Focus: limiti

$+\infty^3 + \infty^2 + 2 = +\infty + \infty = +\infty$

➢ $\lim_{x \to +\infty} (x^3 + x^2 + 2) = +\infty$

$\frac{(-1)^2 - 1}{-1+1} = \frac{+1-1}{-1+1} = \frac{0}{0}$

➢ $\lim_{x \to -1} \frac{x^2-1}{x+1} = \frac{0}{0}$ → forma indeterminata

Scomponiamo il limite: $\lim_{x \to -1} \frac{(x+1)(x-1)}{x+1} = \lim_{x \to -1} (x-1) = -2 \neq \frac{0}{0}$

➢ $\lim_{x \to 7} e^{\frac{1}{x}} = e^{\lim_{x \to 7} \frac{1}{x}} = e^{1/7}$

➢ $\lim_{x \to +\infty} \frac{x^3+x^2-1}{2x^2+2} = \frac{+\infty}{+\infty}$ → forma indeterminata

Scomponiamo il limite: $\lim_{x \to +\infty} \frac{\cancel{x^3}(1+\frac{1}{x}-\frac{1}{x^3})}{\cancel{x^3}(\frac{2}{x}+\frac{2}{x^3})} = \frac{+1+0-0}{0+0} = \frac{+1}{0} = +\infty$

Esercizio esemplificativo sul limite notevole: $\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x} = e$

DIMOSTRAZIONE

- $\lim_{x\to+\infty}\left(1+\frac{a}{x}\right)^{x}, a \in \mathbb{R}$

Il limite notevole originale veniva da $1^{\infty} = e$. Anche questa variazione viene da 1^{∞}, ma avendo una a non posso usare la forma del limite notevole: devo ricondurlo a tale forma:

$y = \frac{x}{a} \to x = ya$

$$\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x}{a}}\right)^{x} = \lim_{x\to+\infty}\left(1+\frac{1}{y}\right)^{x}, y = \frac{x}{a} \Rightarrow x \to +\infty : y = +\frac{\infty}{a} = +\infty$$

y

$$\lim_{x\to+\infty}\left(1+\frac{1}{y}\right)^{ya} \Leftrightarrow a > 0, a \in \mathbb{R}^{+}$$

$$\lim_{x\to+\infty}\left[\left(1+\frac{1}{y}\right)^{y}\right]^{a} = e^{a}$$ la stessa cosa vale per $a < 0, \in \mathbb{R}^{-}$

$e^{a}, \quad a = 1/3 \to e^{1/3}$

Esercizio esemplificativo sul limite notevole: $\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x} = e$

- $\lim_{x\to+\infty}\left(1+\frac{1}{3x}\right)^{x}$

Altra variazione dello stesso limite notevole. Chiamo $3x = y$:

$$\lim_{x\to+\infty}\left(1+\frac{1}{3x}\right)^{x} = \lim_{y\to+\infty}\left[\boxed{\left(1+\frac{1}{y}\right)^{y}}\right]^{\frac{1}{3}} = e^{\frac{1}{3}}$$

e

Se $x \to +\infty$, anche $3x \to +\infty$

Lo si può capire anche scrivendo: $\lim_{x\to+\infty}\left(1+\frac{1/3}{x}\right)^{x}$

In questo modo, $1/3$ sarebbe la a, che va come esponente di e

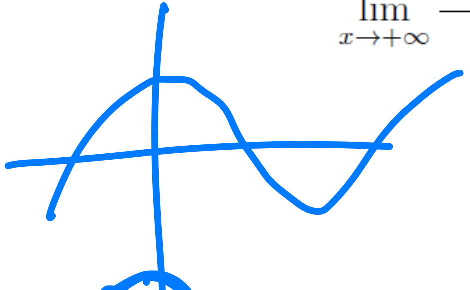
Focus: limiti

➢ $\lim\limits_{x\to+\infty} \frac{x+2+\cos x}{3-x} = -1$

$$\lim_{x\to+\infty} \frac{x+2+\cos x}{3-x} = \lim_{x\to+\infty} \frac{\cancel{x}\left(1+\underbrace{\frac{2}{x}}_{\downarrow\, 0}+\underbrace{\frac{\cos x}{x}}_{\downarrow\, 0}\right)}{\cancel{x}\left(\underbrace{\frac{3}{x}}_{\downarrow\, 0}-1\right)} = -1$$

$\to 1$

-1

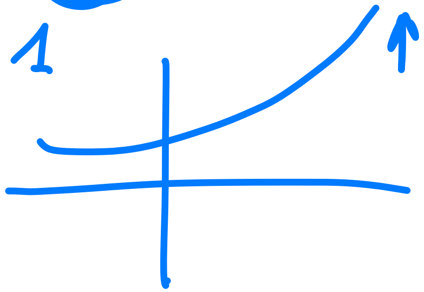


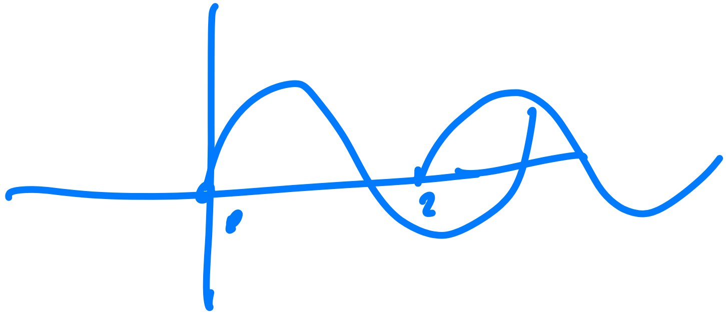
➢ $\lim\limits_{x\to 0} e^x(\cos x - 2\sin x) = 1$

$1(1-0) = 1$

Prodotto di funzioni continue: il limite si valuta calcolando il valore della funzione in quel punto

∞





➢ $\lim\limits_{x\to-\infty} \frac{x^2+\sin e^x}{2x} = -\infty$

$$\lim_{x\to-\infty} \frac{x^2+\sin e^x}{2x} = \lim_{x\to-\infty} \frac{x^2\left(1+\underbrace{\frac{\sin e^x}{x^2}}_{\downarrow\, 0}\right)}{2x} = -\infty$$

