$$f(x) = \frac{x^2-1}{x-2}$$

① DOMINIO $\longrightarrow$ $x-2 \neq 0 \rightarrow x \neq 2 \rightarrow D_f = \{\forall x \in \mathbb{R} : x \neq 2\}$

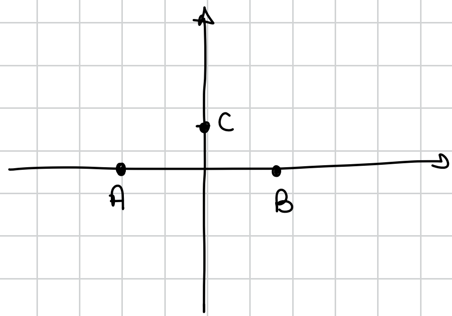
$\mathbb{R} - \{2\}$

② INTERSEZIONI $\rightarrow \begin{cases} y = \dfrac{x^2-1}{x-2} \\ x = 0 \end{cases}$ $\qquad \begin{cases} y = \dfrac{x^2-1}{x-2} \\ \boxed{y=0} \end{cases}$

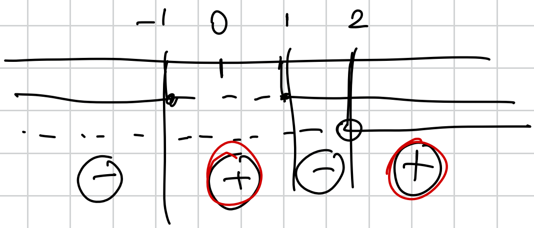
$$y = \frac{0^2-1}{0-2} \qquad \frac{x^2-1}{x-2} = 0 \quad x = \pm 1$$

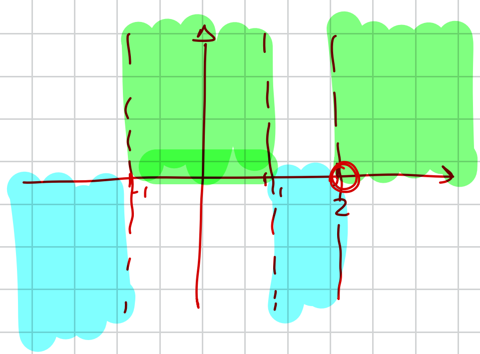
$$y = 1/2$$

$A(-1,0) \quad B(1,0) \quad C(0, 1/2)$



③ SEGNO $\longrightarrow$ $\dfrac{x^2-1}{x-2} \geq 0 \rightarrow \begin{cases} x^2-1 \geq 0 \rightarrow x \leq -1,\ x \geq 1 \\ x-2 > 0 \rightarrow x > 2 \end{cases}$



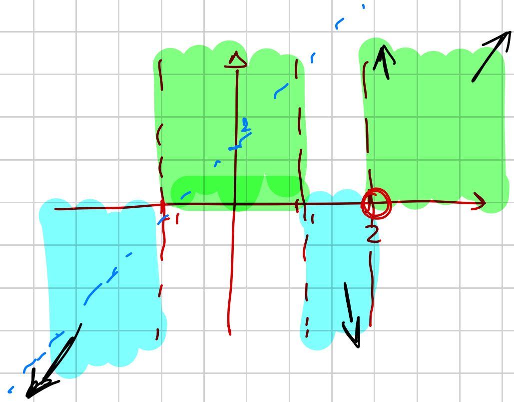


4 ASINTOTI

$$\lim_{x\to+\infty} \frac{x^2-1}{x-2} = \lim_{x\to+\infty} \frac{x^{\cancel{2}}\left(1-\cancel{\frac{1}{x^2}}\right)}{\cancel{x}\left(1-\cancel{\frac{2}{x}}\right)} = +\infty$$

$$\lim_{x\to-\infty} \frac{x^2-1}{x-2} = -\infty$$

$$\left.\begin{array}{l} \lim_{x\to2^+} \frac{x^2-1}{x-2} = \frac{3}{0^+} = +\infty \\ \lim_{x\to2^-} \frac{x^2-1}{x-2} = \frac{3}{0^-} = -\infty \end{array}\right] \text{AS. VERT.}$$



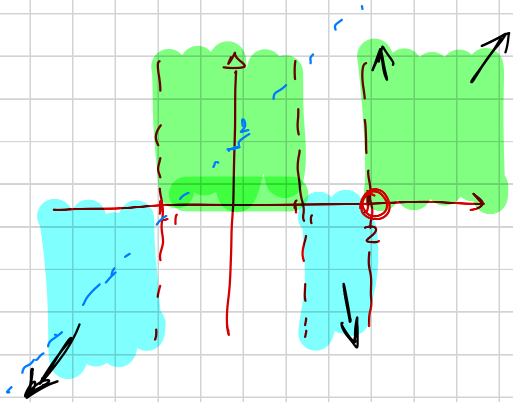
AS. OBL. $\to$ $y = x+2$

$$\lim_{x\to+\infty} \frac{x^2-1}{x-2}\cdot\frac{1}{x} = 1 = m$$

$$\lim_{x\to+\infty} \frac{x^2-1}{x-2} - x = 2$$

5) MASSIMI E MINIMI

$$f(x)=\frac{x^2-1}{x-2} \to f'(x)=\frac{x^2-4x+1}{(x-2)^2}$$



$$\frac{x^2-4x+1}{(x-2)^2}=0 \Leftrightarrow \text{punti staz.?}$$

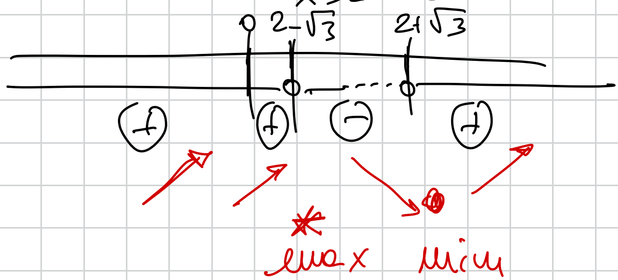
$$x_1=2+\sqrt{3} \qquad x_2=2-\sqrt{3}$$

