Lezione 3 11/03/2025

martedì 11 marzo 2025 14:06

Lezione #3

11/03/2025

CINEMATICA

$\vec{r}$; $\Delta\vec{r}$; $\vec{v}_M = \frac{\Delta\vec{r}}{\Delta t} \to \vec{v}_{IST} = \frac{d\vec{r}}{dt}$

accelerazione:

$$\vec{a}_M = \frac{\Delta\vec{v}}{\Delta t}$$

$$[\vec{a}_M] = \frac{[v]}{[t]} = \frac{m}{s}\,\frac{1}{s} = m/s^2$$

$$\vec{a}_{ist} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

se $\vec{a}_M = \frac{\Delta\vec{v}}{\Delta t}$ $\qquad a_M$ = COSTANTE se e solo se

$\vec{a}$ { Modulo, Direz., Verso

MOTO UNIF. ACCELERATO IN DUE DIMENSIONI

↓

$\vec{a}$ = COSTANTE $\qquad \vec{a}_{IST} = \vec{a}_M = \vec{a}$

$$\begin{cases} t_{IN} = 0 \\ t_{FIN} = t \end{cases} \qquad \begin{cases} \vec{r}_{IN} = \vec{r}_0 \\ \vec{r}_{FIN} = \vec{r} \end{cases} \qquad \begin{cases} \vec{v}_{IN} = \vec{v}_0 \\ \vec{v}_{FIN} = \vec{v} \end{cases} \qquad a_{IN} = a_{FIN} = a$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t - 0}$$

$$\vec{v} = \vec{v}_0 + \vec{a}\, t \qquad \begin{cases} v_x = v_{0x} + a_x t \\ v_y = v_{0y} + a_y t \end{cases} \qquad \vec{v} = (v_x ; v_y)$$

$$\vec{v}_M = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r} - \vec{r}_0}{t - 0}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_M\, t \leftarrow$$

$$\left(\vec{v}_M = \frac{\vec{v}_0 + \vec{v}}{2} \right)$$ Valore medio

$$\vec{r} = \vec{r}_0 + \underbrace{\left(\frac{\vec{v}_0 + \vec{v}}{2} \right)}_{\vec{v}_M} t$$

$$\vec{r} = \vec{r}_0 + \frac{1}{2} \vec{v}_0\, t + \frac{1}{2} \vec{v}\, t$$

$\vec{v} \hookrightarrow (\vec{v}_0 + \vec{a}\, t)$

$$= \vec{r}_0 + \frac{1}{2} \vec{v}_0 t + \underbrace{\frac{1}{2} \vec{v}_0 t + \frac{1}{2} \vec{a}\, t^2}_{\frac{1}{2} (\vec{v}_0 + \vec{a}t) t}$$

$$\frac{1}{2}(\vec{v}_0 + \vec{a}t)\,t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \qquad \begin{cases} x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \end{cases}$$

EQ^{NI} MOTO UNIF. ACC. IN DUE DIR.

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \qquad \begin{cases} x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \end{cases}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \begin{cases} v_x = v_{0x} + a_x t \\ v_y = v_{0y} + a_y t \end{cases}$$

Quando $\vec{a} = \vec{g}$

↳ accelerazione di gravità

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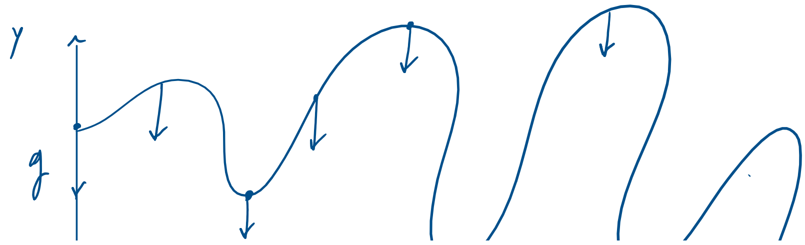
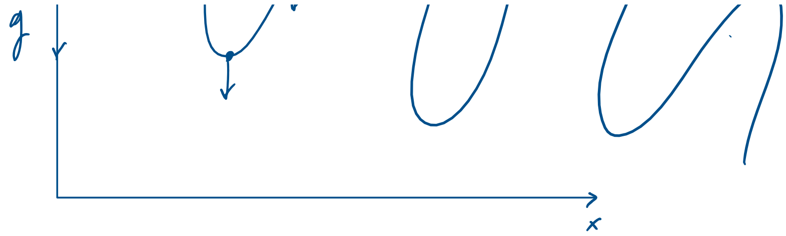
Moto in caduta libera in due dimensioni

