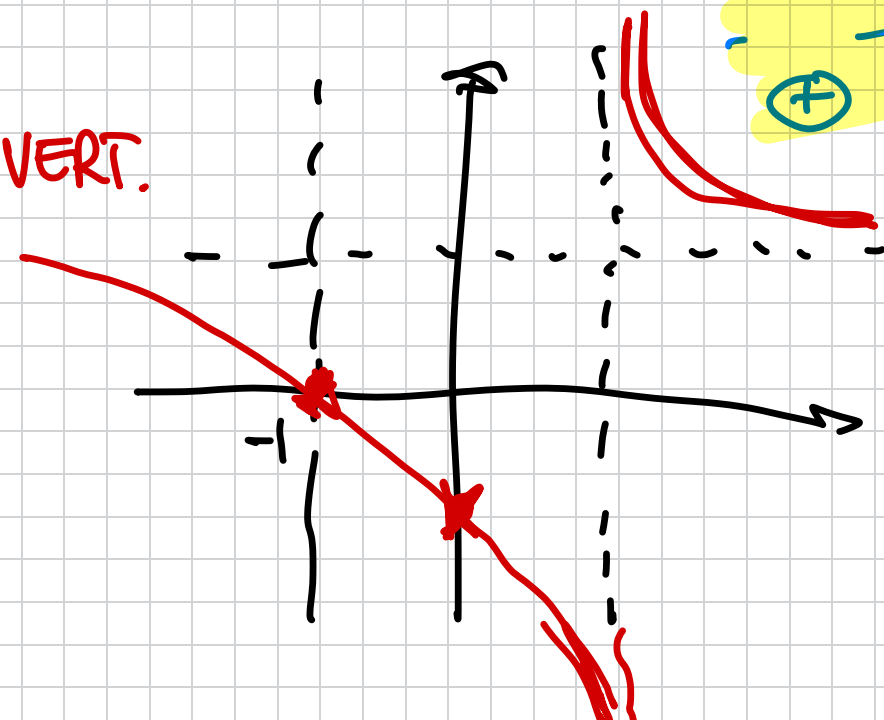
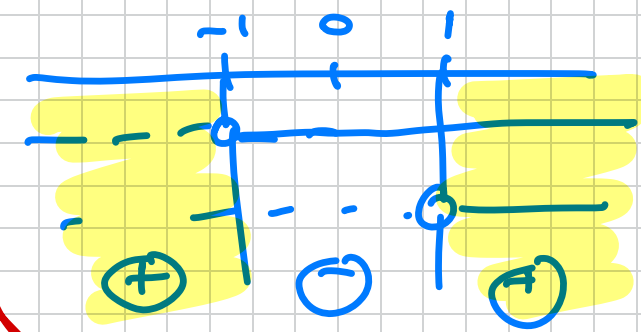
$$\lim_{x \rightarrow 1^+} \frac{x+1}{x-1} = \frac{2}{0^+} = +\infty$$

AS. VERT.



$$x+1 > 0 \rightarrow x > -1$$

$$x-1 > 0 \rightarrow x > 1$$

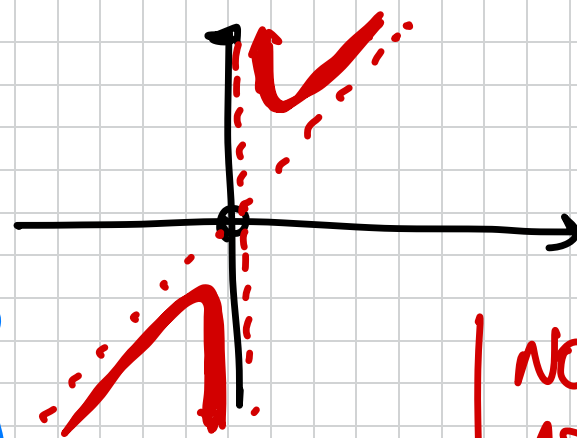


$$f(x) = \frac{x^2+3}{x}$$

$$\Delta = x \neq 0 \quad \pm \infty, 0$$

$$\lim_{x \rightarrow \pm \infty} \frac{x^2+3}{x} = \frac{\infty}{\infty} \rightarrow \lim_{x \rightarrow \pm \infty} \frac{x^2+3}{x} = +\infty$$

$$\sim x^2 \left(1 + \frac{3}{x^2}\right) = x \rightarrow \pm \infty$$



NO  
AS.  
ORIZZ.

$$\lim_{x \rightarrow \pm \infty} \frac{x^2+3}{x} \cdot \frac{1}{x} \Rightarrow \lim_{x \rightarrow \pm \infty} \frac{x^2+3}{x^2} = 1 = \text{cu}$$

AS. OBL.

$$y = x$$

$$\lim_{x \rightarrow \pm \infty} \frac{x^2+3}{x} - x = \infty - \infty \rightarrow \lim_{x \rightarrow \pm \infty} \frac{x^2+3-x^2}{x} = \frac{\pm 3}{\pm \infty} = 0 = 0$$

$$\lim_{x \rightarrow 0^+} \frac{x^2+3}{x} = +\infty \rightarrow \lim_{x \rightarrow 0^-} \frac{x^2+3}{x} = -\infty$$

AS  
VERT.

$$f(x) = \frac{x^2 - 4}{x + 1}$$