$$y_1=4+2\sqrt{3}\cong 7.46$$

$$y_2=4-2\sqrt{3}\cong 0.54$$

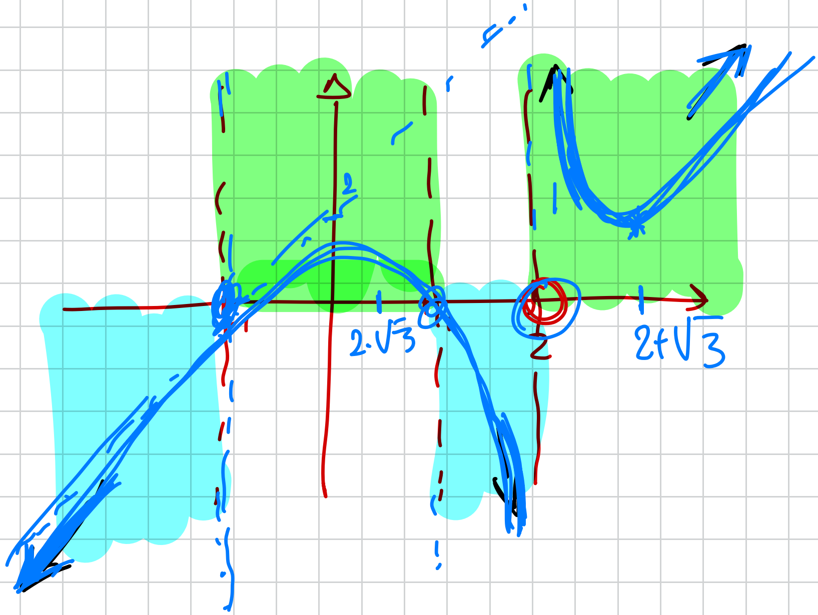
$$\frac{x^2-4x+1}{(x-2)^2}>0 \quad \begin{cases} x^2-4x+1>0 \\ (x-2)^2>0 \to \text{sempre} \end{cases}$$

$$x^2-4x+1>0 \quad \begin{cases} x<2-\sqrt{3} \\ x>2+\sqrt{3} \end{cases}$$



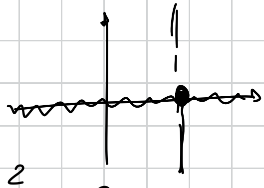
6) CONCAVITÀ

$$f''(x)=\frac{6x-12}{(x-2)^4} \to =0,\ \boxed{x=2} \to$$ NO FLESSO, $\notin D_f$



2
2-√3
2
2+√3

$f(x) = x^3 - 2x^2 - x + 2$



① DOMINIO $\longrightarrow \mathbb{R}$

② INTERSEZ. $\rightarrow \begin{cases} y = x^3 - 2x^2 - x + 2 \\ y = 0 \end{cases}$

$\underbrace{x^3 - 2x^2 - x + 2 = 0}$

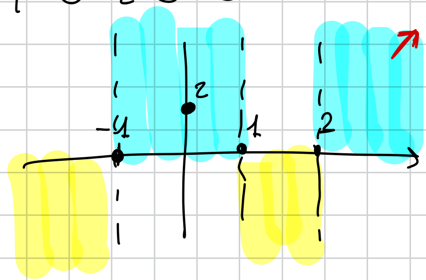
$\begin{cases} y = x^3 - 2x^2 - x + 2 \\ x = 0 \end{cases}$

$\underbrace{\qquad}$

$y = 0^3 - 2 \cdot 0^2 - 0 + 2$

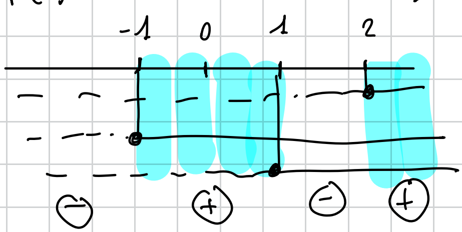
A $(0,2)$

B $(-1,0)$ C $(1,0)$ D $(2,0)$



③ SEGNO $f(x)$

$f(x) \geq 0 \qquad x^3 - 2x^2 - x + 2 \geq 0 \rightarrow (x-2)(x+1)(x-1) \geq 0$



$x - 2 \geq 0 \rightarrow x \geq 2$

$x + 1 \geq 0 \rightarrow x \geq -1$

$x - 1 \geq 0 \rightarrow x \geq 1$

④ ASINTOTI

ORIZZ. ?

$\lim_{x \to -\infty} x^3 - 2x^2 - x + 2 = -\infty$

$\lim_{x \to +\infty} \left(x^3 - \cancel{2x^2} - \cancel{x} + \cancel{2}\right) = +\infty - \infty - \infty + 2 =$

$= \boxed{+\infty} \longrightarrow$ NO AS ORIZZ

OBLIQUI ?

$\lim_{x \to \pm\infty} \frac{x^3 - 2x^2 - x + 2}{x} = \lim \frac{\cancel{x^3}^{\,2}\left(1 - \frac{2}{x}^{\nearrow 0} - \frac{1}{x^2}^{\nearrow 0} + \frac{2}{x^3}^{\nearrow 0}\right)}{\cancel{x}} = \lim_{x \to \pm\infty} x^2 = +\infty$

$\neq l$

NO AS. OBLIQUO

$f(-0.21)$

$f(1.5)$

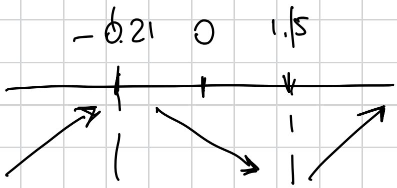
$(f \cdot g)' = f'g + fg'$

MAX & MIN

$f'(x) = 3x^2 - 4x - 1 \geq 0$

$x \leq -\dfrac{\sqrt{7}+2}{3} \cong -0.21$

$x \geq \dfrac{\sqrt{7}+2}{3} \cong 1.5$

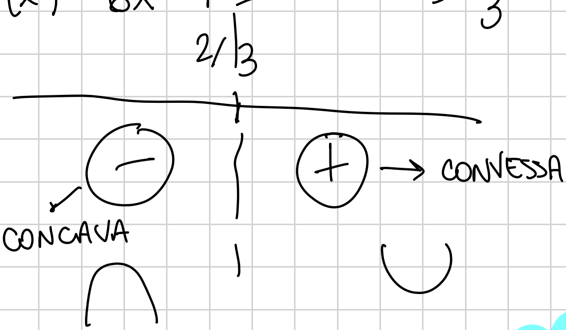


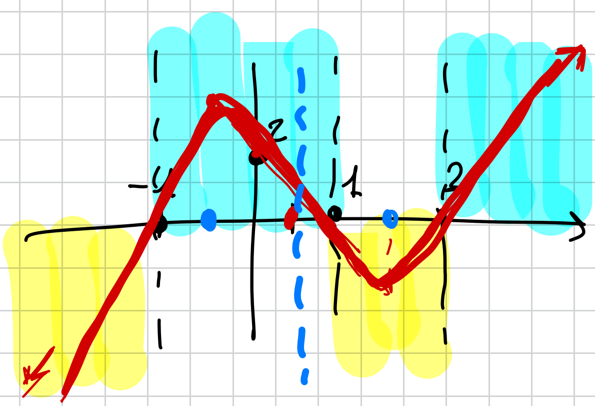
$\rightarrow$ $-0.21 \rightarrow$ max

$1.5 \rightarrow$ min

CONCAVITÀ

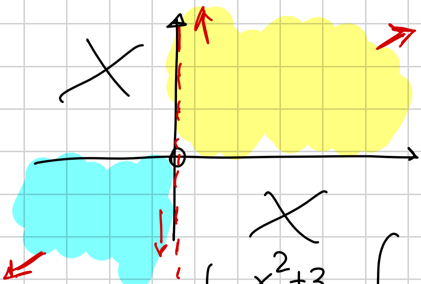
$f''(x) = 6x - 4 \geq 0 \rightarrow x \geq \frac{2}{3} \cong 0.67$





$$f(x) = \frac{x^2+3}{x}$$

DOMINIO $\to x \neq 0 \to D_f = \{x \in \mathbb{R} : x \neq 0\} = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$



INTERSEZ. $\to \begin{cases} y = \frac{x^2+3}{x} \\ y = 0 \end{cases}$ $\quad \begin{cases} y = \frac{x^2+3}{x} \\ x = 0 \end{cases}$

$$\frac{x^2+3}{x} = 0$$

no soluz.

no intersez. con x

$$y = \frac{0^2+3}{0} \to$$ no soluz.

y no intersez.

