MANAGERIAL ECONOMICS

Corrado Pasquali

UNITE

5. QUICK RECAP ON CHOICE UNDER UNCERTAINTY

The sample space

When dice are rolled, we say that the set

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

of all possible outcomes is a sample space.

Events

The events that can result from rolling the dice are identified with the subsets of Ω. Thus the event that the dice shows an even number is the set $E = \{2, 4, 6\}$

Probability measures

A *probability measure* is a function defined on the set S of all possible events.

The number $\text{prob}(E)$ is said to be the probability of the event E.

To qualify as a probability measure, the function $\text{prob} : S \to [0, 1]$ must satisfy three properties.

First property

The first property is that $\mathrm{prob}(\emptyset) = 0$. Since $\emptyset$ is the set with no elements, this means that the probability of the impossible event that nothing at all will happen is zero.

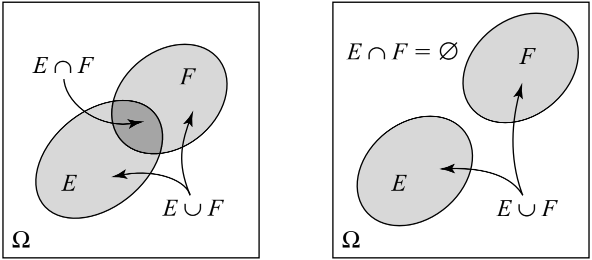
Second property

The second property is that $\mathrm{prob}(\Omega) = 1$, which means that the probability of the certain event that something will happen is 1.

Third property

The third property says that the probability that one or the other of two events will occur is equal to the sum of their separate probabilities – provided that the two events can't both occur simultaneously.

The set $E \cap F$ represents the event that both events E and F occur at the same time. So $E \cap F = 0$ means that E and F can't occur simultaneously, as in the Figure:



Third property

The set $E \cup F$ represents the event that at least one of E or F occurs.

So the third property can be expressed formally by writing

$$E \cap F = 0 \rightarrow \mathrm{prob}(E \cup F) = \mathrm{prob}(E) + \mathrm{prob}(F)$$

Third property

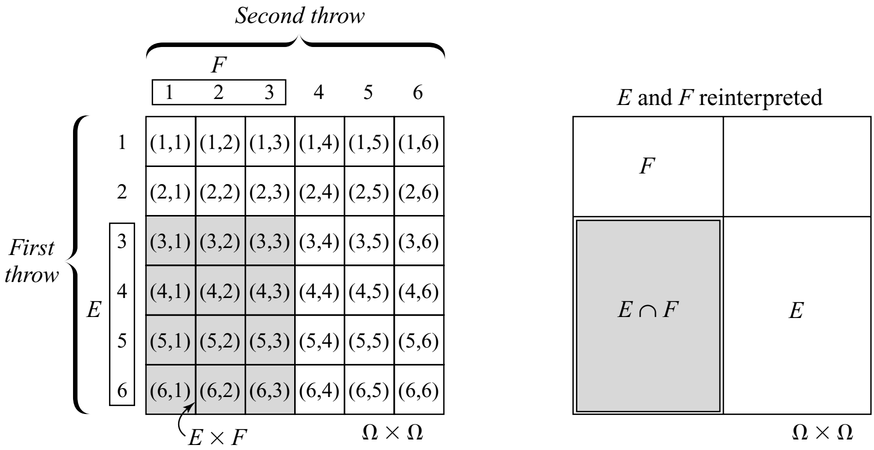
A fair dice is equally likely to show any of its faces when rolled, and so $\mathrm{prob}(1) = \mathrm{prob}(2) = \ldots = \mathrm{prob}(6) = 1/6$. The probability of the event $E = \{2, 4, 6\}$ that an even number will appear is therefore:

$$\mathrm{prob}(E) = \mathrm{prob}(2) + \mathrm{prob}(4) + \mathrm{prob}(6) = 1/6 + 1/6 + 1/6 = 1/2$$

Independent rolls

If A and B are sets, then $A \times B$ is the set of all pairs (a, b) with $a \in A$ and $b \in B$. Next slide shows the sample space $\Omega^2 = \Omega \times \Omega$ obtained when two independent rolls of the dice are observed.

Independent rolls



(a) (b)

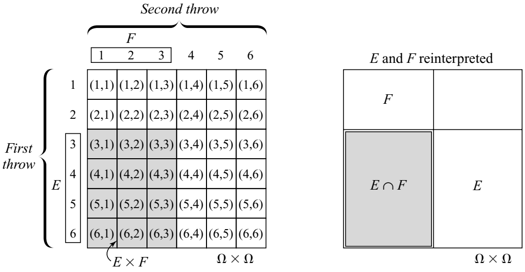
Independent rolls

There are $36 = 6 \times 6$ possible outcomes in $\Omega \times \Omega$. If the two dice are rolled independently, each outcome is equally likely. The probability of each is therefore $1/36$ So the probability of $E \times F$ must be:

$$\mathrm{prob}(E \times F) = 12/36 = 1/3$$

Notice that $\mathrm{prob}(E) = 2/3$ and $\mathrm{prob}(F) = 1/2$. Thus,

$$\mathrm{prob}(E \times F) = \mathrm{prob}(E) \times \mathrm{prob}(F)$$



(a) (b)

Independent rolls

The equation

$$\mathrm{prob}(E \times F) = \mathrm{prob}(E) \times \mathrm{prob}(F)$$

holds whenever E and F are independent events. The conclusion is usually expressed as:

$$\mathrm{prob}(E \cap F) = \mathrm{prob}(E)\mathrm{prob}(F)$$

which says that the probability that two independent events will both occur is the product of their separate probabilities.

Conditional probability

After you observe that an event F has happened, your knowledge base changes. The only states of the world that are now possible lie in the set F.

You must therefore replace Ω by F, which is the new world in which your future decision problems will be set.

The new probability $\mathrm{prob}(E|F)$ you assign to an event E after learning that F has occurred is called the conditional probability of E given F.