$$\boxed{g = 9{,}81\ m/s^2}$$

$$\vec{a} = (0; -g)$$

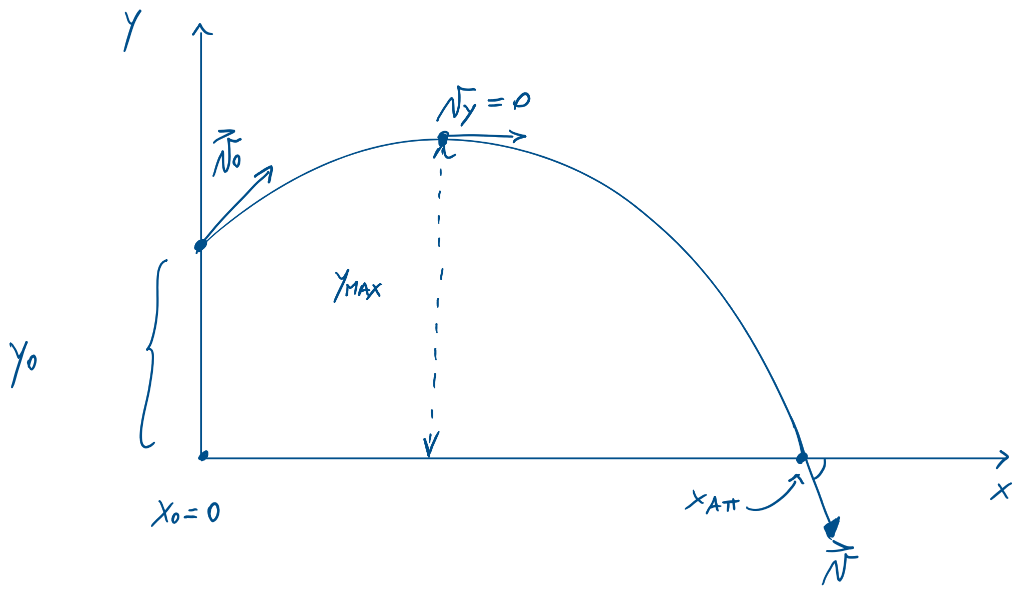


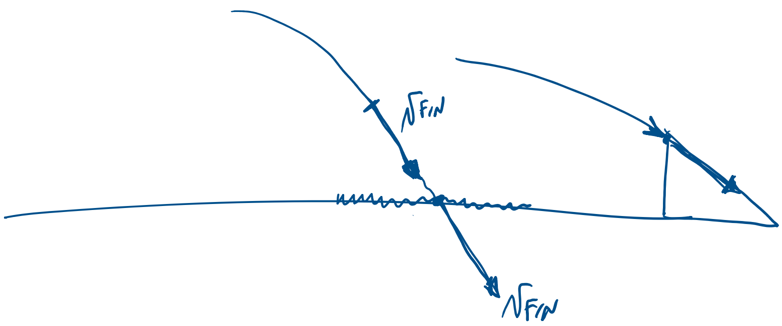
$\vec{a} = (0; -g)$

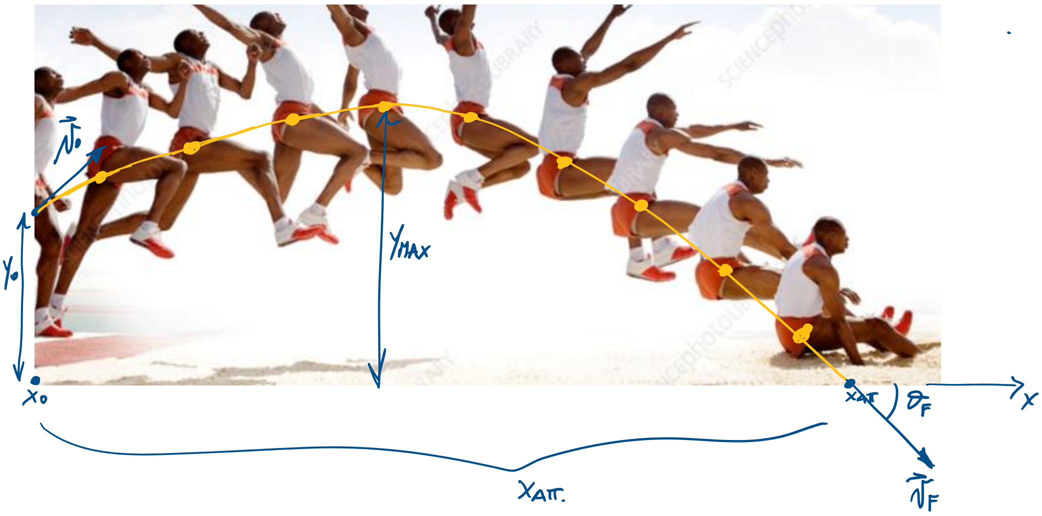


$$\begin{cases} x = x_0 + v_{0x} t + \cancel{\tfrac{1}{2} a_x t^2} = x_0 + v_{0x} t \Leftarrow v.\ cost. \quad (a_x = 0) \\ y = y_0 + v_{0y} t + \tfrac{1}{2} a_y t^2 = y_0 + v_{0y} t - \tfrac{1}{2} g t^2 \Leftarrow acc. < 0 \end{cases}$$

$$\begin{cases} v_x = v_{0x} + \cancel{a_x t} = v_{0x} \\ v_y = v_{0y} + a_y t = v_{0y} - g t \end{cases}$$







Moto in caduta libera $\Rightarrow$ x moto rettilineo uniforme ($v_x = cost.$)

$$\begin{cases} v_x = v_{0x} \\ x = x_0 + v_{0x}\, t \end{cases}$$