SEGNO $\quad f(x) > 0 \quad \frac{x^2+3}{x} > 0$ $\quad x^2+3>0 \to$ sempre

$x > 0$

$\to$ quando $x > 0$

ASINTOTI

$$\lim_{x \to 0^+} \frac{x^2+3}{x} = \frac{3}{0^+} = +\infty \qquad \lim_{x \to 0^-} \frac{x^2+3}{x} = \frac{3}{0^-} = -\infty$$

AS. VERT. $x = 0$

$$\lim_{x \to +\infty} \frac{x^2+3}{x} = +\infty \qquad \lim_{x \to -\infty} \frac{x^2+3}{x} = -\infty$$

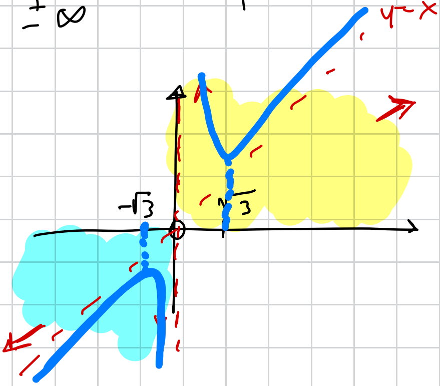
$\hookrightarrow \frac{\cancel{x^2}\left(1+\frac{3}{x^2}\right)}{\cancel{x}}$ (con $\frac{3}{x^2} \to 0$) $\lim_{x \to +\infty}$ $x = +\infty$

NO AS. ORIZZ.

$$\lim_{x\to\pm\infty} \frac{x^2+3}{x}\cdot\frac{1}{x} = \lim_{x\to\pm\infty}\frac{x^2+3}{x^2} = \frac{1}{1} = 1 = m$$

$$\lim_{x\to\pm\infty} \frac{x^2+3}{x} - x = \lim_{x\to\pm\infty}\frac{x^2+3-x^2}{x} = \frac{3}{\pm\infty} = 0 = q$$

AS. OBL. $\to$ $y = x$



MAX & MIN

$$(f/g)' = \frac{f'g - fg'}{g^2}$$

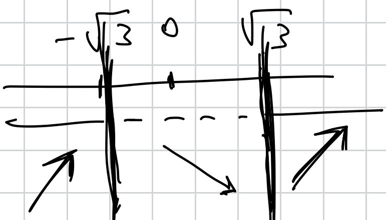
$f'(x) =$

$$\frac{\frac{d}{dx}(x^2+3)\cdot x - (x^2+3)\cdot\frac{dx}{dx}}{x^2} = \frac{2x\cdot x-(x^2+3)}{x^2} = \frac{2x^2-x^2-3}{x^2} = \frac{x^2-3}{x^2}$$

$$\frac{x^2}{x^2} - \frac{3}{x^2} \Rightarrow \boxed{1-\frac{3}{x^2}}$$

$$f'(x) = 1-\frac{3}{x^2} > 0 \to \frac{x^2-3}{x^2} > 0$$

$$\begin{cases} x > \sqrt{3} \\ x < -\sqrt{3} \end{cases}$$



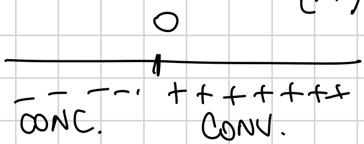
$\max x = -\sqrt{3}$

$\min = +\sqrt{3}$

$$f''(x) = \frac{d}{dx}\left(\frac{x^2-3}{x^2}\right) = \frac{\frac{d(x^2-3)}{dx}\cdot x^2 - (x^2-3)\cdot\frac{d(x^2)}{dx}}{(x^2)^2} = \frac{2x\cdot x^2-(x^2-3)2x}{(x^2)^2} = \frac{\cancel{2x^3}-\cancel{2x^3}+6x}{(x^2)^2}$$

$$f''(x) = \frac{6x}{x^4} = \frac{6}{x^3}$$

$$\frac{6}{x^3} \geq 0 \to x^3 > 0 \to x > 0$$



$$f(x)=\frac{x^2+2x-15}{x^2-9}=\frac{(x+5)(x-3)}{(x+3)(x-3)}$$

DOMINIO

$(x+3)(x-3)\neq 0$

$x+3\neq 0 \rightarrow x\neq -3$

$x-3\neq 0 \rightarrow x\neq +3$

$D_f = x\in\mathbb{R}: x\neq -3, +3$

$\mathbb{R}-\{-3, 3\}$

$(-\infty,-3)\cup(-3,3)\cup(3,+\infty)$

$$f(x)=\frac{x+5}{x+3}$$

INTERSEZIONE

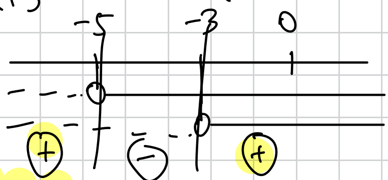
$\begin{cases} x=0 \\ y=\dfrac{x+5}{x+3} \end{cases} \rightarrow y=\dfrac{0+5}{0+3}=\dfrac{5}{3}$

$A\left(0, \frac{5}{3}\right)$

$\begin{cases} y=0 \\ y=\dfrac{x+5}{x+3} \end{cases} \rightarrow \dfrac{x+5}{x+3}=0 \rightarrow x+5=0 \rightarrow x=-5$

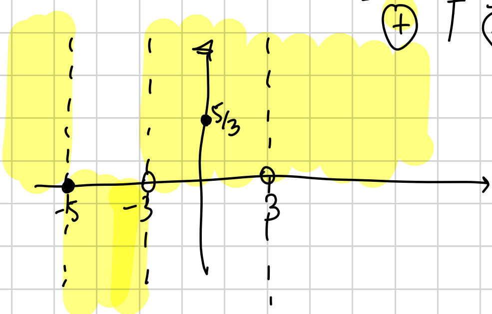
$B(-5, 0)$

SEGNO $\rightarrow f(x)>0 \rightarrow \dfrac{x+5}{x+3}>0 \rightarrow \begin{cases} x+5>0 \rightarrow x>-5 \\ x+3>0 \rightarrow x>-3 \end{cases}$