Focus: limiti

- $\displaystyle\lim_{x\to+\infty}\frac{x+3\sqrt{x}}{2x-5\sqrt{x}}=\frac{1}{2}$

$$\lim_{x\to+\infty}\frac{x+3\sqrt{x}}{2x-5\sqrt{x}}=\lim_{x\to+\infty}\frac{x\left(1+\underbrace{\frac{3}{\sqrt{x}}}_{\downarrow\atop 0}\right)}{x\left(2+\underbrace{\frac{5}{\sqrt{x}}}_{\downarrow\atop 0}\right)}=\frac{1}{2}$$

- $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\left(x-\frac{\pi}{2}\right)^2}=-\frac{1}{2}$

$y = x-\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\left(x-\frac{\pi}{2}\right)^2}\overset{x-\frac{\pi}{2}=y}{=}\lim_{y\to0}\frac{\sin\left(y+\frac{\pi}{2}\right)-1}{y^2}$$

$$=\lim_{y\to0}\frac{\sin y\cos\frac{\pi}{2}+\sin\frac{\pi}{2}\cos y-1}{y^2}=\lim_{y\to0}\frac{\cos y-1}{y^2}=-\frac{1}{2}$$

- $\displaystyle\lim_{x\to0}\frac{\sin x-2\log(1+x)}{x+\sin x}=-\frac{1}{2}$

$$\lim_{x\to0}\frac{\sin x-2\log(1+x)}{x+\sin x}=\lim_{x\to0}\frac{x\left(\overbrace{\frac{\sin x}{x}}^{1}-2\overbrace{\frac{\log(1+x)}{x}}^{e}\right)}{x\left(1+\underbrace{\frac{\sin x}{x}}_{1}\right)}=-\frac{1}{2}$$

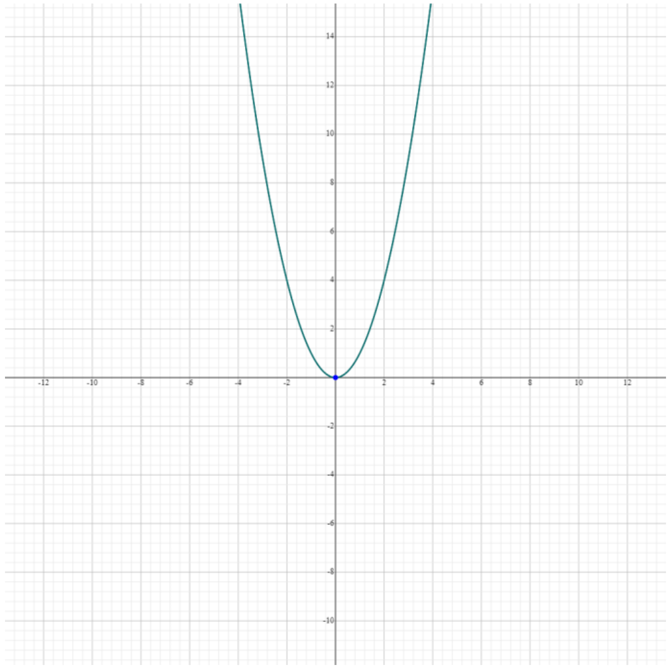
Ricorda: limiti notevoli → $\displaystyle\lim_{x\to0}\frac{\sin x}{x}=1$ $\qquad\displaystyle\lim_{x\to0}\frac{\log_a(1+x)}{x}=\log_a e$

Esercizio. Valutare dominio, continuità e limiti.

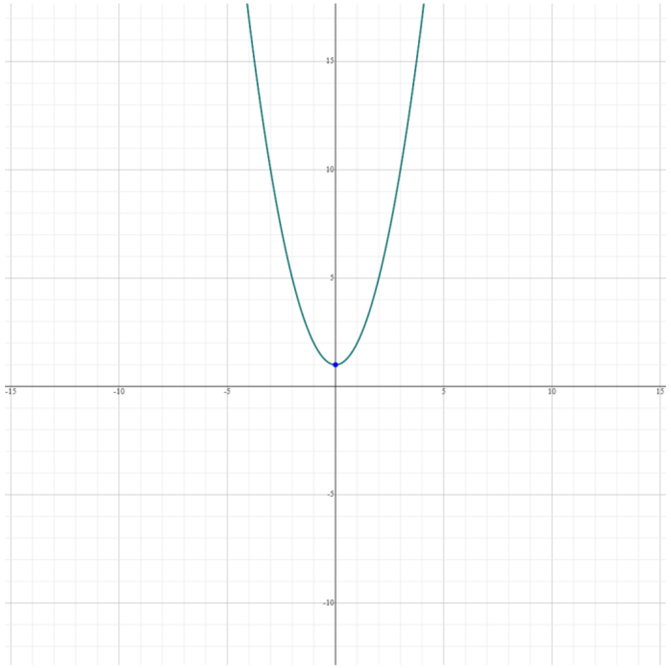
$f(x) = \sqrt{x^2+1}$

$x^2+1 \geq 0 \quad x^2 \geq -1 \rightarrow$ sempre

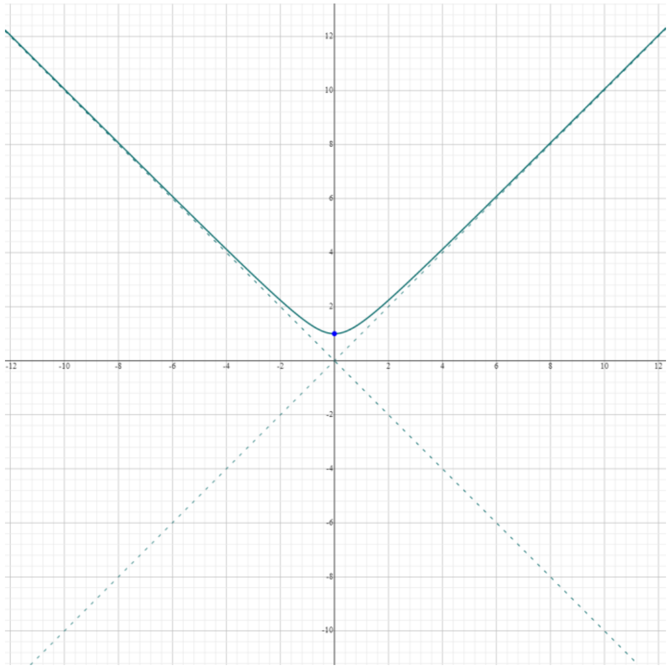
- $D_f = \mathbb{R}$
- la parabola è continua, la radice è continua, quindi la composizione i due funzioni continue dà una funzione continua