Conditional probability

For example, we know that $\text{prob}(4) = \frac{1}{6}$ when a fair dice is rolled. If we learn that the outcome was even, this probability must be adjusted.

The event $F = \{2, 4, 6\}$ that the outcome is even contains three equally likely states. The probability of rolling a 4, given that F has occurred, is therefore $\frac{1}{3}$. Thus,

$$\text{prob}(4|F) = \frac{1}{3}$$

The principle on which this calculation is based is embodied in the formula:

$$\text{prob}(E|F) = \text{prob}(E \cap F)/\text{prob}F$$

Lotteries

A bookie may offer you odds of 3:4 against an even number being rolled with a fair dice.

If you take the bet, you win $3 if an even number appears and lose $4 if an odd number appears.

Accepting this bet is equivalent to choosing or accepting to participate in a lottery.

Lotteries

Accepting this bet is equivalent to choosing the lottery L.

The top row shows the possible final outcomes or prizes, and the bottom row shows the respective probabilities with which each prize is awarded.

The lottery M of Figure 3 has three prizes. You have five chances in every twelve of winning the big prize of $24.

L =

$3	−$4
$\frac{1}{2}$	$\frac{1}{2}$

(a)

M =

−$4	$24	$3
$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{3}$

(b)

Random variables

Lotteries are nothing but *random variables*. A random variable is a function:

$$X : \Omega \mapsto \mathbb{R}$$

Random variables

The lottery L is equivalent to the random variable $X : \Omega \mapsto \mathbb{R}$ defined by:

$$X(\Omega) = \begin{cases} 3 & \text{if } \omega = 0\ 2,\ 4,\ 6 \\ -4 & \text{if } \omega = 1{,}3,\ 5 \end{cases} \tag{1}$$

If you take the bet represented by the random variable X, your probability of winning \$3 is $\mathrm{prob}(X = 3) = \mathrm{prob}(2, 4, 6) = \frac{1}{2}$ Your probability of losing \$4 is $\mathrm{prob}(X = -4) = \mathrm{prob}(1, 3, 5) = \frac{1}{2}$

Expected value

The expectation or expected value $\mathbb{E}X$ of a random variable X is defined by:

$$\mathbb{E}X = \sum k \, \mathrm{prob}(X = k) \tag{2}$$

where the summation extends over all values of k for which $\mathrm{prob}(X = k)$ isn't zero.

Expected value

Your expected dollar winnings in the lottery L are

$$\mathbb{E}(a) = 3 \times \frac{1}{2} + (-4) \times \frac{1}{2} = -\frac{1}{2} \quad (3)$$

L =

$3	−$4
$\frac{1}{2}$	$\frac{1}{2}$

(a)

M =

−$4	$24	$3
$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{3}$

(b)

If you bet over and over again on the roll of a fair die, winning $3 when the outcome is even and losing $4 when the outcome is odd, you are therefore likely to lose an average of about 50 cents per bet in the long run.

Expected value

The expected dollar value of lottery M is:

$$\mathbb{E}(b) = (-4) \times \frac{1}{4} + 24 \times \frac{5}{12} + 3 \times \frac{1}{3} = 10 \qquad (4)$$

If you repeatedly paid \$3 for a ticket in this lottery, you would be likely to win an average of about \$7 per trial in the long run.

$\mathbf{L} =$

\$3	−\$4
$\frac{1}{2}$	$\frac{1}{2}$

(a)

$\mathbf{M} =$

−\$4	\$24	\$3
$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{3}$

(b)

Risky choices

How do we describe a player's preferences over lotteries that involve more than two prizes?

A naive approach would be to replace all the prizes in the lotteries by their worth to the player in money.

Wouldn't a rational person then simply prefer whichever of two lotteries has the larger dollar expectation?

. . . meet the St.Petersburg paradox!

St. Petersburg paradox

A fair coin is tossed until it shows heads for the first time. If the first head appears on the k^{th} trial, you win $\$2^k$. How much should you be willing to pay in order to participate in this lottery?

prize	\$2	\$4	\$8	\$16	...	$\$2^k$	...
coin sequence	*H*	*TH*	*TTH*	*TTTH*	...	*TT...TH*	...
probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	...	$\left(\frac{1}{2}\right)^k$	...

St. Petersburg paradox

Since each toss of the coin is independent, the probability of winning $\$2^k$ is calculated as shown below for the case $k = 4$:

$$\mathrm{prob}(TTTH) = \mathrm{prob}(T) \times \mathrm{prob}(T) \times \mathrm{prob}(T) \times \mathrm{prob}(H) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

St. Petersburg paradox

The expectation in dollars of the St. Petersburg lottery L is therefore:

$$\mathbb{L} = 2\mathrm{prob}(H) + 4\mathrm{prob}(TH) + 8\mathrm{prob}(TTH) + \ldots$$

$$2 \times \frac{1}{2} + 4 \times \frac{1}{4} + 8 \times \frac{1}{8} \ldots$$

$$1 + 1 + 1 \ldots$$

which implies that the expected dollar value is infinite.

So: should we go sell off all that we own to participate in the lottery?

Expected utility theory

An adequate theory needs to recognize that the extent to which one is willing to bear risk is as much a part of her preference profile.

This is exactly the core of Von Neumann and Morgenstern expected utility theory.

Olga's utility

Suppose that Olga's utility for money is given by the Von Neumann and Morgenstern utility function $u : \mathbb{R}_+ \mapsto \mathbb{R}$ defined by:

$$u(x) = 4\sqrt{x}$$

then, her expected utility for the St. Petersburg lottery L of Figure 6 is given by:

$$\begin{aligned}
\mathbb{E}u(L) &= \frac{1}{2}u(2) + \left(\frac{1}{2}\right)^2 u(2^2) + \left(\frac{1}{2}\right)^3 + u(2^3) + \dots \\
&= 4\left\{\frac{1}{2}\sqrt{2} + \left(\frac{1}{2}\right)^2 \sqrt{2^2} + \left(\frac{1}{2}\right)^3 \sqrt{2^3} + \dots\right\} \\
&= \frac{4}{\sqrt{2}}\left\{1 + \left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)^2 + \dots\right\} \\
&= \frac{4}{\sqrt{2}-1} \approx 4 \times 3.42
\end{aligned}$$

Olga is thus indifferent between the lottery L and \$$X$ iff their utilities are the same.