Esempio moto in caduta libera:

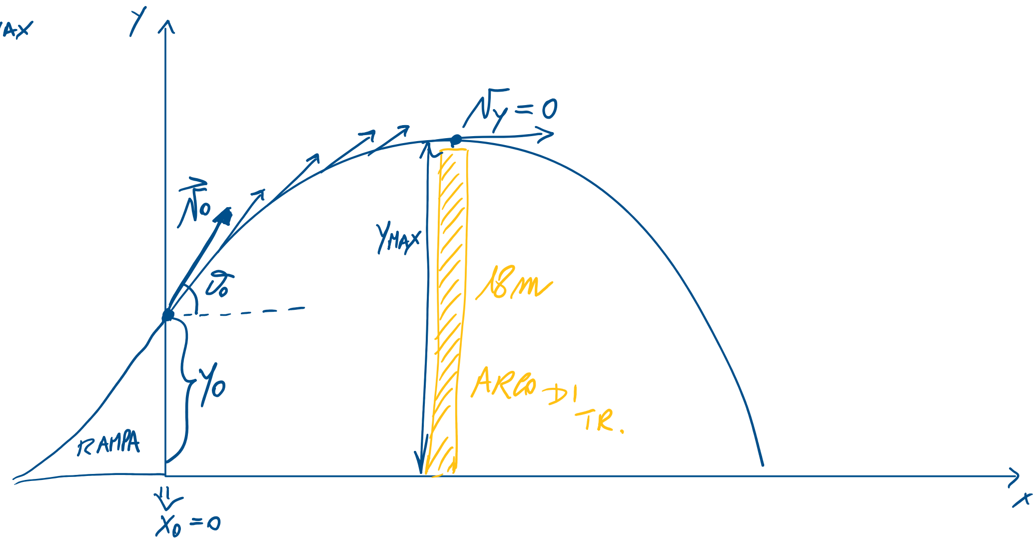
Esercizio:

A capodanno 2007 lo stuntman Robbie Madison tentò di stabilire un nuovo record a Las Vegas cercando di superare una replica dell'Arco Di Trionfo alta 18 m. Sapendo che si lanciò con una velocità iniziale pari a v0 = 90 km/h da una rampa alta y0 = 3 m e inclinata con un angolo θ_0 = 45°, calcolare:

1. Altezza massima raggiunta. Riesce a superare l'Arco?
2. La distanza di atterraggio
3. Il modulo, direzione e verso della sua velocità finale (all'atterraggio)

(Lo stesso Madison nell'impatto col terreno, si lacerò la mano tra pollice e indice e dichiarò che non avrebbe mai ripetuto tale impresa neppure per 10 milioni di dollari)