$f(x) = x^2$



$f(x) = x^2 + 1$

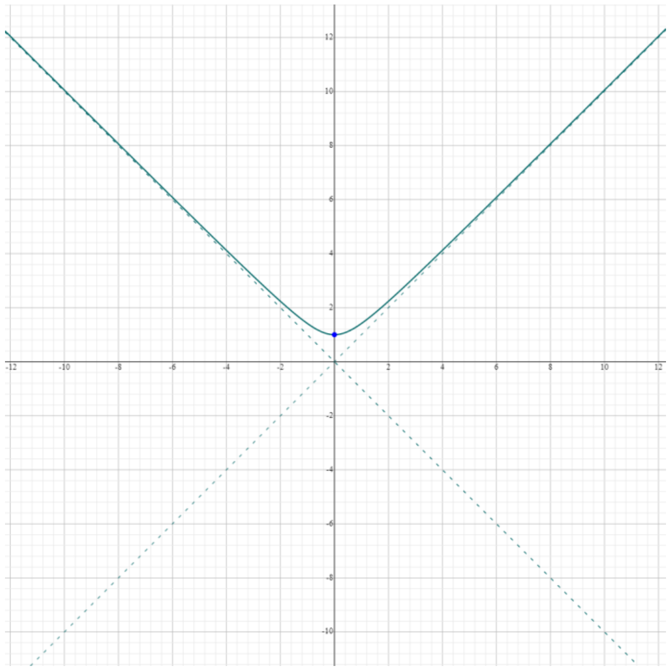


$f(x) = (x^2+1)^{\frac{1}{2}} = \sqrt{x^2+1}$

Esercizio. Valutare dominio, continuità e limiti.

$f(x) = \sqrt{x^2 + 1}$

- $D_f = \mathbb{R}$
- la parabola è continua, la radice è continua, quindi la composizione i due funzioni continue dà una funzione continua



Ricordando:

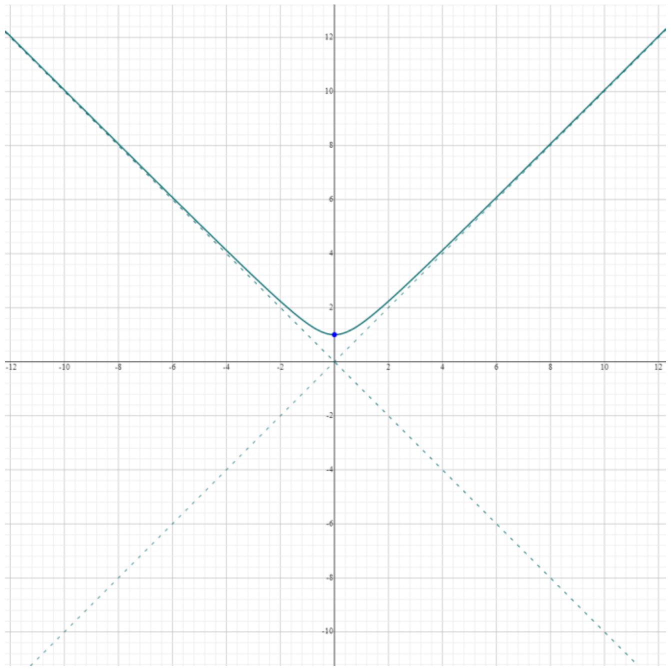
- Se $\lim_{x\to\pm\infty} f(x) = k$ → asintoto orizzontale
- Se $\lim_{x\to c} f(x) = \pm\infty$ → asintoto verticale
- Se $\lim_{x\to\pm\infty} f(x) = \pm\infty$ → asintoto obliquo, con
  - $m = \lim_{x\to+\infty} \frac{f(x)}{x}$
  - $q = \lim_{x\to+\infty} [f(x) - mx]$

$f(x) = \sqrt{x^2+1} \to \lim_{x\to\pm\infty} \sqrt{x^2+1} = +\infty$ → as. obliquo!

Esercizio. Valutare dominio, continuità e limiti.

$f(x) = \sqrt{x^2+1}$

- $D_f = \mathbb{R}$
- la parabola è continua, la radice è continua, quindi la composizione i due funzioni continue dà una funzione continua



$f(x) = \sqrt{x^2+1} \to \lim_{x\to\pm\infty} \sqrt{x^2+1} = +\infty$ → as. obliquo!

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to+\infty} \sqrt{\frac{x^2+1}{x^2}} = \lim_{x\to+\infty} \sqrt{1+\frac{1}{x^2}} = 1$$

$$q = \lim_{x\to+\infty_{-}} [f(x) - mx)] = \lim_{x\to+\infty_{-}} \sqrt{x^2+1} - 1x = \infty - \infty \to f.i.$$

$$= \lim_{x\to+\infty} \sqrt{x^2+1} - 1x \cdot \frac{\sqrt{x^2+1}+1x}{\sqrt{x^2+1}+1x} = \lim_{x\to+\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \frac{1}{+\infty+\infty} = \frac{1}{+\infty} = 0^+$$

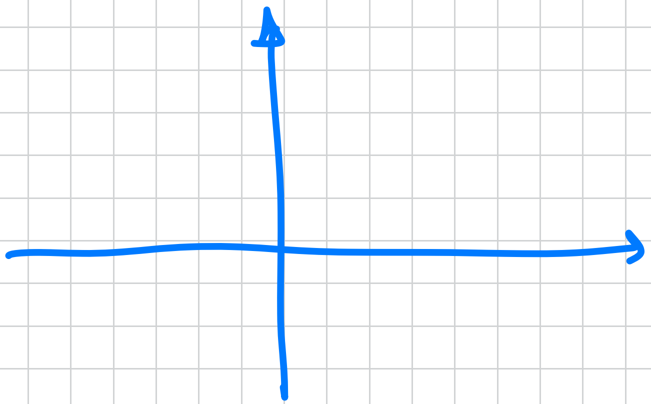
$$m = 1, q = 0 \to y = 1x + 0 = x$$

cosa accade per $-\infty$? $\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to+\infty} -\sqrt{1+\frac{1}{x^2}} = -1 \to y = -x$