So, X is the dollar equivalent of the lottery L iff

$$u(X) = \mathbb{E}u(L)$$

$$4\sqrt{X} \approx 4 \times 3.42$$

$$X \approx (3.242)^2 = 11.70$$

Remember: the "dollar equivalent" is the smallest amount in dollars for which the agent would be willing to forego enjoying the prize.

Risk attitude

Thus Olga won't pay more than $ 11.70 to participate in the St. Petersburg lottery - which is a lot less than the infinite amount she would pay if her Von Neumann and Morgenstern utility function were $u(x) = x$.

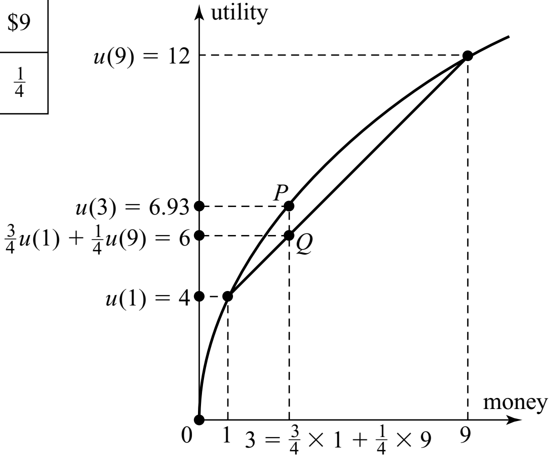
We will see that the reason we get such a different result is that Olga's new Von Neumann and Morgenstern utility function makes her risk averse instead of risk neutral.

Risk attitude

Consider now lottery M:

$\mathbf{M} =$

\$1	\$9
$\frac{3}{4}$	$\frac{1}{4}$



Risk attitude

The dollar expectation of M is:

$$\mathbb{E}M = \frac{3}{4} \times 1 + \frac{1}{4} \times 9 = 3$$

If Olga's Von Neumann and Morgenstern utility for \$x continues to be $u(x) = 4\sqrt{x}$, then her expected utility for M is:

$$\mathbb{E}u(M) = \frac{3}{4}u(1) + \frac{1}{4}u(9) = \frac{3}{4} \times 4\sqrt{1} + \frac{1}{4} \times 4\sqrt{9} = 6$$

It follows that:

$$u(\mathbb{E}M) = u(3) = 4\sqrt{3} \approx 6.93$$

and so Olga would rather not participate in the lottery if she can have its expected dollar value for certain instead.

If Olga would always sell a ticket for a lottery with money prizes for an amount equal to its expected dollar value, she is risk averse over money.

If she would always buy a ticket for a lottery for an amount equal to its expected dollar value, then she is risk loving. If she is always indifferent between buying and selling, she is risk neutral.

9. ADVERSE SELECTION

Remember Gresham's law?

Imagine an economy in which the currency consists of gold coins.

The holder of a coin is able to shave a bit of gold from it in a way that is undetectable without careful measurement.

The gold so obtained can then be used to produce new coins.

Remember Gresham's law?

Imagine that some of the coins have been shaved in this fashion, while others have not.

Then someone taking a coin in trade for goods will assess positive probability that the coin being given her has been shaved, and thus less will be given for it than if it was certain not to be shaved.

The holder of an unshaved coin will therefore withhold the coin from trade; only shaved coins will circulate.

This unhappy situation is known as Gresham's law: bad money drives out good.

Let's move to cars

- In Akerlof's context, Gresham's law is rephrased as "Bad used cars drive out good." It works as follows.

Akerlof's market for lemons

- Suppose there are two types of used cars: peaches and lemons.
- A peach, if it is known to be a peach, is worth \$3,000 to a buyer' and \$2,500 to a seller. (We will assume the supply of cars is fixed and the supply of possible buyers is infinite, so that the equilibrium price in the peach market will be \$3,000.)
- A lemon, on the other hand, is worth \$2,000 to a buyer and \$1,000 to a seller. There are twice as many lemons as peaches.

Akerlof's market for lemons

- If buyers and sellers both had the ability to look at a car and see whether it was a peach or a lemon, there would be no problem: Peaches would sell for \$3,000 and lemons for \$2,000.
- Or if neither buyer nor seller knew whether a particular car was a peach or .a lemon, we would have no problem (at least, assuming risk neutrality, which we will to avoid complications): A seller, thinking she has a peach with probability $1/3$ and a lemon with probability $2/3$, has a car that (in expectation) is worth \$1,500.
- A buyer, thinking that the car might be a peach with probability $1/3$ and a lemon with probability $2/3$, thinks that the car is worth on average \$2,333.33.
- Assuming inelastic supply of cars and perfectly elastic demand, the market clears at \$2,333.33.

Akerlof's market for lemons

- Unhappily, it isn't like this with used cars.
- The seller, having lived with the car for quite a while, knows whether it is a peach or a lemon. Buyers typically can't tell.
- If we make the extreme assumption that buyers can't tell at all, then the peach market breaks down.

Akerlof's market for lemons

- To see this, begin by assuming that cars are offered for sale at any price above \$1,000.
- All the lemons will be offered for sale.
- But only if the price is above \$2,500 will any peaches appear on the market. Hence at prices below \$2,500 and above \$1,000, rational buyers will assume that the car must be a lemon.
- Why else would the seller be selling?

Akerlof's market for lemons

- Given this, the buyers conclude that the car is worth only $2,000.
- And at prices above $2,500, the car has a 2/3 chance of being a lemon, hence is worth $2,333.33.
- There is no demand at prices above $2,000, because:
  * above $2,333.33, there is no demand whatsoever- no buyer is willing to pay that much;
  * below $2,500 there is only demand starting at $2,000, since buyers assume that they must be getting a lemon.

