$$f(x) = \frac{x^2-1}{x-2}$$

① DOMINIO $\longrightarrow$ $x-2 \neq 0 \rightarrow x \neq 2 \rightarrow D_f = \{\forall x \in \mathbb{R} : x \neq 2\}$

$\mathbb{R} - \{2\}$

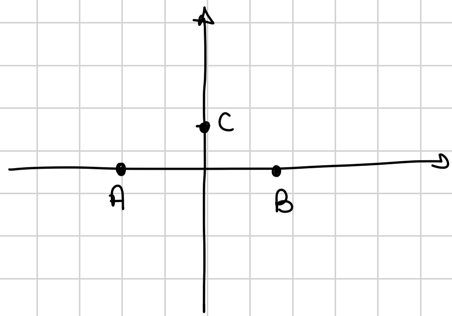
② INTERSEZIONI $\rightarrow \begin{cases} y = \dfrac{x^2-1}{x-2} \\ x = 0 \end{cases}$ $\qquad \begin{cases} y = \dfrac{x^2-1}{x-2} \\ \boxed{y=0} \end{cases}$

$$y = \frac{0^2-1}{0-2}$$

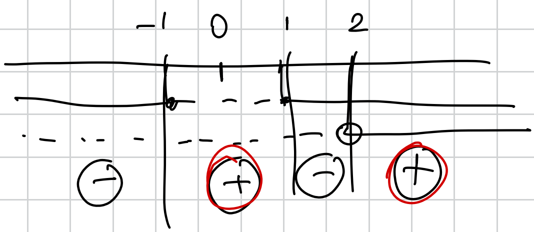
$$y = 1/2$$

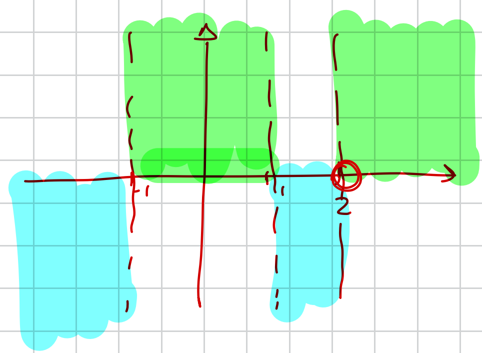
$$\frac{x^2-1}{x-2} = 0 \quad x = \pm 1$$

$A(-1,0)$ $\quad B(1,0)$ $\quad C(0, 1/2)$

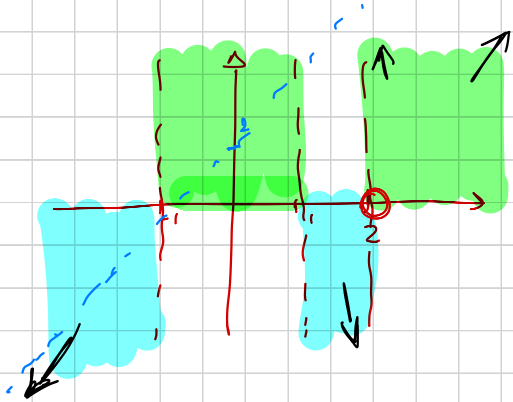


③ SEGNO $\longrightarrow$ $\dfrac{x^2-1}{x-2} \geq 0 \rightarrow \begin{cases} x^2-1 \geq 0 \longrightarrow x \leq -1,\ x \geq 1 \\ x-2 > 0 \longrightarrow x > 2 \end{cases}$





4 ASINTOTI



$$\lim_{x\to+\infty} \frac{x^2-1}{x-2} = \lim_{x\to+\infty} \frac{\cancel{x^2}\left(1-\cancel{\frac{1}{x^2}}\right)}{\cancel{x}\left(1-\cancel{\frac{2}{x}}\right)} = +\infty$$

$$\lim_{x\to-\infty} \frac{x^2-1}{x-2} = -\infty$$

$$\left.\begin{aligned} &\lim_{x\to 2^+} \frac{x^2-1}{x-2} = \frac{3}{0^+} = +\infty \\ &\lim_{x\to 2^-} \frac{x^2-1}{x-2} = \frac{3}{0^-} = -\infty \end{aligned}\right] \text{AS. VERT.}$$