So che le x che prendo sono negative

$f(x) = x(x^2-1)^2$

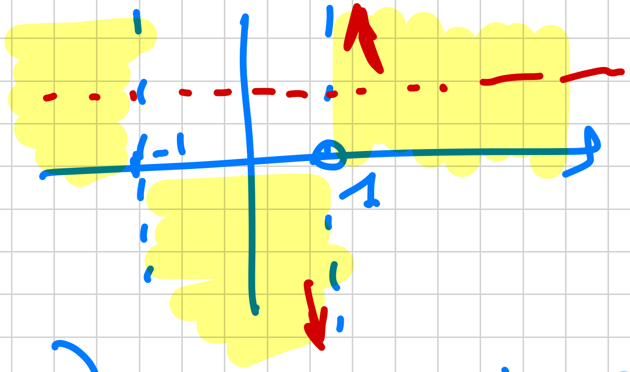
$D_f ? \to \mathbb{R}$



$\lim_{x \to \pm\infty} x(x^2-1)^2 = \pm\infty \to$ NO ORIZZ.

$\lim_{x \to \pm\infty} \frac{\cancel{x}(x^2-1)^2}{\cancel{x}} = +\infty \neq m \to$ NO OBLIQUO

$f(x) = \frac{x+1}{x-1}$



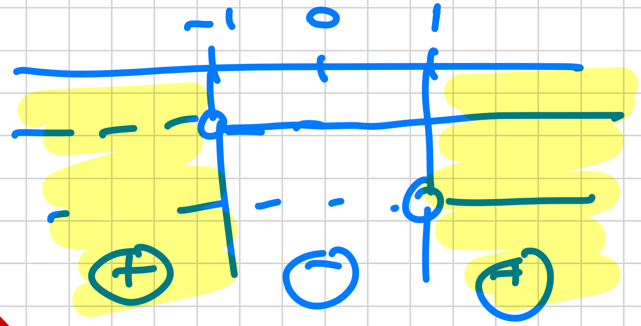
$x-1 \neq 0 \quad x \neq 1$

$D_f = x \in \mathbb{R} : x \neq 1 \;;\; \mathbb{R} - \{1\} \;;\; (-\infty, 1) \cup (1, +\infty)$

$\frac{x+1}{x-1} > 0$

$x+1 > 0 \rightarrow x > -1$

$x-1 > 0 \rightarrow x > 1$

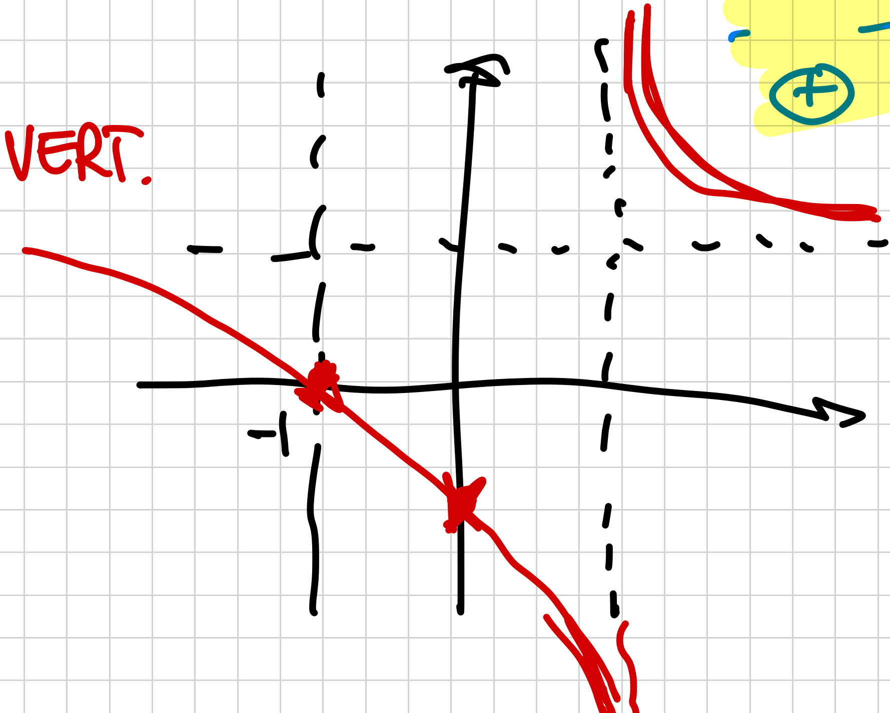


$\lim_{x \to \pm\infty} \frac{x+1}{x-1} = 1$ AS. ORIZZ.

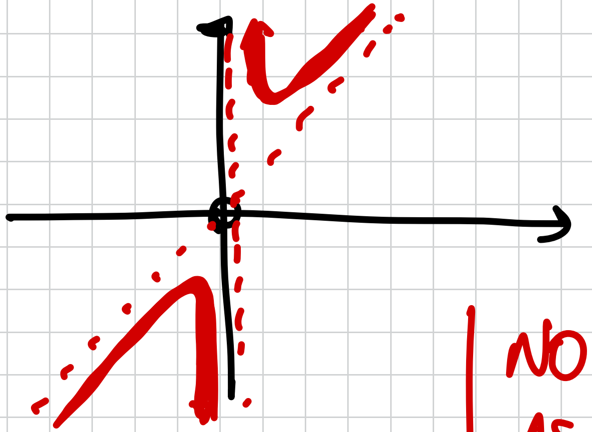
$\lim_{x \to 1^-} \frac{x+1}{x-1} = \frac{1+1}{1-1} = \frac{2}{0^-} = -\infty$

$\lim_{x \to 1^+} \frac{x+1}{x-1} = \frac{2}{0^+} = +\infty$

AS. VERT.