$f(x)>0$ quando:

$x<-5, x>-3$



ASINTOTI

$$\lim_{x\to -3^+} \frac{x+5}{x+3} = \frac{-3+5}{-3+3} = \frac{2}{0^+} = +\infty \longrightarrow \text{AS. VERT. DX}$$

$$\lim_{x\to -3^-} \frac{x+5}{x+3} = \frac{-3+5}{-3+3} = \frac{2}{0^-} = -\infty \longrightarrow \text{AS. VERT. SX}$$

$x=-3$
AS. VERT.

$$\lim_{x\to 3^+} \frac{x+5}{x+3} = \frac{3+5}{3+3} = \frac{8}{6} = \frac{4}{3}$$

$$\lim_{x\to 3^-} \frac{x+5}{x+3} = \frac{3+5}{3+3} = \frac{4}{3}$$

$\longrightarrow$ no as. verticale

$$\lim_{x\to -\infty} \frac{x+5}{x+3} = \frac{\infty}{\infty}^{\text{F.I.}} \longrightarrow \frac{\cancel{x}\left(1+\cancel{\frac{5}{x}}^{\,0}\right)}{\cancel{x}\left(1+\cancel{\frac{3}{x}}_{\,0}\right)} = \frac{1}{1} = 1$$

$$\lim_{x\to +\infty} \frac{x+5}{x+3}$$

$y=1$
AS. ORIZZ.

NO AS. OBLIQUI

MAX & MIN

$$\left(\frac{f}{g}\right)' = \frac{f'g - f\cdot g'}{g^2}$$

$$f'(x) = \frac{d}{dx}\left(\frac{x+5}{x+3}\right) = \frac{\frac{d(x+5)}{dx}(x+3) - (x+5)\cdot\frac{d(x+3)}{dx}}{(x+3)^2} = \frac{1(x+3)-(x+5)1}{(x+3)^2}$$

$$= \frac{\cancel{x}+3-\cancel{x}-5}{(x+3)^2} = \frac{-2}{(x+3)^2}$$

$$f'(x)>0 \rightarrow \frac{-2}{(x+3)^2}>0 \longrightarrow \text{mai} \longrightarrow f(x) \text{ è sempre decresc.}$$

$f(x)<0$ sempre

CONCAVITÀ

$$f''(x) = \frac{d}{dx}\left(-\frac{2}{(x+3)^2}\right) \rightarrow (f/g)' = \frac{f'g - fg'}{g^2} \qquad g = (m \circ n)' \quad m'(n) \cdot n'$$

$$= \frac{\cancel{\frac{d(-2)}{dx}}^{=0} \cancel{(x+3)^2} - (-2) \cdot \frac{d(x+3)^2}{dx}}{(x+3)^4} \qquad y = x+3 \quad y^2 \rightarrow 2y^{2-1} = 2y \quad 2(x+3)$$

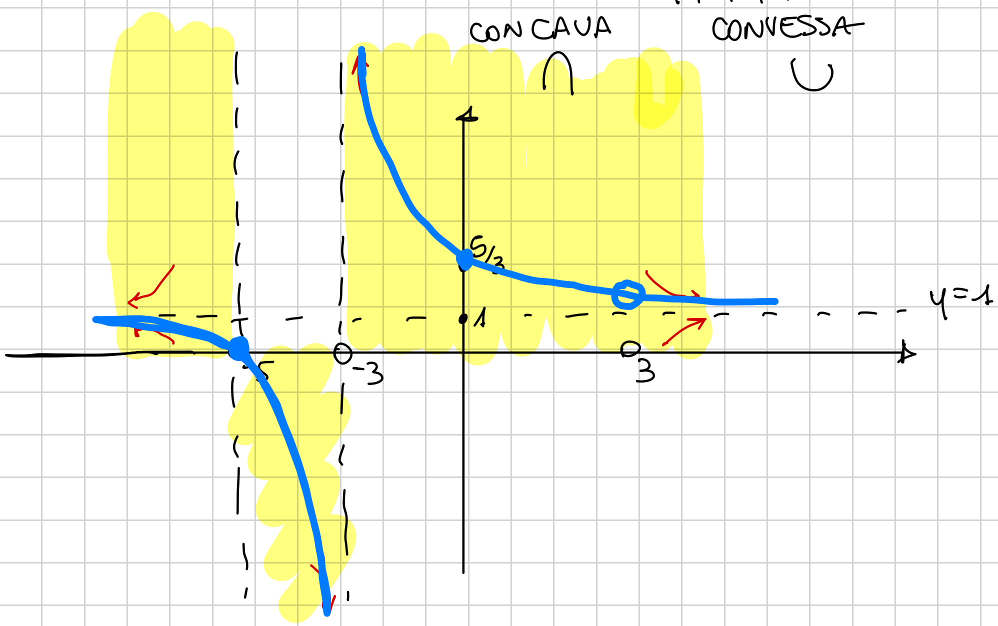
$$= \frac{+2 \cdot 2(x+3)}{(x+3)^4} = \frac{4\cancel{(x+3)}}{(x+3)^{\cancel{4}3}} = \frac{4}{(x+3)^3} = f''(x)$$

$$f'' > 0 \rightarrow \frac{4}{(x+3)^3} > 0 \rightarrow (x+3)^3 > 0 \rightarrow x+3 > 0 \rightarrow x > -3$$

-3

– – – – – – + + + + + + + +

CONCAVA ∩ CONVESSA ∪



$$f(x) = \frac{7}{x^2+1} - 3$$

DOMINIO $\rightarrow$ $x^2+1 \neq 0$ $\quad x^2 \neq -1 \rightarrow$ sempre $\rightarrow D_f = \mathbb{R}$

INTERSEZIONI $\rightarrow$ $\begin{cases} x=0 \\ y = \frac{7}{x^2+1} - 3 \end{cases}$ $\qquad \begin{cases} y=0 \\ y = \frac{7}{x^2+1} - 3 \end{cases} \rightarrow \frac{7}{x^2+1} - 3 = 0$

$\downarrow$

$y = \frac{7}{0+1} - 3 = 4$

$(\cancel{x^2+1})\ \frac{7}{\cancel{x^2+1}} = 3\ (x^2+1)$

$7 = 3x^2 + 3$

$3x^2 - 4 = 0$

$x = \pm \frac{2}{\sqrt{3}}$

$* \ \frac{2}{\sqrt{3}} \simeq 1.15$

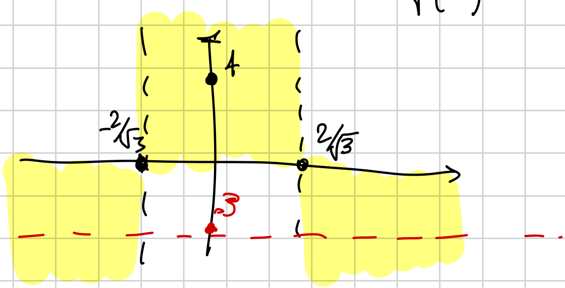
1) $A(0, 4)$
2) $B\left(-\frac{2}{\sqrt{3}}, 0\right)$
3) $C\left(\frac{2}{\sqrt{3}}, 0\right)$