1) y_{MAX}



y_{MAX}

Per calcolare $y_{MAX} \Rightarrow v_y = 0$

$$v_y = v_{0y} - g t$$

$$0 = v_{0y} - g\, t_{MAX}$$

$$g\, t_{MAX} = v_{0y}$$

$$t_{MAX} = v_{0y}/g$$

1^a Poss. t_{MAX} calcolo numericamente

$$v_{0y} = v_0 \sin\theta_0$$

$$t_{MAX} = \frac{v_0 \sin\theta_0}{g}$$

$$v_0 = 90\ km/h = 90\ \frac{10^3\ m}{60 \cdot 60\ min\ s} = 90\ \frac{10^3}{3{,}6\ 10^3}\ \frac{m}{s}$$

$$v_0 = 25\ m/s$$

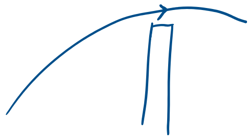
$$t_{MAX} = \frac{25 \cdot \sin(45^\circ)}{9{,}81} = 1{,}80200\ s$$

Sostituisco t_{MAX} in y

$$y_{MAX} = y_0 + v_{0y}\, t_{MAX} - \frac{1}{2} g\, t_{MAX}^2$$

$$= 3 + 25 \cdot \sin(45^\circ) \cdot 1{,}80200 - \frac{1}{2} \cdot 9{,}81 \cdot (1{,}80200)^2$$

$$y_{MAX} = 18{,}92\ m > 18\ m$$



2^a STRADA

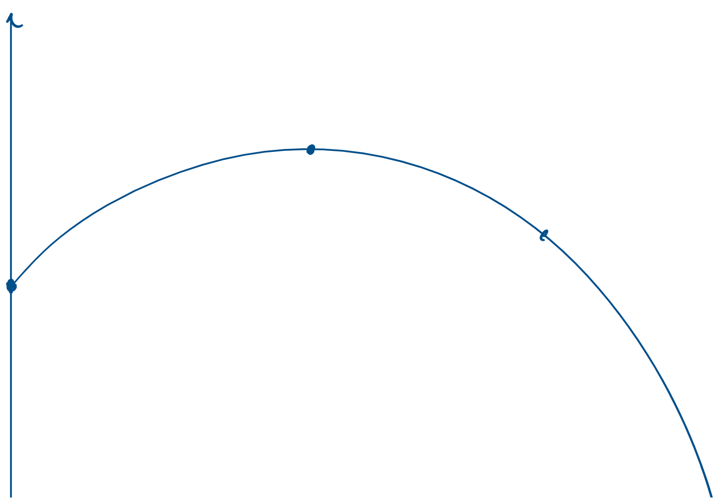
$$t_{max} = \frac{v_0 \sin\vartheta_0}{g} \implies y = y_0 + v_{0y}\left(\frac{v_{0y}}{g}\right) - \frac{1}{2} g \left(\frac{v_{0y}}{g}\right)^2$$

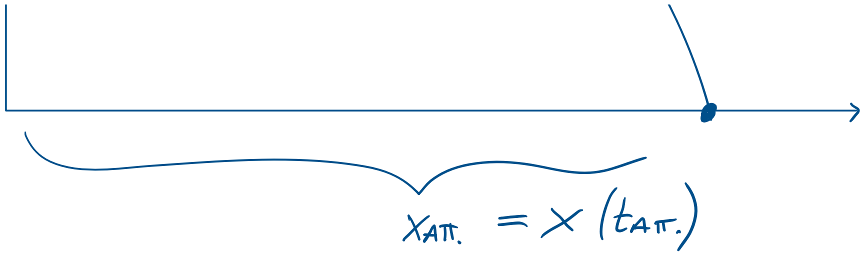
$$= y_0 + \frac{2}{2}\frac{v_{0y}^2}{g} - \frac{1}{2} g \frac{v_{0y}^2}{g^2} = y_0 + \frac{1}{2}\frac{v_{0y}^2}{g}$$

$$y_{max} = y_0 + \frac{1}{2}\frac{v_{0y}^2}{g} = y_0 + \frac{1}{2}\frac{(v_0 \sin\vartheta_0)^2}{g}$$

$$y_{max} = 18{,}92\ m \simeq 20\ m \quad (1\ c.s.)$$

2) x_{att} ?