$f(x)=\dfrac{x^2+3}{x}$ $\quad D = x \neq 0 \quad \pm\infty,\ 0$



$\lim\limits_{x\to\pm\infty} \dfrac{x^2+3}{x} = \dfrac{\infty}{\infty} \to \lim\limits_{x\to\pm\infty} \dfrac{x^2+3}{x} = \pm\infty$

$\hookrightarrow \dfrac{x^{\cancel{2}}\left(1+\frac{3}{x^2}\overset{0}{}\right)}{\cancel{x}} = x \to \pm\infty$

NO AS. ORIZZ.

$\lim\limits_{x\to\pm\infty} \dfrac{x^2+3}{x}\cdot\dfrac{1}{x} \Rightarrow \lim\limits_{x\to\pm\infty} \dfrac{x^2+3}{x^2} = 1 = m$

$\lim\limits_{x\to\pm\infty} \dfrac{x^2+3}{x} - x = \infty-\infty \to \lim\limits_{x\to\pm\infty} \dfrac{\cancel{x^2}+3-\cancel{x^2}}{x} = \dfrac{3}{\pm\infty} = 0 = q$

AS. OBL.

$y=x$

$\lim\limits_{x\to 0^+} \dfrac{x^2+3}{x} = +\infty \to \lim\limits_{x\to 0^-} \dfrac{x^2+3}{x} = -\infty$

AS. VERT.

$$f(x)=\frac{x^2-4}{x+1}$$

DOMINIO $\to$ $x\neq -1$ $\to$ $D_f=\{x\in\mathbb{R}: x\neq -1\}$

$\mathbb{R}-\{-1\}$

$(-\infty,-1)\cup(-1,+\infty)$

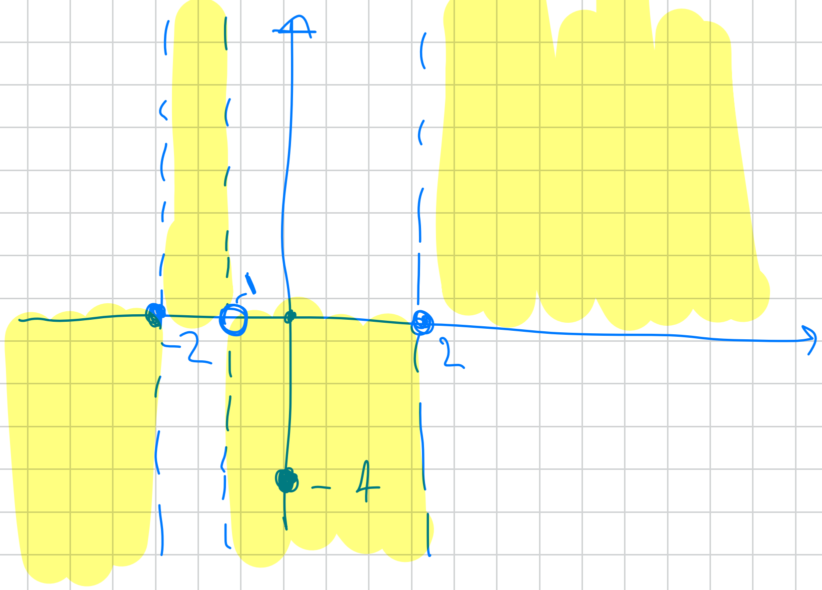
INTERSEZIONI

$$\begin{cases} y=0\\ y=\dfrac{x^2-4}{x+1}\end{cases} \to \frac{x^2-4}{x+1}=0 \to x=\pm 2$$

$B(-2,0)$ $C(2,0)$

$$\begin{cases} x=0\\ y=\dfrac{x^2-4}{x+1}\end{cases} \to y=\frac{0^2-4}{0+1}$$

$y=-4 \to A(0,-4)$

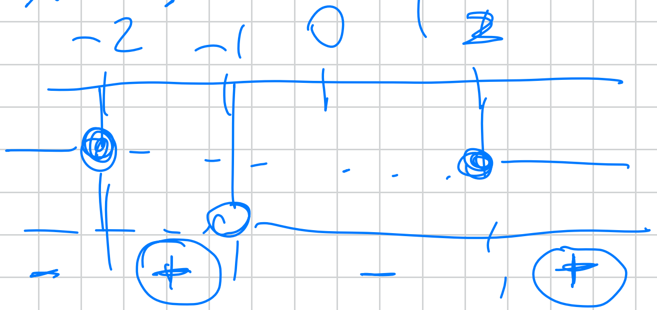


SEGNO

$f(x)>0$

$$\frac{x^2-4}{x+1}\geq 0$$

$$\begin{cases} x^2-4\geq 0\\ x+1>0\end{cases} \qquad \begin{cases} x\leq -2,\ x\geq 2\\ x>-1\end{cases}$$



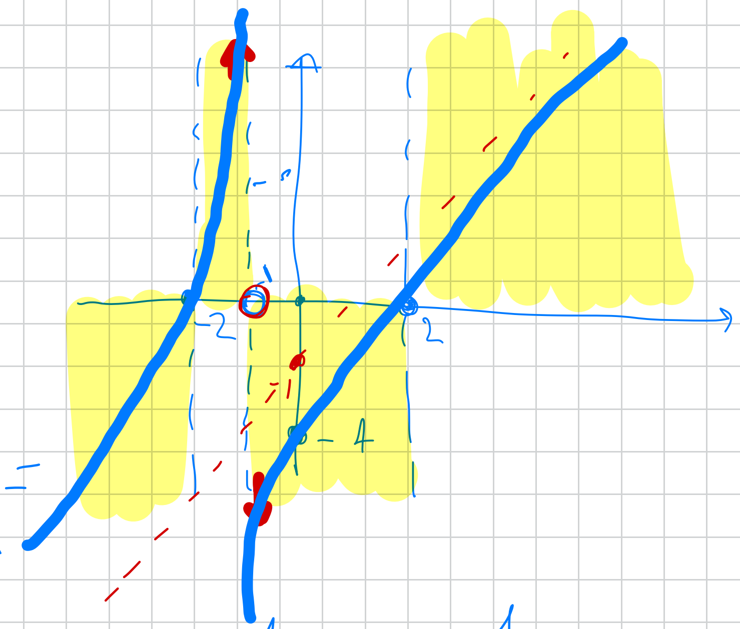
ASINTOTI

$$\lim_{x\to\pm\infty} \frac{x^2-4}{x+1} = +\infty \longrightarrow \text{no as. oriz.}$$

obliquo?