Akerlof's market for lemons

- So we get as equilibrium: Only lemons are put on the market, at a price of $2,000.
- Further gains from trade are theoretically possible (between the owners of peaches and buyers), but these gains cannot in fact be realized, because buyers can't be sure that they aren't getting a lemon.

Signalling: Spence Model

- Suppose you face two groups of individuals: A and B.
- The As are super smart while the Bs are below average.
- The world we are is is populated with a 50% of As and a 50% of Bs
- Let's say that As productivity is equal to 2.000 and call this y_a
- Bs productivity equals 1.000 and have this called y_b

Signalling: Spence Model

- Suppose a firm is hiring while it can't tell the As from the Bs
- It follows that the firm is expecting a productivity value:

$$\hat{y} = 0,5(2.000) + 0,5(1.000) = 1.500$$

- With a sufficiently competitive labor market, the firm will be willing to pay a wage $w = 1.500$
- This meand the 'below the average guys' are super happy as they are being paid way more their productivity while the smart guys will try and get another job.
- The firm is left with the below the average guys.

- However, let us now suppose that both the As and the Bs might invest in education and acquire a degree.
- Meanwhile, we are assuming that education does not change their productivity levels (which is a kinda heroic assumption)

- Education has a cost (and you know this well, right?)
- Suppose the cost structure be given by the following functions:

$$C_A = 200h$$

and

$$C_B = 500h$$

where h stand for the number of years spent in education.

- Assume now that the firm is willing to pay a higher wage to those guys that actually got a degree
- So that the firm is willing to pay $w = 2.000$ to those with a degree and $w = 1.000$ to those without it.

- We now have something we call a *self selections constraint*: smart guys decide to invest in education iff the benefits they expect to get when they are actually singled out as smart (i.e. $w = 2000$) net of education costs are higher than the benefits they get if they don't get the degree:

$$2.000 - 200h > 1.000$$

- Note: this constraint is met whenever h < 5.

- The very same self selection constraint for the not-smart guys: it must not be worth for them getting the degree and being perceived as smart.
- That is: the higher wage they would receive net of the cost of education must be lower than the lower wage level:

$$2.000 - 500h < 1.000$$

- Note: this constraint is met whenever h > 2
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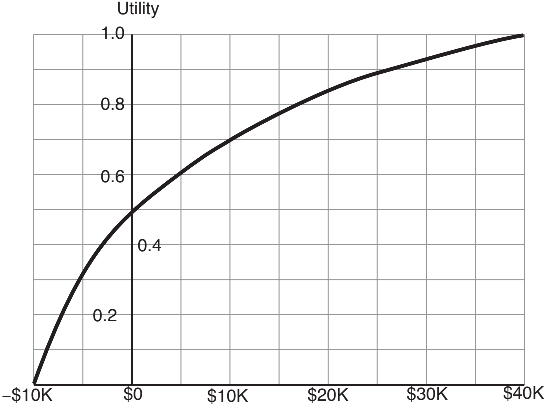
8. ADVERSE SELECTION IN AN EQUITY MARKET

Short review: the Certainty Equivalent

Suppose we model an individual having a utility function like this:



Short review: the Certainty Equivalent

Suppose we offer her a choice among:

- a gamble with prizes \$0 and \$40K, where \$40K has probability 0.4
- a gamble with prizes \$0 or \$20K, where the probability of \$20K is 0.7
- \$15K for sure.

Which one will she take?

Short review: the Certainty Equivalent

First, we read off the graph the utility values of the possible prizes:

$$u(0) = 0.5$$

$$u(15K) = 0.77$$

$$u(20K) = 0.84$$

$$u(40K) = 1$$

Short review: the Certainty Equivalent

The three expected utilities are, respectively:

$$(0.6)(0.5) + (0.4)(1) = 0.7$$

$$(0.3)(0.5) + (0.7)(0.84) = 0.738$$

$$0.77$$

Short review: the Certainty Equivalent

The model says she takes the sure thing.

But we might want to know, How much better is \$15K for sure than the other two gambles?

Short review: the Certainty Equivalent

Take the first gamble, with expected utility level 0.7.

What dollar amount has this utility?

To answer this, you go to the graph, find the utility level 0.7 on the y-axis, go across until you hit the utility function, and drop down: The answer is \$10K.

Short review: the Certainty Equivalent

This number - i.e. the dollar value whose utility equals the expected utility of the first gamble - is her certainty equivalent (henceforth CE) for the first gamble.
It is the amount of money for certain that gives her the same level of (expected) utility or satisfaction as the gamble.
And, for the second gamble, with expected utility 0.738, the reading of the graph suggests a CE of somewhere in the neighborhood of $12K.

Short review: the Certainty Equivalent

Since the for-certain \$15K has a certainty equivalent of \$15K wed say that:

1. This individual is roughly \$3K better off with a sure \$15K than with the second gamble (whose CE $=$ \$12K, roughly)
2. \$5K better off than with the first gamble (whose CE is \$10K, roughly).

Short review: the Certainty Equivalent

WATCH OUT!!!

As long as an individuals utility function u is continuous and increasing every gamble or lottery she faces has a unique certainty equivalent.
$\Rightarrow$ Continuity of u guarantees existence (by the Intermediate Value Theorem) and if u is increasing, there cant be more than one.

The CE is the Selling Price

Suppose an individual owns the following lottery: With probability 0.4, she wins \$40K. With probability 0.6, she wins nothing (that is, \$0.) Suppose she is an expected-utility maximizer, and her utility function is the one represented on our first slide.

Short review: the Certainty Equivalent

We know that her expected utility for this lottery is, roughly, 0.7, and her CE is around \$10K.

Imagine we approached her and asked, “We want to buy that lottery from you. We are willing to pay you \$11K for it. Will you sell?

This is more than her CE for the lottery and she will sell.