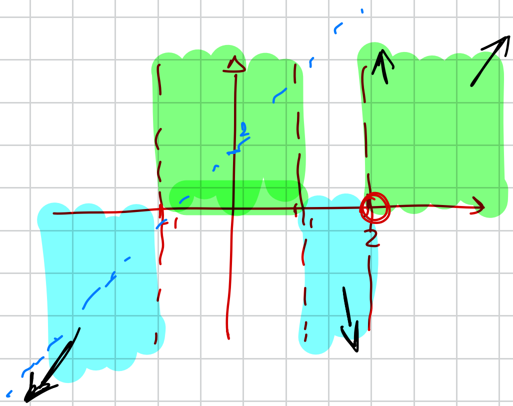
AS. OBL. $\rightarrow y = x+2$

$$\lim_{x\to+\infty} \frac{x^2-1}{x-2} \cdot \frac{1}{x} = 1 = m$$

$$\lim_{x\to+\infty} \frac{x^2-1}{x-2} - x = 2$$

5) MASSIMI E MINIMI

$$f(x)=\frac{x^2-1}{x-2} \to f'(x)=\frac{x^2-4x+1}{(x-2)^2}$$



$$\frac{x^2-4x+1}{(x-2)^2}=0 \Leftrightarrow \text{punti staz.?}$$

$$x_1=2+\sqrt{3} \qquad x_2=2-\sqrt{3}$$

$$y_1=4+2\sqrt{3}\cong 7.46$$

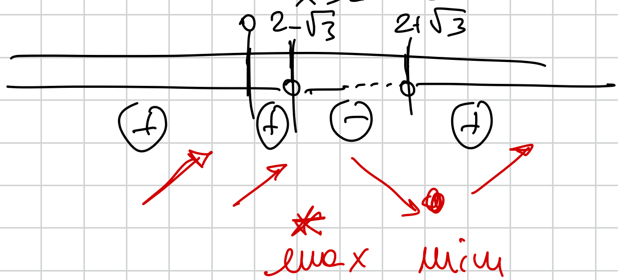
$$y_2=4-2\sqrt{3}\cong 0.54$$

$$\frac{x^2-4x+1}{(x-2)^2}>0$$

$x^2-4x+1>0$

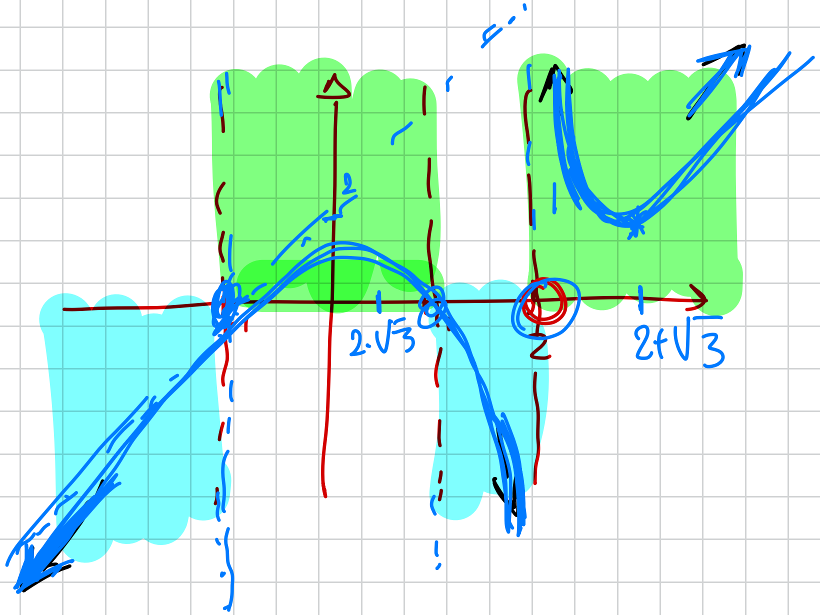
$(x-2)^2>0 \to$ sempre

$x^2-4x+1>0$ → $x<2-\sqrt{3}$; $x>2+\sqrt{3}$

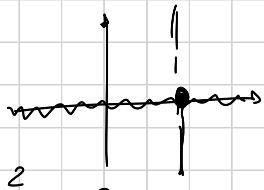


6) CONCAVITÀ

$$f''(x)=\frac{6x-12}{(x-2)^4} \to =0,\ \boxed{x=2} \to$$ NO FLESSO, $\notin D_f$