$$\lim_{x\to\pm\infty} \underbrace{\frac{x^2-4}{x+1}\cdot\frac{1}{x}} = m = +1$$

$$\frac{x^2-4}{x^2+x} = \frac{1}{1} = 1$$



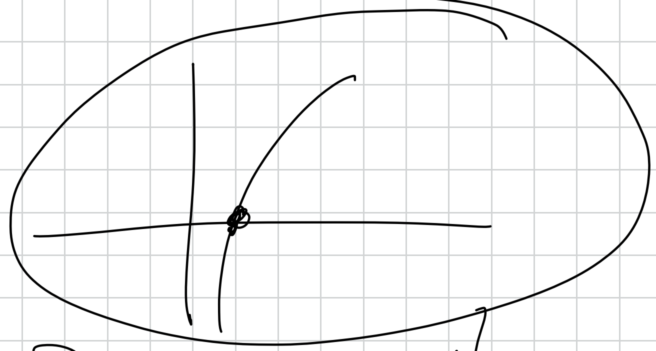
$$\lim_{x\to\pm\infty} \underbrace{\frac{x^2-4}{x+1} - x} = \frac{\cancel{x^2}-4-\cancel{x^2}-x}{x+1} = \frac{-x-4}{x+1} = \frac{-1}{1} = -1 = q$$

$$y = x-1$$

$$\lim_{x\to -1^+} \frac{x^2-4}{x+1} = \frac{(-1^+)^2-4}{-1^{+}+1} = \frac{-3}{0^+} = -\infty$$

$$\lim_{x\to -1^-} \frac{x^2-4}{x+1} = \frac{(-1^-)^2-4}{-1^{-}+1} = \frac{-3}{0^-} = +\infty$$

$f(x) = \frac{x}{\log_e x}$



DOMINIO $\rightarrow \begin{cases} \log x \neq 0 \rightarrow x \neq 1 \\ \underline{x > 0} \end{cases}$

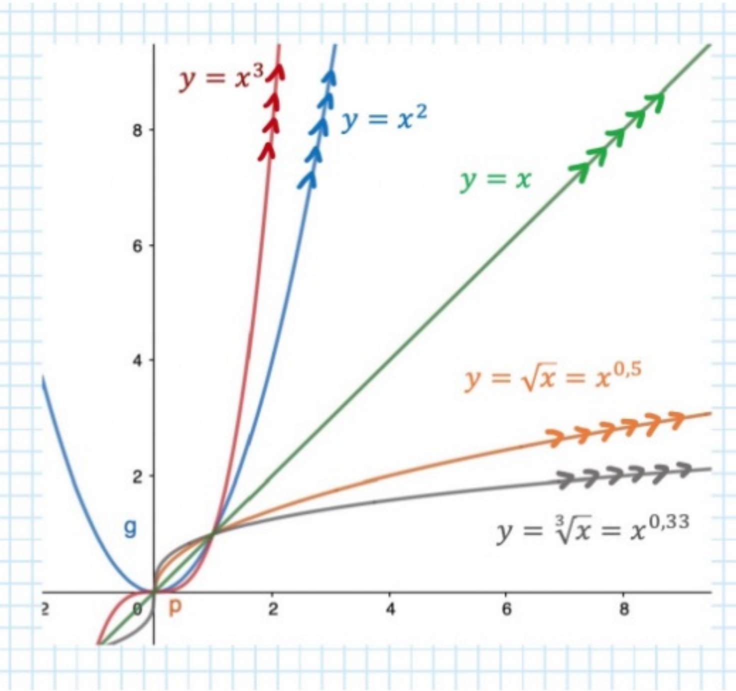
$D_f = \{x \in \mathbb{R} : x > 0,\ x \neq 1\}$

$(0, 1) \cup (1, +\infty)$

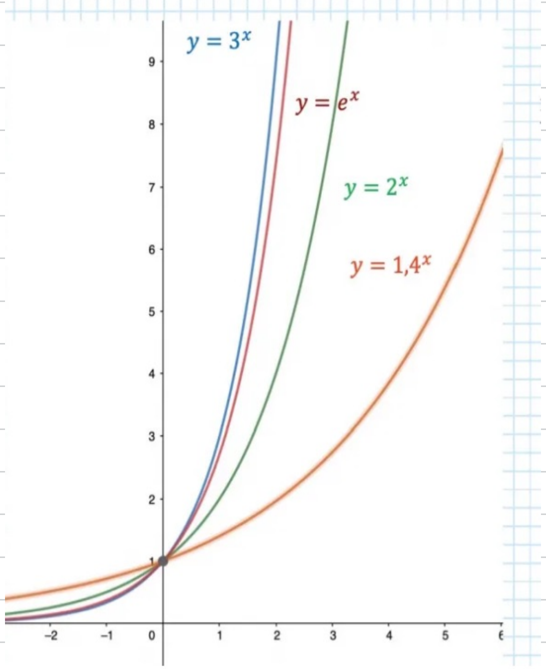
INTERSEZIONI

$\begin{cases} x = 0 \\ y = \frac{x}{\log x} \end{cases} \rightarrow y = \cancel{\frac{0}{\log 0}}$ ∄ intersez con y