SEGNO $\rightarrow f(x) > 0 \rightarrow \frac{7}{x^2+1} - 3 > 0$

$\frac{7 - 3x^2 - 3}{x^2+1} > 0 \qquad \frac{-3x^2+4}{x^2+1} > 0$

$\begin{cases} -3x^2 + 4 > 0 \rightarrow 3x^2 - 4 < 0 \rightarrow \boxed{-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}} \\ x^2 + 1 > 0 \rightarrow \text{sempre} \end{cases}$

$f(x) > 0$ quando



ASINTOTI

$$\lim_{x\to\pm\infty} \frac{4-3x^2}{x^2+1} = \frac{\infty}{\infty} \text{ f.i.} = \frac{-3}{1} = -3 \longrightarrow y=-3 \rightarrow \text{AS. ORIZZ.}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

MONOTONIA

$$f'(x) = \frac{d}{dx}\left(\frac{4-3x^2}{x^2+1}\right) = \frac{\frac{d(4-3x^2)}{dx}\cdot(x^2+1) - (4-3x^2)\cdot\frac{d(x^2+1)}{dx}}{(x^2+1)^2} =$$

$$\frac{-6x(x^2+1)-(4-3x^2)\cdot 2x}{(x^2+1)^2} = \frac{\cancel{-6x^3}-6x-8x+\cancel{6x^3}}{(x^2+1)^2} = \frac{-14x}{(x^2+1)^2}$$

$$f'(x)>0 \rightarrow \frac{-14x}{(x^2+1)^2}>0 \quad \begin{cases} -14x>0 \rightarrow x<0 \\ (x^2+1)^2>0 \rightarrow \text{sempre} \end{cases}$$

0

0 punto di max

$$f(0) = \frac{4-3\cdot 0^2}{0+1} = 4 \longrightarrow \max(0,4)$$

CONCAVITÀ

$$f''(x) = \frac{d\left(\frac{-14x}{(x^2+1)^2}\right)}{dx} = \frac{\frac{d(-14x)}{dx}\cdot(x^2+1)^2-(-14x)\cdot\frac{d(x^2+1)^2}{dx}}{(x^2+1)^4}$$

$$= \frac{-14(x^2+1)^2+14x\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = \frac{-14(x^2+1)^2+14x\cdot 4x(x^2+1)}{(x^2+1)^4}$$

$$= \frac{-14(-3x^2+1)}{(x^2+1)^3}$$

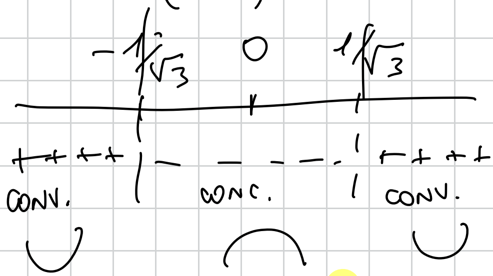
$f'' > 0 \qquad -\dfrac{14(-3x^2+1)}{(x^2+1)^3} > 0$

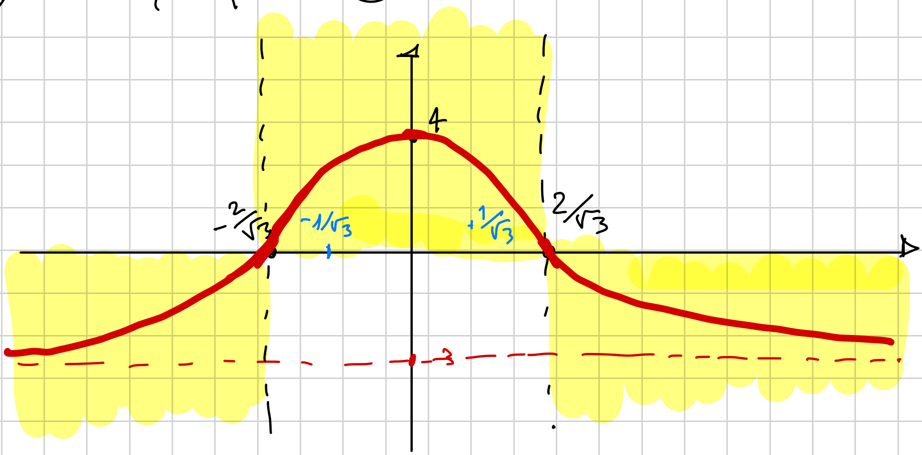
$-3x^2+1 > 0 \to \underline{3x^2-1<0}$

$x^2+1>0 \to$ sempre

$x < -\dfrac{1}{\sqrt{3}}$

$x > \dfrac{1}{\sqrt{3}}$





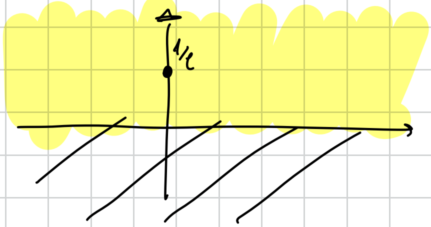
$f(x)=e^{x^2-1}$

DOMINIO $\rightarrow \mathbb{R}$

INTERSEZIONI $\rightarrow$ $\begin{cases} x=0 \\ y=e^{x^2-1} \end{cases} \rightarrow y=e^{0-1}=e^{-1}=\frac{1}{e} \rightarrow A\left(0,\frac{1}{e}\right)$

$\cancel{\begin{cases} y=0 \\ y=e^{x^2-1} \end{cases}}$

SEGNO $\rightarrow f(x)>0 \rightarrow e^{x^2-1}>0 \rightarrow$ sempre



ASINTOTI

$\lim_{x\to\pm\infty} e^{x^2-1} = e^{+\infty} = +\infty \rightarrow$ NO AS. ORIZZ.

$\boxed{+\infty}$

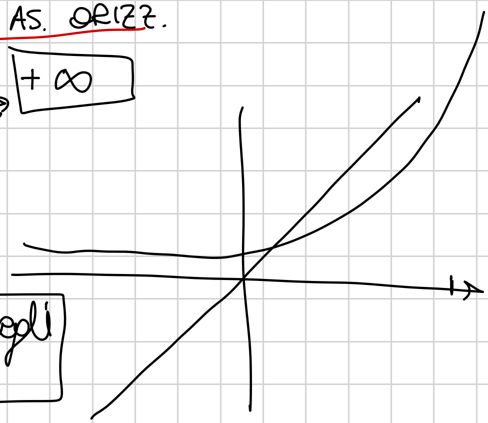
OBLIQUO?