In this sense, her CE is her (least) selling price for this lottery; more generally, the CE of any lottery is the lowest price that an individual will accept in exchange for a lottery that she owns.

A Market in Equity shares

Let us now turn and analyze one of the most meaningful arenas for adverse selection to work: equity shares in entrepreneurial ventures.

A Market in Equity shares

Consider the following market in equity shares in entrepreneurial ventures.

- Each share represents a 1% share in a venture;
- Each venture will pay off either \$50,000 or \$-25,000
- Each venture (of which there are many) is controlled by an entrepreneur.
- Entrepreneurs are all risk-averse expected-utility maximizers and, for simplicity, all of them have the utility function $u(x) = -e^{0.0000211x}$, where x is the proceeds to the entrepreneur from sales of shares in her venture, plus the returns from any share she retains.

A Market in Equity shares

- Half the ventures have probability 0.65 of being successful (returning \$50,000), while the other half have probability 0.35 of success;
- Each entrepreneur knows the probability of success of her own venture, but she cannot directly communicate that information to investors. (. . . would investors trust her anyway?)

A Market in Equity shares

- A price per 1% share in any venture is established by supply-equals-demand at p.
- The different ventures carry no systematic risk, so demand for shares is at the expected monetary value of each share.
- The supply of shares comes from entrepreneurs who, taking p as given, decide how big a fraction of their venture to sell and how big a fraction to retain.

A Market in Equity shares

Our question: Why wont a price per 1% share of $125 be an equilibrium in this market?

Our Analysis

UTTERLY IMPORTANT:

Notice: a randomly selected venture has a 50% chance of having a success probability of 0.65 and a 50% chance of having a success probability of 0.35. That is, the EV of a randomly selected venture is:

$$0.5(0.65(50.000) + 0.5(0.35(50.000) = 25.000$$

But: the average share in the equity market does not do that well:

$$\frac{500 \times 100 + (-250) \times (100)}{200} = 125$$

Our Analysis

Matter of fact: Suppose you are an entrepreneur with one of the good projects.

Question: What fraction of your venture do you wish to sell at a price of \$125 per 1% share?

Certainly not all your shares; your venture has an EMV of

$$(0.65)(\$50,000) + (0.35)(\$25,000) = \$23.750$$

so a 1% share has an EMV of \$237,50 which is way less than 125.

This doesnt mean that such an entrepreneur would not want to possibly sell some of her shares!! But we want to know how many!!

Our Analysis

	A	B	C	D	E	F	G	H	I	J
1	prob success	price per 1% share	share retained	good outcome result	bad outcome result	coeff of risk aversion	good outcome utility	bad outcome utility	expected utility	certainty equivalent
2	0.65	$125.00	50.00%	$31,250.00	-$6,250.00	0.0000211	-0.517174	-1.140966	-0.735501	$14,559.38

Entered as constants:

probability of the success of the project (cell A2);

price per 1% share (B2);

coefficient of risk aversion of the entrepreneur (F2).

Our Analysis

	A	B	C	D	E	F	G	H	I	J
1	prob success	price per 1% share	share retained	good outcome result	bad outcome result	coeff of risk aversion	good outcome utility	bad outcome utility	expected utility	certainty equivalent
2	0.65	$125.00	50.00%	$31,250.00	-$6,250.00	0.0000211	-0.517174	-1.140966	-0.735501	$14,559.38

The share retained by the entrepreneur (in cell C2) is the driving variable; we begin with 50% retained.

Let's now see some computations.

Proceeds: D2 and E2

The net proceeds to the entrepreneur as a function of the price per 1% and the share she retains in the two cases are computed.

1. D2 = revenue from selling 50% shares + good outcome result = $(125 \times 50) + \$25.000 = 31.250$
2. E2 = revenue from selling 50% shares + bad outcome result = $(125 \times 50) + (-12.500) = -\ 6250$.

Utilities: G2 and H2

Proceeds are then converted to utilities for the good and bad case (G2 and H2):

$$u(31.250) = -e^{-0{,}0000211 \times 31.250} = -0.517174$$

and

$$u(-6250) = -e^{-0{,}0000211 \times -6250} = -1.140996$$

Expected Utility: I2

Expected utility is computed using the probability of the good outcome (I2)

$$0.65(-0.517174) + 0.35(-1.140996) = 0.7355117$$

Certainty Equivalent: J2

CE $=-ln(-EU)/\lambda$

So:

$$CE = -ln(0.7355117)/-0.0000211 = 14.559,38$$

Note: the Appendix for the Math Addicts has the rationale for this calculation. It's on harvard.canvas

Our Analysis

We then maximize the certainty equivalent (via Solver) by varying the share retained (C2), and Solver returns panel b:

5										
6	prob success	price per 1% share	share retained	good outcome result	bad outcome result	coeff of risk aversion	good outcome utility	bad outcome utility	expected utility	certainty equivalent
7	0.65	$125.00	39,12%	$27.169,18	-$2.169,18	0.0000211	-0,563679	-1,046833	-0.732783	$14.734,85
8										

The entrepreneur chooses to retain 39.12% of her venture.

Our Analysis

What about an entrepreneur with a project whose probability of success is only 0.35?

We could repeat the spreadsheet analysis just done but, in fact, the answer is obvious.

This entrepreneur knows that the EMV of her project is

$$0.65 \times \$50,000 + 0.35 \times (-\$25,000) = \$1250.$$

Since the market is willing to pay, up front, \$125 per 1% share, and since the entrepreneur is risk averse, this entrepreneur will of course sell 100% of her venture.

Our Analysis

This means that the shares "in the market" are an adverse selection of all the shares there are.

Our Analysis

Suppose there are 100 of these entrepreneurs, 50 of each type. For the 50 entrepreneurs with good projects, they put 61 1% shares into the market. The other 50 entrepreneurs put 100 1% shares into the market.

So the total number of shares in the market is

$$(50)(61) + (50)(100) = 8050,$$

of which 5000, or roughly 62% are shares of bad ventures.