$x_{ATT.} \Rightarrow y = 0$

Se imponiamo $y = 0$

$$\Downarrow$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$y = 0 \Rightarrow \quad 0 = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

↳ risolvere rispetto a t

$$t^2 \underbrace{\left(-\frac{1}{2} g\right)}_{a} + t \underbrace{\left(v_{0y}\right)}_{b} + \underbrace{y_0}_{c} = 0$$

$a = -4,9050$

$b = 17,677$

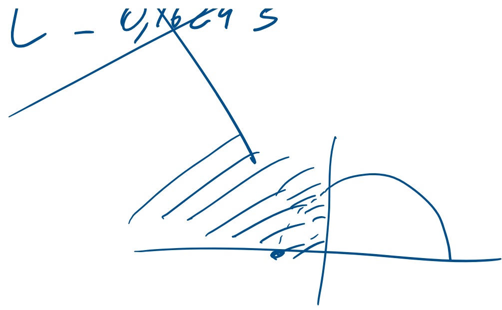
$c = 3$

Formula risolutiva

$$t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

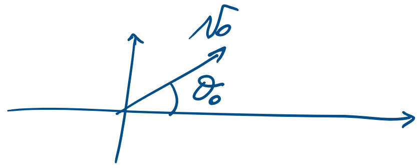
$3,7664 \text{ s}$

~~$-0,1624 \text{ s}$~~



$$\boxed{t_{ATT} = 3{,}7664\ s}$$

$$x = \overset{0}{\overset{\shortparallel}{x_0}} + v_{0x}\, t$$



$v_{0x} = v_0 \cos\vartheta_0$

$$x_{ATT.} = v_{0x}\, t_{ATT}$$

$$x_{ATT.} = v_0 \cos\vartheta_0\, t_{ATT}$$

$$x_{ATT.} = 25 \cdot \cos(45°) \cdot 3{,}7664 =$$

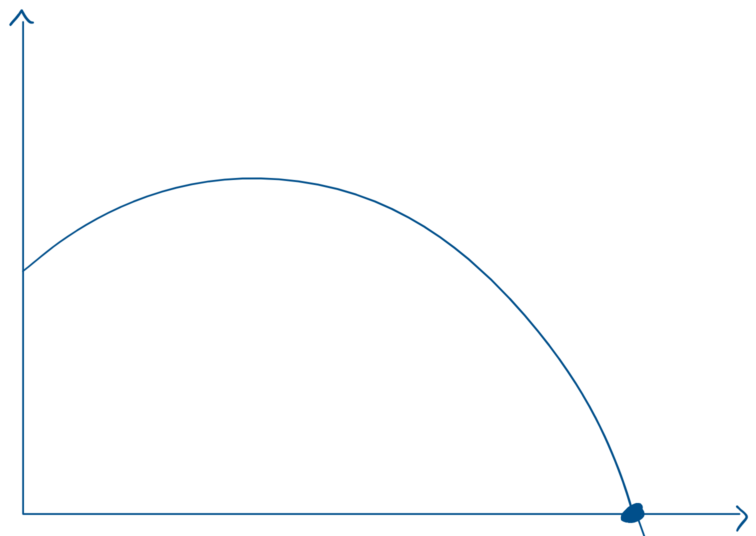
$$\boxed{x_{ATT.} = 66{,}5811\ m \simeq 70\ m\ \ (1\ c.s.)}$$

3) $\vec{v}_F$?

↓

corrisponde $t = t_{ATT.}$

$t_{ATT} = 3{,}7664\ s$



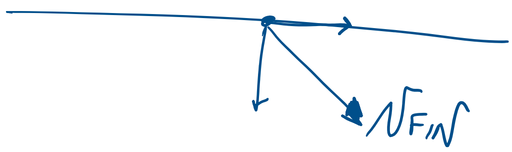
$\vec{v}_{FIN}$

$$\begin{cases} v_x = v_{0x} = v_0 \cos\vartheta_0 = 25 \cos(45°) \\ v_y = v_{0y} - g\, t_{ATT.} = v_0 \sin\vartheta_0 - g\, t_{ATT.} = \end{cases}$$

$$v_y = v_{0y} - g\,t_{ATT.} = v_0 \sin\vartheta_0 - g\,t_{ATT.} =$$

$$= 25 \sin(45°) - 9{,}81 \cdot 3{,}7664$$

$$\begin{cases} v_x = 17{,}6777 \ m/s \\ v_y = -19{,}2707 \ m/s \end{cases}$$



$$v_F = \sqrt{v_x^2 + v_y^2} = 26{,}1505 \ m/s$$

$$\boxed{v_F \simeq 30 \ m/s \ (1\,c.s.)}$$