$\lim_{x\to\pm\infty} \frac{e^{x^2-1}}{x} = \boxed{\frac{e^{\infty}}{\infty}} = \frac{\infty}{\infty}$ f.i.

$\boxed{\text{gerarchia degli infiniti}}$

$+\infty \neq m$

NO AS. OBLIQUO



MONOTONIA

$(f \circ g)' = f'(g) \cdot g'$

$f'(x) > 0 \rightarrow f'(x) = \dfrac{d(e^{x^2-1})}{dx}$

$= \boxed{e^{x^2-1} \cdot 2x}$

$2x \cdot e^{x^2-1} > 0$

$f'(x) = 0 \rightarrow x = 0$ punto stazionario

$\begin{cases} 2x > 0 \rightarrow x > 0 \\ e^{x^2-1} > 0 \rightarrow \text{sempre } + \end{cases}$

0

- - - - - - - + + + + + + + +

$x = 0$ punto di min.

$\hookrightarrow f(0) = 1/e$

$\min(0, 1/e)$

CONCAVITÀ

$(f \cdot g)' = f'g + fg'$ (g = composta $\rightarrow (u \circ u)' = u'(u) \cdot u'$)

$f''(x) = \left(2x \cdot e^{x^2-1}\right)' =$

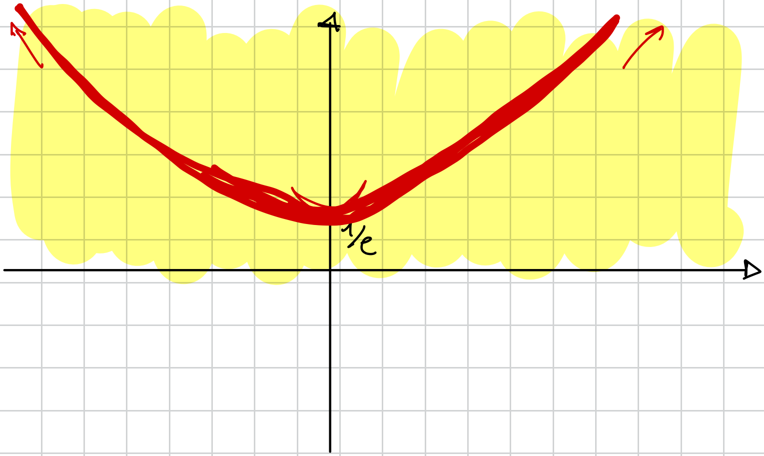
$\dfrac{d(2x)}{dx} \cdot e^{x^2-1} + 2x \cdot \dfrac{d(e^{x^2-1})}{dx}$

$= 2 \cdot e^{x^2-1} + 2x \cdot e^{x^2-1} \cdot 2x = 2e^{x^2-1} + 4x^2 \cdot e^{x^2-1}$

$\boxed{e^{x^2-1}(2+4x^2)}$

$e^{x^2-1}(2+4x^2) > 0 \quad \begin{cases} e^{x^2-1} > 0 \rightarrow \text{sempre} \\ 2+4x^2 > 0 \rightarrow \text{sempre} \end{cases}$

$f''(x) > 0$ sempre $\rightarrow$ sempre convessa

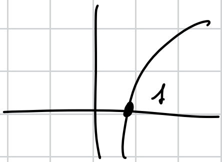

1/e

$f(x) = \ln(x^2+1)$

DOMINIO $\longrightarrow$ $x^2+1>0$ $\quad x^2>-1 \rightarrow$ sempre $\rightarrow$ $D_f = \mathbb{R}$

INTERSEZIONI $\rightarrow$ $\begin{cases} x=0 \\ y=\ln(x^2+1) \end{cases} \rightarrow y=\ln(0+1)=0$

$A(0,0)$

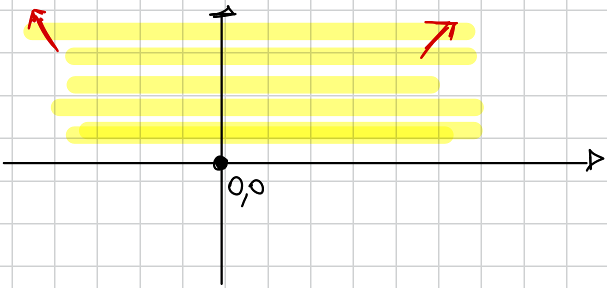


$\begin{cases} y=0 \\ y=\ln(x^2+1) \end{cases} \rightarrow \ln(x^2+1)=0$

$x=0$

SEGNO $\rightarrow$ $f(x)>0 \rightarrow \ln(x^2+1)>0$

$x^2+1>1 \rightarrow x^2>0 \quad x\neq 0$



ASINTOTI

NO ORIZZ.

$$\lim_{x\to+\infty} \ln(x^2+1) = +\infty \quad // \quad \lim_{x\to-\infty} \ln(x^2+1) = +\infty$$

$$\lim_{x\to\pm\infty} \frac{\ln(x^2+1)}{x} = \frac{\infty}{\infty} \quad \text{F.I.}$$

↳ de l'Hôpital: $\lim_{x\to\pm\infty} \dfrac{\frac{1}{x^2+1}\cdot 2x}{1} =$

$$\lim_{x\to\pm\infty} \frac{2x}{x^2+1} = 0 \neq m$$ $\rightarrow$ no as. obliquo!

MONOTONIA

$$(f \circ g)' = f'(g) \cdot g'$$

$$f'(x) > 0 \longrightarrow f' = \frac{d(\ln(x^2+1))}{dx} = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

$$\frac{2x}{x^2+1} > 0 \quad \begin{cases} 2x > 0 \longrightarrow x > 0 \\ x^2+1 > 0 \longrightarrow \text{sempre} \\ x^2 > -1 \end{cases}$$

0

- - - - - - - | + + + + + + + + +

$x = 0$ punto di minimo

CONCAVITÀ

$$f''(x) = \frac{d\left(\frac{2x}{x^2+1}\right)}{dx} = \frac{\frac{d(2x)}{dx} \cdot (x^2+1) - 2x \cdot \frac{d(x^2+1)}{dx}}{(x^2+1)^2} =$$