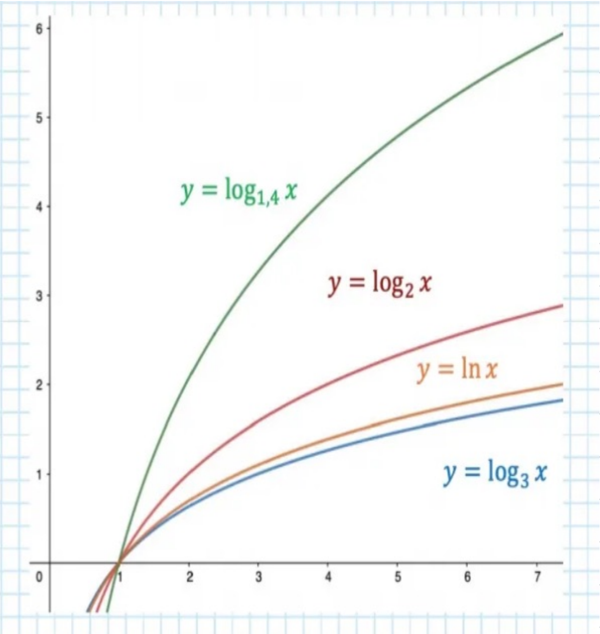
$\begin{cases} y = 0 \\ y = \frac{x}{\log x} \end{cases} \rightarrow \frac{x}{\log x} = 0 \rightarrow x = 0 \rightarrow$ mai $\rightarrow$ ∄ intersez. con x



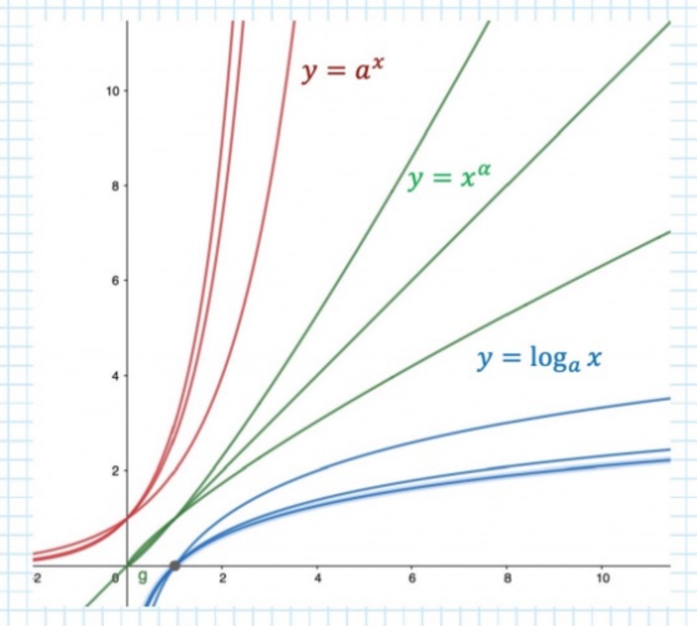
$y = x^3$
$y = x^2$
$y = x$
$y = \sqrt{x} = x^{0,5}$
$y = \sqrt[3]{x} = x^{0,33}$
g
p



$y = 3^x$
$y = e^x$
$y = 2^x$
$y = 1{,}4^x$

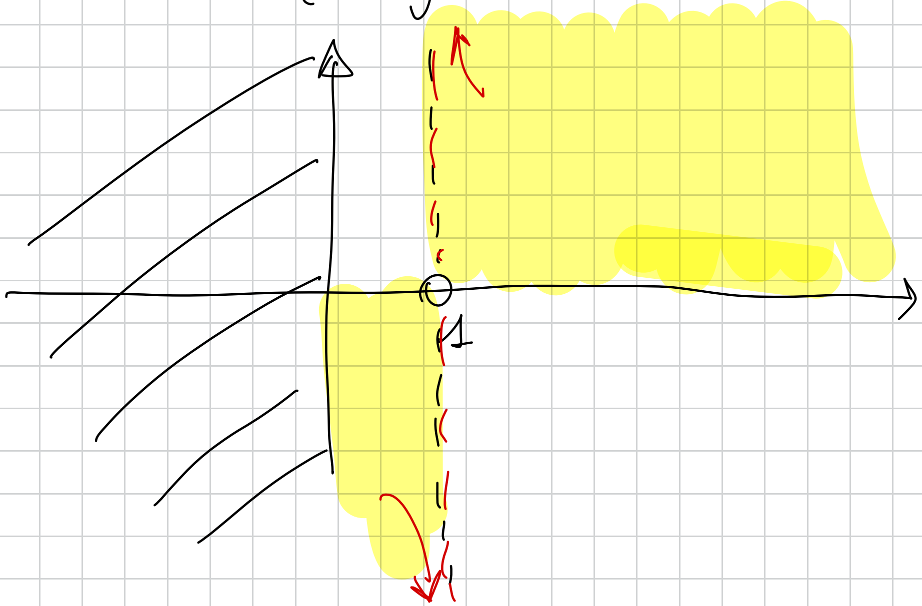


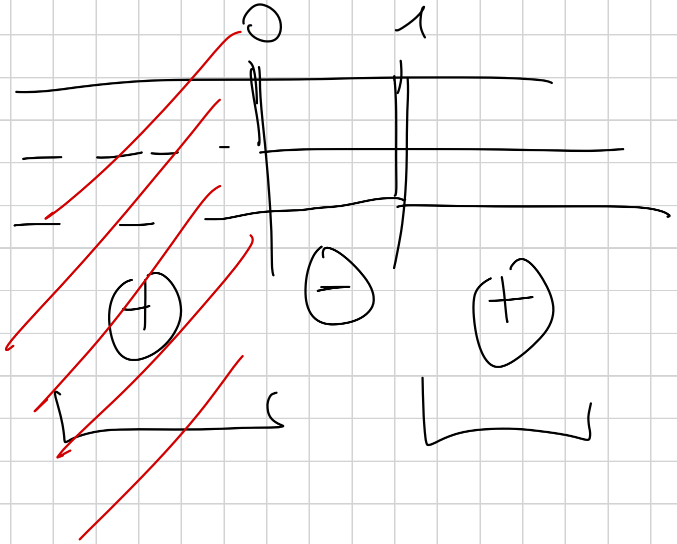
$y = \log_{1,4} x$
$y = \log_2 x$
$y = \ln x$
$y = \log_3 x$



$y = a^x$
$y = x^\alpha$
$y = \log_a x$
g

$f(x)>0 \qquad \frac{x}{\log x}>0 \qquad \begin{cases} x>0 \\ \log x>0 \end{cases} \qquad \begin{cases} x>0 \\ x>1 \end{cases}$