If you purchase one of these shares at random, the chance youll get a good outcome is not 50% but

$$(0.62)(0.35) + (0.38)(0.65) = 0.4635,$$

Our Analysis

And so the EMV from a 1% share randomly bought in the market is:

$$(0.4635)(500) + (0.5365)(250) = \$97.64.$$

Paying $125 for one of these shares is way too much!

In particular EMV $< p$

So, Where Does Supply Equal Demand?

Let's try and follow these steps and answer these questions:

i) For each price p, find out what fraction of shares each type of entrepreneur will retain and what fraction they will put in the market.

So, Where does Supply Equals Demand?

ii) Use the answers found in step 1 to compute the EMV of an average 1% share in the market.

So, Where does Supply Equals Demand?

iii) The assumption on the demand side of the market is that there is enough investors to soak up all the shares provided, if they are priced at their EMV or less. If p is less than the EMV of the "average" share in the market, competition among investors will push the price up to that EMV. If p exceeds the EMV, investors won't buy any shares at all.

So, Where does Supply Equals Demand?

Answer: supply will equal demand at the price p where the answer to step 2 is the price p used in step 1.

So, Where does Supply Equals Demand?

Note well: investors, either through insight and knowledge or, more likely, through experience, understand how the price p will affect the selection of shares in the market and, therefore, the EMV of a randomly selected share.

So, Where does Supply Equals Demand?

Note well: as there are only two types of entrepreneur and, as long as the price p is above \$12.50, entrepreneurs with bad projects will want to sell 100% of their projects.

So we only need to discover the share retained by entrepreneurs with good projects.

So, Where does Supply Equals Demand?

So, let's Excel some more and replicate – in successive rows – the row of computations in our first table (on slide 20) varying the price per 1% share.

9	prob success	price per 1% share	% of share retained	proportion of good shares	average probability of a good outcome	EMV of Average 1% share
10	0.65	125	39,12%	0.378	0.463	$97.65
11	0.65	120	40,80%	0.371	0.461	$96.17
12	0.65	115	42,49	0.365	0.459	$94.65
13	0.65	110	44,18	0.358	0.457	$93.11
14	0.65	105	45,86	0.351	0.455	$91.52
15	0.65	100	47,56	0.344	0.453	$89.90
16	0.65	95	49,25	0.336	0.45	$88.25
17	0.65	90	50,95	0.329	0.448	$86.55
18	0.65	85	52,65	0.321	0.446	$84.80
19	0.65	80	54,36	0.313	0.444	$83.01
20	0.65	75	56,07	0.305	0.441	$81.17
21	0.65	70	57,79	0.296	0.439	$79.29

So, Where does Supply Equals Demand?

Let's find the proportion of shares the good-project entrepreneurs would retain as the price per 1% share changes;

In the third column we find the results of the row-by-row optimization.

Note that as the price per 1% share decreases, the share retained by good entrepreneurs increases, which means (fourth column) that the proportion of shares in the market that are from good projects decreases.

Hence, the average probability of a good outcome and the EMV of a randomly selected 1% share of a venture (selected from those shares that supplied to the market) decreases.

So, Where does Supply Equals Demand?

The final column shows the EMV of an average 1% share in the market.

The equilibrium is where the EMV of an average 1% share in the market equals the price of that 1% share.

Between p = \$85 and p = \$80, the EMV of the average 1% share "catches up" to the price per share.

That is our equilibrium price p !!!

Now to the BIG QUESTION

Can we find a different equilibrium, where entrepreneurs are paid a higher price for their shares if they "prove" they have good projects by holding on to a fraction of their ventures?

You tell why this is THE big question!

Does this have anything to do with signals and discrimination?

hint: YES!

Does this sound any similar to education in Spence Model?

hint: YES!

A different equilibrium

Trivial but true: savvy investors when going to buy a share in a venture, observe that shares in ventures whose entrepreneurs retain a signficant fraction of their ventures do very well.

So investors compete for shares in these ventures, bidding the price of 1% shares in such ventures to $237.50 (the EMV of a 1% share in a venture that succeeds with probability 0.65).

And, at the same time, they shun shares in ventures whose entrepreneurs put 100% of their ventures on the market, since those ventures succeed only 35% of the time: the price of 1% shares in those ventures falls to $12.50.

A different equilibrium

There is, however, a problem with this: as the price of shares in ventures where the entrepreneur retains a large share of her venture rise toward $237.50 per share, the entrepreneurs want to sell off an increasing share of their ventures.

When the price reaches $237.50, they want to sell off 100% of their ventures (why?), and they can no longer be distinguished from the bad-venture entrepreneurs.

A different equilibrium

So, to make this work, investors must structure their offers as follows: if an entrepreneur is willing to retain X% of her venture (for some X still to be determined), investors will pay her \$237.50 per 1% share.

If the entrepreneur is unwilling to retain this much, she is paid only \$12.50 per 1% share.

A different equilibrium

The key to this signaling or separating equilibrium is the answer to the following question: How big must X be, so that entrepreneurs with bad projects dont want to pretend to be good entrepreneurs, in order to get the much better price for their shares?

This equilibrium only works if good entrepreneurs are willing to retain $X\%$ to prove they are good, but for bad entrepreneurs, sending this signal is more costly than it is worth.

A different equilibrium

Problem thus becomes: how do we find X?

Let's start with bad ventures entrepreneurs.

We know that If they fail to retain X% of their ventures, they will sell 100% of their ventures at \$12.50 per 1%, for a net \$1250.

Have a look at the table in the next slide: it shows the certainty equivalents for a bad-project entrepreneur as she retains an increasing fraction of her project, supposing she can sell shares at \$237.50 per 1%.