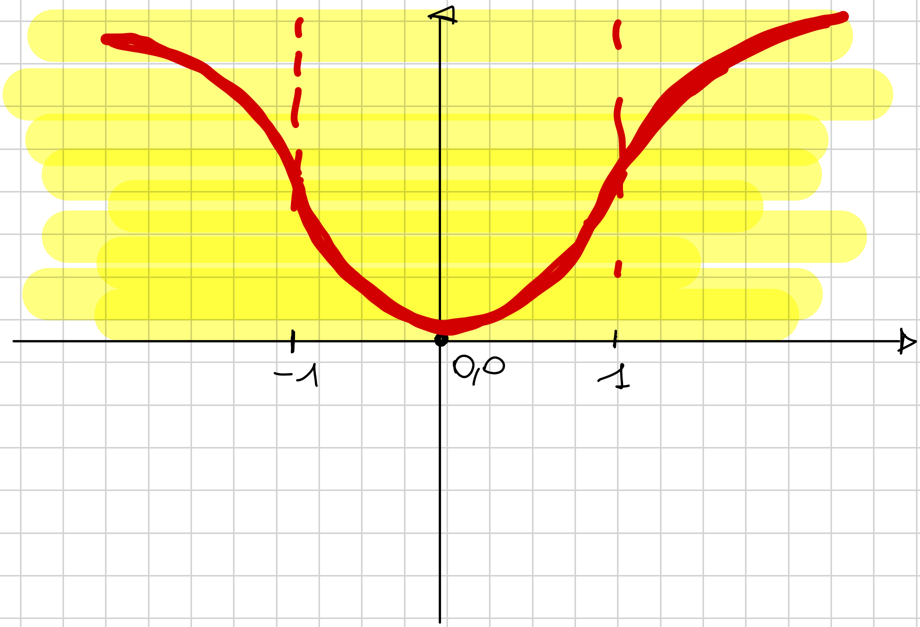
$$f'' \quad \frac{-2x^2+2}{(x^2+1)^2} > 0$$

$$-2x^2+2 > 0 \longrightarrow 2x^2-2 < 0 \longrightarrow -1 < x < 1$$

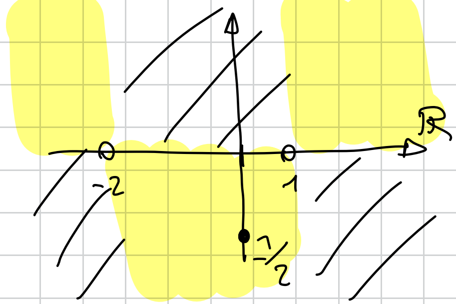
-1   0   1

- - - - - + + + + + + + - - - - -

$\cap \quad \cup \quad \cap$


-1
0,0
1

$$f(x) = \frac{1}{x^2+x-2}$$



DOMINIO

$$x^2+x-2 \neq 0 \rightarrow x \neq 1 \ , \ x \neq -2$$

$$D_f = \{x \in \mathbb{R} : x \neq 1, x \neq -2\}$$

$$(-\infty, -2) \cup (-2, 1) \cup (1, +\infty)$$

INTERSEZIONI

$y \neq 0$

↳ no intersez. x

$$\frac{P_n}{Q_m} = 0 \rightarrow \begin{matrix} P_n = 0 \\ Q_m \neq 0 \end{matrix} \qquad \frac{1 \rightarrow \neq 0}{Q_m \neq 0}$$

$$\begin{cases} x = 0 \\ y = \frac{1}{x^2+x-2} \end{cases} \rightarrow y = \frac{1}{0+0-2} = -\frac{1}{2} \longrightarrow A\left(0, -\frac{1}{2}\right)$$

SEGNO

$$f(x) > 0 \rightarrow \frac{1}{x^2+x-2} > 0 \rightarrow x^2+x-2 > 0 \rightarrow \begin{matrix} x < -2 \\ x > 1 \end{matrix}$$

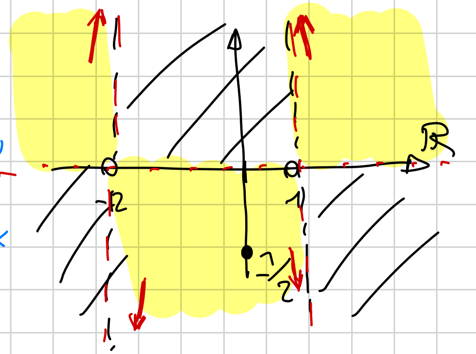
ASINTOTI

$$\lim_{x\to\pm\infty} \frac{1}{x^2+x-2} = \frac{1}{\infty} = 0 \rightarrow \text{AS. ORIZZ. } y=0$$

$$\lim_{x\to-2^+} \frac{1}{x^2+x-2} = \frac{1}{(-2^+)^2-2^+-2} = -\infty$$

$$\lim_{x\to-2^-} \frac{1}{x^2+x-2} = \frac{1}{0^-} \overset{1/0}{=} +\infty$$

AS. VERTIC. $x = -2$



$$\lim_{x\to+1^+} \frac{1}{x^2+x-2} = \frac{1}{1^2+1-2} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to+1^-} \frac{1}{x^2+x-2} = \frac{1}{1^2+1-2} = \frac{1}{0^-} = -\infty$$

AS. VERT. $x = +1$

NO AS. OBLIQUI

MONOTONIA

$$f'(x) = \frac{d\left(\frac{1}{x^2+x-2}\right)}{dx} \rightarrow \left(f/g\right)' = \frac{f'g - fg'}{g^2}$$

$$= \frac{\cancel{\frac{d(1)}{dx}\cdot(x^2+x-2)} - 1\cdot\frac{d(x^2+x-2)}{dx}}{(x^2+x-2)^2}$$

$$= -\frac{(2x+1)}{(x^2+x-2)^2} \rightarrow f'(x)=0 \rightarrow \text{con } x=-\frac{1}{2} \rightarrow \text{punto stazion.}$$

$f'(x)>0 \rightarrow$ $\quad x<-\frac{1}{2}$

$-\frac{1}{2}$

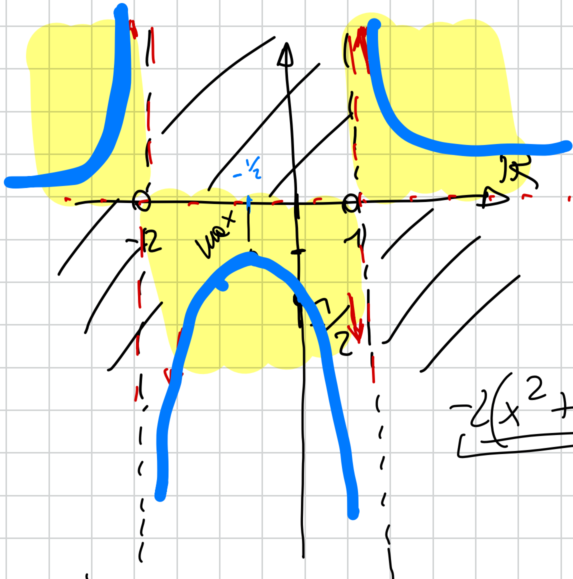
+ + + + + + − − − − − −

↗ ↘

$f'(x)<0 \rightarrow x>-\frac{1}{2}$

punto di max

max $\left(-\frac{1}{2}, -\frac{4}{9}\right)$



$$2(x^2+x-2)\cdot(2x+1)$$

$$-2(x^2+x-2) + |4x(x^2+x-2)|$$

CONCAVITÀ

$$f''(x) = \frac{d\left(\frac{-(2x+1)}{(x^2+x-2)^2}\right)}{dx} \to \frac{-\frac{d(+2x+1)}{dx}\cdot(x^2+x-2)^2 + (2x+1)\cdot\ \frac{d}{dx}}{(x^2+x-2)^4}$$

$$= \frac{6(x^2+x+1)}{(x^2+x-2)^3} \neq 0 \quad \text{?FLESSO?} \qquad 1^2-4\cdot1\cdot1 \to \Delta<0$$