2
2-√3
2
2+√3

$f(x) = x^3 - 2x^2 - x + 2$



① DOMINIO $\longrightarrow \mathbb{R}$

② INTERSEZ. $\rightarrow \begin{cases} y = x^3 - 2x^2 - x + 2 \\ y = 0 \end{cases}$

$\underbrace{x^3 - 2x^2 - x + 2 = 0}$

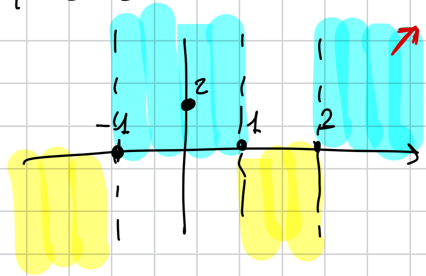
$\begin{cases} y = x^3 - 2x^2 - x + 2 \\ x = 0 \end{cases}$

$\underbrace{\qquad}$

$y = 0^3 - 2 \cdot 0^2 - 0 + 2$

A $(0,2)$

B $(-1,0)$ C $(1,0)$ D $(2,0)$



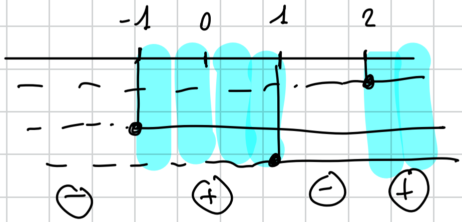
③ SEGNO $f(x)$

$f(x) \geq 0 \quad x^3 - 2x^2 - x + 2 \geq 0 \rightarrow (x-2)(x+1)(x-1) \geq 0$

$x - 2 \geq 0 \rightarrow x \geq 2$

$x + 1 \geq 0 \rightarrow x \geq -1$

$x - 1 \geq 0 \rightarrow x \geq 1$



④ ASINTOTI

ORIZZ. ?

$\lim_{x \to +\infty} x^3 - \cancel{2x^2} - \cancel{x} + \cancel{2} = +\infty - \infty - \infty + 2 =$

$= \boxed{+\infty} \longrightarrow$ NO AS ORIZZ

$\lim_{x \to -\infty} x^3 - 2x^2 - x + 2 = -\infty$

OBLIQUI ?

$\lim_{x \to \pm\infty} \frac{x^3 - 2x^2 - x + 2}{x} = \lim \frac{x^{\cancel{3}\,2}\left(1 - \frac{2}{x} - \frac{1}{x^2} + \frac{2}{x^3}\right)}{\cancel{x}} = \lim_{x \to \pm\infty} x^2 = +\infty$