A different equilibrium

% of share retained	expected utility	certainty equivalent
0	-0,6058	$23,750.00
10	-0,6371	$21,367.23
20	-0,6736	$18,728.56
30	-0,7157	$15,849.78
40	-0,7642	$13.00
50	-0,8194	$9,441.00
60	-0,8821	$5,946.81
70	-0,9530	$2,283.21
71	-0,9605	$1,908.20
72	-0,9682	$1,531.69
73	-0,9760	$1,153.69
74	-0,9838	$774.22

A different equilibrium

% of share retained	expected utility	certainty equivalent
0	-0,6058	$23,750.00
10	-0,6371	$21,367.23
20	-0,6736	$18,728.56
30	-0,7157	$15,849.78
40	-0,7642	$13.00
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70	-0,9530	$2,283.21
71	-0,9605	$1,908.20
72	-0,9682	$1,531.69
73	-0,9760	$1,153.69
74	-0,9838	$774.22

The important rows are the rows for 72% and 73% retained.

Recall that a bad-project entrepreneurs certainty equivalent for selling 100% of her venture at $12.50 per 1% share is $1250.

So if, by retaining 72%, she could get the “good price” of $237.50 per 1%, she would rather do that.

But if it takes retaining 73% to get the “good price”, she would rather settle for selling 100% of her venture for $12.50 per 1%.

A different equilibrium

This provides us with an alternative to the pooling equilibrium, where all shares sell for around \$85.

Investors are willing to pay \$237.50 per 1% share to any entrepreneur who retains 73% of her venture.

They are willing to pay \$12.50 per share to entrepreneurs who are not willing to do this.

A different equilibrium

The bad-project entrepreneurs settle for selling 100% of their ventures, while the good-project entrepreneurs are willing to retain 73%;

You can compute that this gives each of the good-project entrepreneurs a certainty equivalent of $16,089.20.

A different equilibrium

And since the two types separate themselves by their choice of how much to retain, the prices of \$237.50 in one case and \$12.50 in the other are market-equilibrium prices.

A different equilibrium

Which of these two market equilibria are preferred by the entrepreneurs?

The bad-project entrepreneurs prefer to be pooled; in the pooling equilibrium they get around \$85 per 1% share, for a net \$8500, versus the \$1250 they get in the separating equilibrium.

The good-project entrepreneurs, on the other hand, prefer the separating equilibrium in which they net a certainty equivalent of \$16,089.20; if all shares go for \$85, a good-project entrepreneur would want to retain 52.65% of her project, for a certainty equivalent of \$12,569.69.

15. FINANCING, MANAGING AND INFORMATIONAL ASYMMETRIES

Adam Smith gone wrong ?

The directors of such companies, however, being the managers rather of other people's money than of their own, it cannot well be expected, that they should watch over it with the same anxious vigilance with which the partners in a private copartnery frequently watch over their own.
Like the stewards of a rich man, they are apt to consider attention to small matters as not for their master's honour, and very easily give themselves a dispensation from having it.
Negligence and profusion, therefore, must always prevail, more or less, in the management of the affairs of such a company.
It is upon this account that joint stock companies for foreign trade have seldom been able to maintain the competition against private adventurers.

Agency and capital market

Classic Modigliani and Miller proposition: firm's financial structure is irrelevant to enterprise value of the firm.

Assumptions: in the absence of taxes, bankruptcy costs, agency costs, asymmetric information, and in an efficient market.

What happens if we drop those assumptions ?

Financial structure is deeply affected by info asymmetries

Financial structure has a (huge) impact on main actors.

What happens if we drop those assumptions ?

External investors (principal) can either be shareholders or creditors.

They lend money to managers (agent) with no perfect info relative e.g. to managers' behaviors, risk attitude. . .

A toy example

A manager/owner: manages and entirely owns the firm.

He can either go debt or equities.

First case: he goes equities

As soon as equities are on the mkt, he is no longer the only claimant to profit.

Ownership gets thus separated from control.

If he only owns a, say, 10% of the shares, his own decisions will only impact his income by a 10%

His effort will thus be somehow "limited", not optimal.

On the other hand, opportunistic behavior somehow gets an incentive: he spend firm's money in luxury hotels, super cars, bespoke suits...

That is because he bears the costs (in terms of lower profits) only up to a a 10% but enjoys all the benefits.

...and the smaller his share, the stronger opportunism hits!

Second case: he goes debt

As debtors are remunerated on a fixed basis (interests) and interests are not bound to profit. . .

. . . he remains the sole claimant to profit. . .

. . . thus no agency problems around and he would eventually bear any inefficient use of resources.

Second case: he goes debt

But: if the quota of the manager own resources invested in the firm is small and debt is big, he will have strong incentives for way too risky projects (potentially huge profits, potentially huge losses).

That is: he would totally appropriate possible gains but only partially bear the costs of failure.

For a good outcome: he would beef up the whole cake net of interests

For a bad outcome: firms goes bankrupt and creditors bear all the costs (role for limited liability)

S&L case.

Three main ideas I

First, managers, lenders, and shareholders may all have different interests and financial arrangements may affect how different those interests are and what decisions management will make.

Three main ideas II

Second, managers may be better informed than investors about the firm's prospects, so the financial decisions they make may affect investors' beliefs and therefore the price of the firm's shares.

Three main ideas III

Third, financial securities are not just claims to part of a firm's net receipts; they also give the security holder certain rights.

Jensen and Meckling

J&M single out a trade-off relative to firms decisions on its financial structure between:

⇒ agency costs (deriving from managers potential opportunism)

⇒ agency costs (deriving from debtors tendency to undertake too risky projects)

In addition:

⇒ they show that (and how) financial decisions have an impact on firms' organization and investment decisions.

Agency Costs

- ⇒ Monitoring Costs: Expenses incurred by the principal to oversee the agent's actions (e.g., audits, performance reviews).
- ⇒ Bonding Costs: Costs incurred by the agent to reassure the principal they will act in the principal's best interest (e.g., contractual commitments, insurance).
- ⇒ Residual Loss: The loss incurred when agents' decisions do not fully align with the principal's goals, even after monitoring and bonding.

Ownership and Incentives

⇒ Jensen and Meckling highlight that ownership structure influences agency costs.

⇒ When managers have an ownership stake in the firm (e.g., through shares), their interests align more closely with those of shareholders, reducing agency problems.