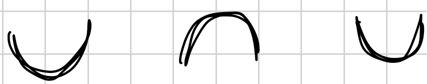
$$f''>0 \quad \frac{6(x^2+x+1)}{(x^2+x-2)^3}>0 \to \quad x<-2,\ x>1$$

$$\hookrightarrow x^2+x+1>0 \to \forall x\in\mathbb{R}$$

$$x^2+x-2>0 \to \boxed{x<-2,\ x>1}$$

$-2 \qquad\qquad 1$

$+ + + + + \quad - - - - - \quad + + + + +$

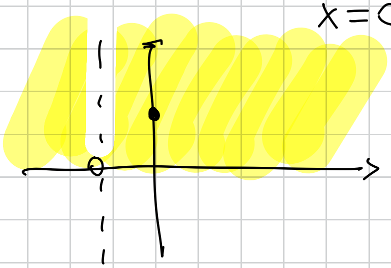


$$f(x) = e^{\frac{x}{x+1}}$$

DOMINIO → $x \neq -1$ $\quad D_f = \{x \in \mathbb{R} : x \neq -1\}$

INTERSEZIONI → $y \neq 0$

$x = 0 \rightarrow y = 1 \quad A(0,1)$



$$\frac{\infty}{\infty} \Rightarrow \frac{1}{1} = 1$$

SEGNO $f(x) > 0 \rightarrow \forall x \in \mathbb{R},\ x \neq -1$

$$\lim_{x\to\pm\infty} \frac{x}{x+1} = 1$$

ASINTOTI → $\displaystyle\lim_{x\to\pm\infty} e^{\frac{x}{x+1}} = e^{\lim_{x\to\pm\infty}\frac{x}{x+1}} = e^{1} = e$

$$\lim_{x\to -1^+} e^{\frac{x}{x+1}} = 0$$

$$\lim_{x\to -1^-} e^{\frac{x}{x+1}} = +\infty$$

MONOTONIA

$f'(x) = 0$

> 0

< 0

$$\rightarrow \frac{d\left(e^{\frac{x}{x+1}}\right)}{dx} = e^{\frac{x}{x+1}} \cdot \frac{\frac{dx}{dx}\cdot(x+1) - x\cdot\frac{d(x+1)}{dx}}{(x+1)^2}$$

$$= e^{\frac{x}{x+1}} \cdot \frac{1}{(x+1)^2} = \frac{e^{\frac{x}{x+1}}}{(x+1)^2}$$

$f'(x) > 0 \quad \forall x \in D$

↳ sempre crescente

CONCAVITÀ

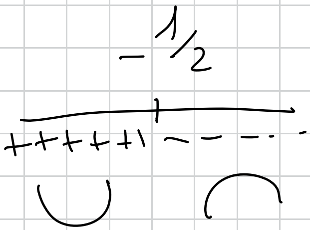
$f''(x) \gtrless 0$

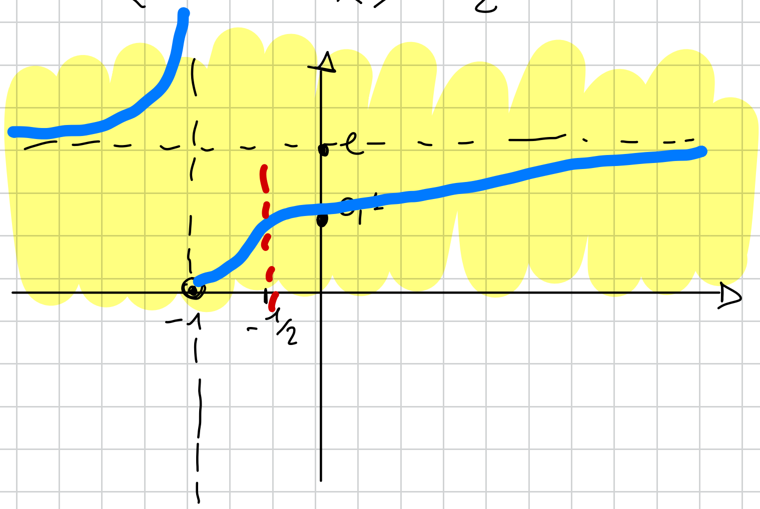
$$\frac{d\left(e^{\frac{x}{x+1}} \cdot \frac{1}{(x+1)^2}\right)}{dx} \qquad \frac{1}{(x+1)^2} \to (x+1)^{-2} \qquad (f\cdot g)' = f'g + fg'$$

$$f'' = -\frac{e^{\frac{x}{x+1}}(2x+1)}{(x+1)^4} = 0 \to x = -\frac{1}{2} \to \text{flesso}$$

$$-\frac{e^{\frac{x}{x+1}}(2x+1)}{(x+1)^4} > 0 \to x < -\frac{1}{2}$$

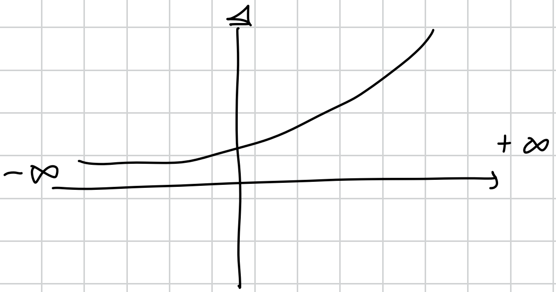
$$< 0 \to x > -\frac{1}{2}$$





$$e^{\lim_{x \to -1^+} \left(\frac{x}{x+1}\right)} = \frac{-1^+}{-1^+ + 1} = \frac{-1}{0^+} = -\infty \longrightarrow e^{-\infty} = 0$$

$$e^{\lim_{x \to -1^-} \left(\frac{x}{x+1}\right)} = \frac{-1^-}{-1^- + 1} = \frac{-1}{0^-} = +\infty \longrightarrow e^{+\infty} = +\infty$$



$$f(x) = x^3 \cdot (\ln x - 1)$$

$$f(x) = 1 - \sqrt{\frac{x}{x+1}}$$

$$x^3 - 3x + 2 > 0$$

$$(x-1)\frac{(x^3-3x+2)}{x-1} \rightarrow (x-1)(x^2+x-2) \rightarrow (x-1)(x-1)(x+2) =$$

$$(x-1)^2(x+2)$$