$\neq l$

NO AS. OBLIQUO

$f(-0.21)$

$f(1.5)$

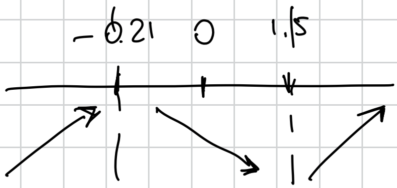
$(f \cdot g)' = f'g + fg'$

MAX & MIN

$f'(x) = 3x^2 - 4x - 1 \geq 0$

$x \leq -\dfrac{\sqrt{7}+2}{3} \cong -0.21$

$x \geq \dfrac{\sqrt{7}+2}{3} \cong 1.5$

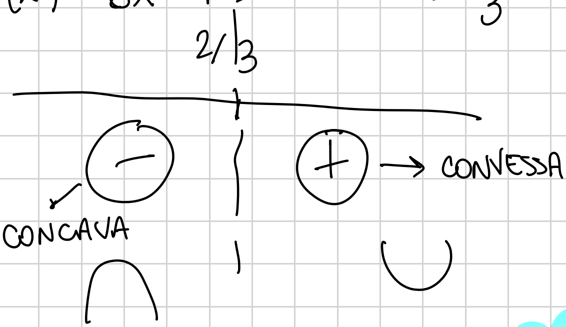


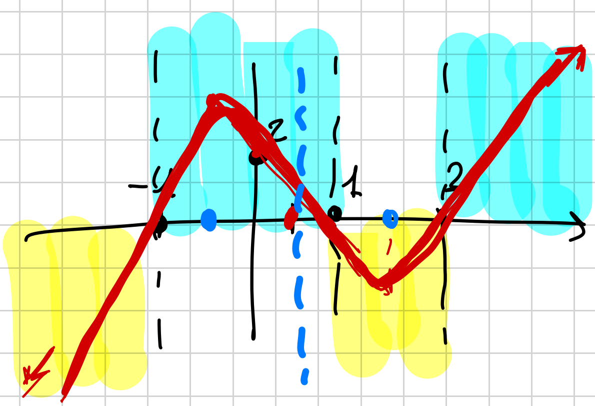
$\longrightarrow$ $-0.21 \rightarrow$ max

$1.5 \rightarrow$ min

CONCAVITÀ

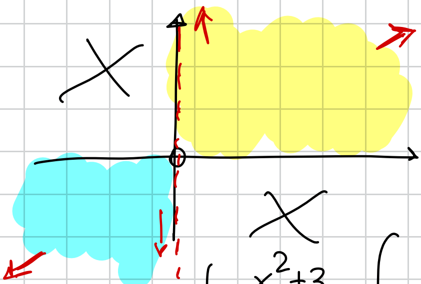
$f''(x) = 6x - 4 \geq 0 \rightarrow x \geq 2/3 \cong 0.67$





$$f(x) = \frac{x^2+3}{x}$$

DOMINIO $\rightarrow x \neq 0 \rightarrow D_f = \{x \in \mathbb{R} : x \neq 0\} = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$



INTERSEZ. $\rightarrow \begin{cases} y = \frac{x^2+3}{x} \\ y = 0 \end{cases}$ $\quad \begin{cases} y = \frac{x^2+3}{x} \\ x = 0 \end{cases}$

$\frac{x^2+3}{x} = 0$ $\quad y = \frac{0^2+3}{0} \rightarrow$ no soluz.

y no intersez.

no soluz.

no intersez. con x

SEGNO $\quad f(x) > 0 \quad \frac{x^2+3}{x} > 0 \quad$ $x^2+3>0 \rightarrow$ sempre

$x > 0$

quando $x > 0$

ASINTOTI

$\lim_{x \to 0^+} \frac{x^2+3}{x} = \frac{3}{0^+} = +\infty \quad \lim_{x \to 0^-} \frac{x^2+3}{x} = \frac{3}{0^-} = -\infty$ AS. VERT. $x = 0$

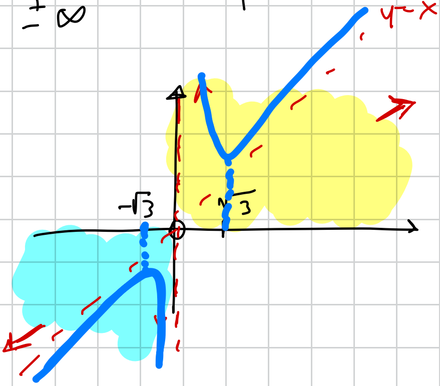
$\lim_{x \to +\infty} \frac{x^2+3}{x} = +\infty \quad \lim_{x \to -\infty} \frac{x^2+3}{x} = -\infty$ NO AS. ORIZZ.