ASINTOTI

$\lim_{x\to 0^+} \frac{x}{\log x} = \nexists$ AS. VERT. $x=0$

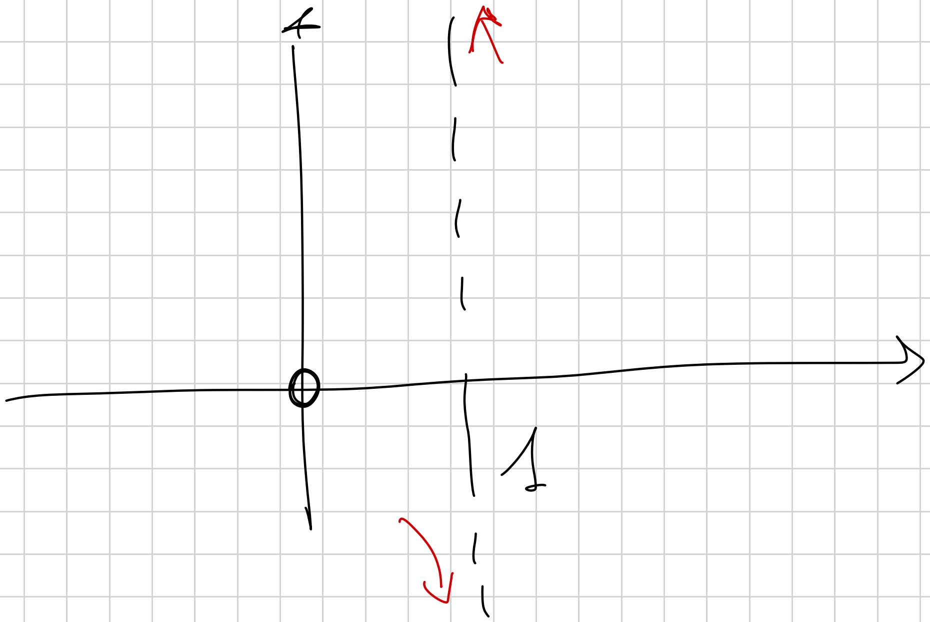
$\lim_{x\to 1^-} \frac{x}{\log x} = -\infty \qquad \lim_{x\to 1^+} \frac{x}{\log x} = +\infty$ AS. VERT. $x=1$

$\lim_{x\to+\infty} \frac{x}{\log x} = \frac{+\infty}{+\infty} = +\infty$

non esiste ORIZZ.

gerarchie degli infiniti

$$\lim_{x \to +\infty} \frac{x}{\log x} \cdot \frac{1}{x} = \frac{\cancel{x}}{\cancel{x}\log x} = \frac{1}{\infty} = 0 \neq m \rightarrow \text{no obliquo}$$

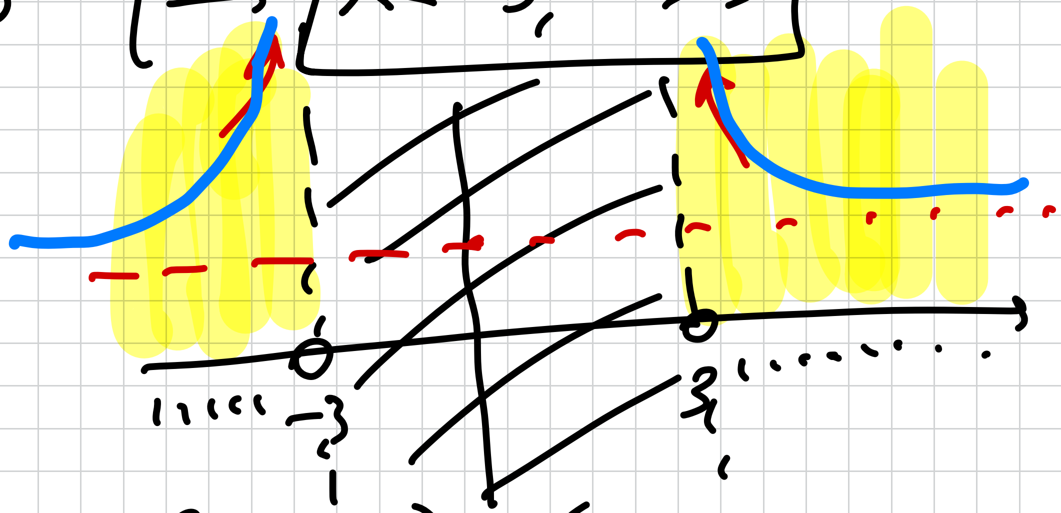


$$f(x) = \sqrt{\frac{x^2}{x^2-9}}$$

DOMINIO $\longrightarrow$ $\dfrac{x^2}{x^2-9} \geq 0$ $\quad \begin{cases} x^2 \geq 0 & \rightarrow \text{sempre} \rightarrow \forall x \in \mathbb{R} \\ x^2-9 > 0 & \rightarrow \boxed{x<-3, \ x>3} \end{cases}$

$\Downarrow$

$x^2 - 9 \neq 0$



INTERSEZIONI

$\begin{cases} x = 0 \\ y = \sqrt{\dfrac{x^2}{x^2-9}} \end{cases} \longrightarrow$ no!, $\quad \begin{cases} y = 0 \\ y = \sqrt{\dfrac{x^2}{x^2-9}} = 0 \end{cases} \longrightarrow x^2 = 0 \rightarrow \cancel{x=0} \longrightarrow$ fuori dal C.E.

SEGNO

$f(x) > 0 \quad \sqrt{\dfrac{x^2}{x^2-9}} > 0$

$$\lim_{x\to\pm\infty}\sqrt{\frac{x^2}{x^2-9}} = 1 \qquad y=1 \text{ as. orizz.}$$

$$\lim_{x\to -3^-}\sqrt{\frac{x^2}{x^2-9}} = \lim\sqrt{\frac{x^2}{(x+3)(x-3)}} = \frac{(-3^-)^2}{(-3^-+3)(-3^--3)} = \frac{9}{0^-(-6)} = +\infty$$

$$\lim_{x\to 3^+}\sqrt{\frac{x^2}{(x+3)(x-3)}} = \frac{9}{(3+3)(3^+-3)} = \frac{9}{+6\cdot 0^+} = +\infty$$

$x=-3$ AS VERT. SX

$x=3$ AS VERT. DX