⇒ However, as external ownership increases, conflicts may arise because external shareholders cannot directly control managerial behavior.

Capital Structure

The model also explores how the mix of debt and equity financing affects agency costs. For example:

- ⇒ Debt creates a commitment to repay, which can reduce the free cash flow available to managers for personal agendas.
- ⇒ High levels of debt, however, may increase the risk of financial distress.

Applications

The Jensen-Meckling model has profound implications in:

- ⇒ Corporate Governance: Designing systems to mitigate agency problems.
- ⇒ Incentive Structures: Creating compensation plans that align managers' goals with shareholders'.
- ⇒ Optimal Capital Structure: Balancing debt and equity to minimize agency costs.

This model laid the foundation for much of modern corporate finance theory and continues to influence how firms approach governance and decision-making.

J&M Analitically

The Jensen and Meckling model introduces a mathematical framework to quantify agency costs and explore how they relate to ownership structure, capital structure, and firm behavior.

Let's go on and discuss a simplified explanation of its mathematical underpinnings!

Agency Costs and Ownership Structure

The model assumes that a firm?s total value is affected by agency costs that arise from conflicts of interest between owners and managers. Let:

- ⇒ V: Total value of the firm.
- ⇒ A: Agency costs, which reduce the firm's value.

The firm's value net of agency costs is:

$$V_{net} = V - A \tag{5}$$

Ownership and Agency Costs

If α represents the fraction of ownership held by the manager, agency costs, A decrease as α increases because the manager has more incentive to act in the firm's interest:

$$A = f(\alpha) \text{ where } \frac{dA}{d\alpha} < 0 \qquad (6)$$

Manager's Utility Function

The manager (agent) derives utility from:

⇒ Personal consumption of perquisites P (e.g., perks, misuse of firm resources, Lambos, luxury hotels).

⇒ Financial returns from their ownership stake α.

Let:

⇒ W_m be Manager's wealth (a combination of personal consumption and firm ownership).

⇒ R be Return generated by the firm

The manager's utility is:

$$U_m = \alpha R - P + g(P) \tag{7}$$

where $g(P)$ represents the personal satisfaction (non-monetary benefit) from consuming perquisites.

Trade-off Between Perquisites and Firm Value

As P increases:

- $\Rightarrow$ Firm value V decreases
- $\Rightarrow$ Managerial utility U_m may initially increase but eventually diminishes due to diminishing marginal returns from $g(P)$.

Firm Value and Capital Structure

The model incorporates debt D and equity E financing. Debt can act as a disciplining mechanism for managers because it limits free cash flow.

Firm Value with Debt: the value of the firm is:

$$V = V_0 - \text{Agency Costs} - \text{Bankruptcy Costs} \tag{8}$$

Where V_0 is the value of the firm without agency and bankruptcy costs.

Optimization

The manager's problem is to maximize their utility while considering the firm's ownership structure and financing constraints:

$$max_{\alpha,P} \quad U_m = \alpha R - P + g(P) \tag{9}$$

subject to:

$$V_{net} = V - \alpha \tag{10}$$

and:

$$D + E = V \tag{11}$$

By solving this optimization problem, the model derives:

⇒ The optimal ownership share α^*

⇒ The optimal level of debt D^*

⇒ The corresponding agency costs A^*

Key Insights from the Model I

- $\Rightarrow$ Increasing managerial ownership α reduces agency costs but may limit diversification for managers.
- $\Rightarrow$ Debt financing reduces agency costs by limiting free cash flow but introduces bankruptcy risks.
- $\Rightarrow$ Firms balance ownership and capital structure to minimize total costs and maximize firm value.

This mathematical framework provides the foundation for analyzing the trade-offs in corporate governance and capital structure decisions.

Key Insights from the Model II

Reconceptualizing the Firm

⇒ The firm is viewed as a nexus of contracts among various stakeholders (e.g., owners, managers, creditors).

⇒ The firm's boundaries, organizational structure, and governance mechanisms are determined by the need to minimize agency costs.

Optimal Ownership Structure

Reconceptualizing the Firm

- $\Rightarrow$ Increasing managerial ownership (e.g., through stock options or equity compensation) aligns managers' incentives with shareholders, reducing agency costs.
- $\Rightarrow$ However, excessive managerial ownership can lead to entrenchment (managers resist changes that could benefit the firm but harm their personal interests).

Key Insights from the Model III

Capital Structure and Debt Discipline

- ⇒ The model demonstrates how debt can act as a disciplining mechanism:
  - ⇒ It reduces free cash flow available for managers to misuse.
  - ⇒ The obligation to service debt imposes financial discipline.
- ⇒ However, high debt levels can lead to financial distress, introducing bankruptcy costs.
- ⇒ Firms must balance agency costs of equity (managerial opportunism) against the costs of debt (financial distress) to determine the optimal capital structure.

Key Insights from the Model III

Corporate Governance Mechanisms

⇒ The model underscores the importance of governance mechanisms to align interests and reduce agency costs:

⇒ Monitoring Mechanisms: Board oversight, audits, and external regulations.

⇒ Incentive Structures: Performance-based pay, stock options, and profit-sharing.

⇒ Market for Corporate Control: Threat of takeovers as a discipline for poorly performing managers.

Key Insights from the Model III

Specific applications

- ⇒ Small vs. Large Firms: Agency costs are generally lower in smaller firms where owners and managers often overlap but rise in larger, publicly traded firms.
- ⇒ Industry-Specific Capital Structures: Industries with predictable cash flows (e.g., utilities) can afford higher debt levels, while volatile industries (e.g., technology) rely more on equity.

Key Insights from the Model III

Behavioral considerations

⇒ Jensen and Meckling's work assumes rational actors, but subsequent studies have incorporated behavioral insights:

- ⇒ Overconfidence: Managers may overestimate their ability to generate returns, increasing risk-taking.
- ⇒ Short-termism: Misaligned incentives can lead to decisions that prioritize short-term profits over long-term value.