$\hookrightarrow \frac{\cancel{x^2}\left(1+\frac{3}{x^2}\right)}{\cancel{x}}$ $\lim_{x \to +\infty}$ $x = +\infty$

$$\lim_{x\to\pm\infty} \frac{x^2+3}{x}\cdot\frac{1}{x} = \lim_{x\to\pm\infty}\frac{x^2+3}{x^2} = \frac{1}{1} = 1 = m$$

$$\lim_{x\to\pm\infty} \frac{x^2+3}{x} - x = \lim_{x\to\pm\infty} \frac{\cancel{x^2}+3-\cancel{x^2}}{x} = \frac{3}{\pm\infty} = 0 = q$$

AS. OBL. $\rightarrow$ $y = x$



MAX & MIN

$$(f/g)' = \frac{f'g - fg'}{g^2}$$

$f'(x) =$

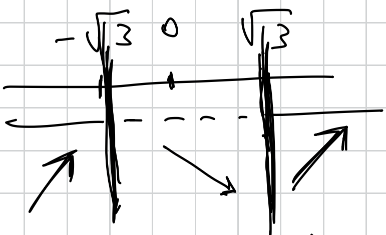
$$\frac{\frac{d}{dx}(x^2+3)\cdot x - (x^2+3)\cdot\frac{dx}{dx}}{x^2} = \frac{2x\cdot x-(x^2+3)}{x^2} = \frac{2x^2-x^2-3}{x^2} = \frac{x^2-3}{x^2}$$

$$\frac{x^2}{x^2} - \frac{3}{x^2} \Longrightarrow \boxed{1-\frac{3}{x^2}}$$

$$f'(x) = 1-\frac{3}{x^2} > 0 \rightarrow \frac{x^2-3}{x^2} > 0$$

$x > \sqrt{3}$

$x < -\sqrt{3}$



$max\,x = -\sqrt{3}$

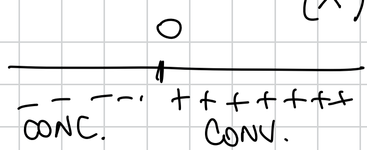
$min = +\sqrt{3}$

$$f''(x) = \frac{d}{dx}\left(\frac{x^2-3}{x^2}\right) = \frac{\frac{d(x^2-3)}{dx}\cdot x^2 - (x^2-3)\cdot\frac{d(x^2)}{dx}}{(x^2)^2} = \frac{2x\cdot x^2-(x^2-3)2x}{(x^2)^2}$$

$$= \frac{\cancel{2x^3} - \cancel{2x^3} + 6x}{(x^2)^2}$$

$$f''(x) = \frac{6x}{x^4} = \frac{6}{x^3}$$

$$\frac{6}{x^3} \geq 0 \rightarrow x^3 > 0 \rightarrow x > 0$$



$$f(x)=\frac{x^2+2x-15}{x^2-9}=\frac{(x+5)(x-3)}{(x+3)(x-3)}$$

DOMINIO

$(x+3)(x-3)\neq 0$

$x+3\neq 0 \rightarrow x\neq -3$

$x-3\neq 0 \rightarrow x\neq +3$

$D_f = x\in\mathbb{R}: x\neq -3, +3$

$\mathbb{R}-\{-3, 3\}$

$(-\infty,-3)\cup(-3,3)\cup(3,+\infty)$

$$f(x)=\frac{x+5}{x+3}$$

INTERSEZIONE

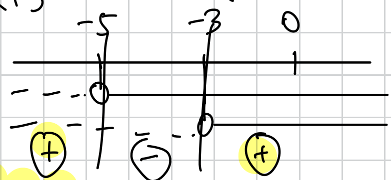
$\begin{cases} x=0 \\ y=\dfrac{x+5}{x+3} \end{cases} \rightarrow y=\dfrac{0+5}{0+3}=\dfrac{5}{3}$

$A\left(0, \frac{5}{3}\right)$

$\begin{cases} y=0 \\ y=\dfrac{x+5}{x+3} \end{cases} \rightarrow \dfrac{x+5}{x+3}=0 \rightarrow x+5=0 \rightarrow x=-5$

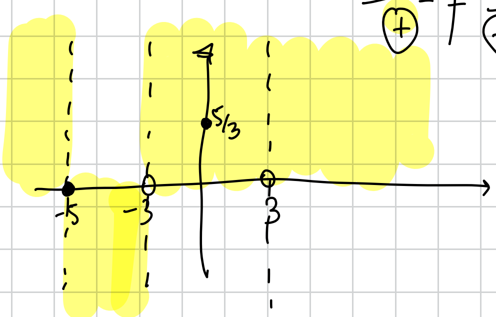
$B(-5, 0)$

SEGNO $\rightarrow f(x)>0 \rightarrow \dfrac{x+5}{x+3}>0 \rightarrow \begin{cases} x+5>0 \rightarrow x>-5 \\ x+3>0 \rightarrow x>-3 \end{cases}$



$f(x)>0$ quando:

$x<-5, \; x>-3$



ASINTOTI

$$\lim_{x\to -3^+} \frac{x+5}{x+3} = \frac{-3+5}{-3+3} = \frac{2}{0^+} = +\infty \longrightarrow \text{AS. VERT. DX}$$

$$\lim_{x\to -3^-} \frac{x+5}{x+3} = \frac{-3+5}{-3+3} = \frac{2}{0^-} = -\infty \longrightarrow \text{AS. VERT. SX}$$

$x=-3$ AS. VERT.

$$\lim_{x\to 3^+} \frac{x+5}{x+3} = \frac{3+5}{3+3} = \frac{8}{6} = \frac{4}{3}$$

$$\lim_{x\to 3^-} \frac{x+5}{x+3} = \frac{3+5}{3+3} = \frac{4}{3}$$

$\longrightarrow$ no as. verticale

$$\lim_{x\to -\infty} \frac{x+5}{x+3} = \frac{\infty}{\infty}\ \text{F.I.} \longrightarrow \frac{\cancel{x}\left(1+\cancel{5/x}^{\,0}\right)}{\cancel{x}\left(1+\cancel{3/x}_{\,0}\right)} = \frac{1}{1} = 1$$

$$\lim_{x\to +\infty} \frac{x+5}{x+3}$$

$y=1$ AS. ORIZZ.

NO AS. OBLIQUI

MAX & MIN

$$\left(\frac{f}{g}\right)' = \frac{f'g - f\cdot g'}{g^2}$$

$$f'(x) = \frac{d}{dx}\left(\frac{x+5}{x+3}\right) = \frac{\frac{d(x+5)}{dx}(x+3) - (x+5)\cdot\frac{d(x+3)}{dx}}{(x+3)^2} = \frac{1(x+3)-(x+5)1}{(x+3)^2}$$

$$= \frac{\cancel{x}+3-\cancel{x}-5}{(x+3)^2} = \frac{-2}{(x+3)^2}$$

$$f'(x)>0 \rightarrow \frac{-2}{(x+3)^2}>0 \longrightarrow \text{mai} \longrightarrow f(x)\ \text{è sempre decresc.}$$

$f'(x)<0$ sempre

CONCAVITÀ

$$f''(x) = \frac{d}{dx}\left(-\frac{2}{(x+3)^2}\right) \rightarrow (f/g)' = \frac{f'g - fg'}{g^2}$$

$g = (m \circ n)'$

$m'(n) \cdot n'$

$$= \frac{\cancel{\frac{d(-2)}{dx}}^{=0}\cancel{(x+3)^2} - (-2)\cdot\frac{d(x+3)^2}{dx}}{(x+3)^4}$$

$y = x+3$

$y^2 \rightarrow 2y^{2-1} = 2y$

$2(x+3)$

$$= \frac{+2\cdot 2(x+3)}{(x+3)^4} = \frac{4\cancel{(x+3)}}{(x+3)^{\cancel{4}3}} = \frac{4}{(x+3)^3} = f''(x)$$

$$f''>0 \rightarrow \frac{4}{(x+3)^3}>0 \rightarrow (x+3)^3>0 \rightarrow x+3>0 \rightarrow x>-3$$

-3

$- - - - - - | + + + + + + + +$

CONCAVA $\cap$ CONVESSA $\cup$