

- 9** $x^2 + 2x + 1 \geq 0$; $x^2 - x + 4 > 0$. $[\forall x \in \mathbb{R}; \forall x \in \mathbb{R}]$
- 10** $-x^2 + 2x - 5 > 0$; $-9x^2 + 12x - 4 \geq 0$. [nessun valore di x ; $x = \frac{2}{3}$]
- 11** $x^2 - 4x + 4 > 0$; $x^2 + 4x + 4 \leq 0$. $[x \neq 2; x = -2]$
- 12** $4x^2 - 4x + 1 < 0$; $x^2 + x + 3 > 0$. [nessun valore di x ; $\forall x \in \mathbb{R}$]
- 13** $2x^2 + 3x - 2 \geq 0$; $3x^2 + 5x - 2 < 0$. $[x \leq -2 \vee x \geq \frac{1}{2}; -2 < x < \frac{1}{3}]$
- 14** $x^2 + x + \sqrt{3} > 0$; $2x^2 - x + \sqrt{2} < 0$. $[\forall x \in \mathbb{R}; \text{nessun valore di } x]$
- 15** $x^2 - 8x + 16 > 0$; $x^2 + 5x + 7 \leq 0$. $[x \neq 4; \text{nessun valore di } x]$
- 16** $4x^2 - 12x + 9 \geq 0$; $4x^2 - 9 > 0$. $[\forall x \in \mathbb{R}; x < -\frac{3}{2} \vee x > \frac{3}{2}]$
- 17** $2x^2 + 7 > 0$; $5x - x^2 > 0$. $[\forall x \in \mathbb{R}; 0 < x < 5]$
- 18** $x^2 - 16 \geq 0$; $-x^2 + 8x - 12 > 0$. $[x \leq -4 \vee x \geq 4; 2 < x < 6]$
- 19** $-x^2 > 0$; $6x^2 - 5x + 1 \geq 0$. [nessun valore di x ; $x \leq \frac{1}{3} \vee x \geq \frac{1}{2}$]
- 20** $9x^2 + 8x - 1 < 0$; $7x - 2x^2 < 0$. $[-1 < x < \frac{1}{9}; x < 0 \vee x > \frac{7}{2}]$
- 21** $x^2 - 10x + 25 < 0$; $4 - x^2 \leq 0$. [nessun valore di x ; $x \leq -2 \vee x \geq 2$]
- 22** $2x^2 + 16x + 32 > 0$; $x^2 + 10x + 25 \leq 0$. $[x \neq -4; x = -5]$
- 23** $-x^2 + 8x - 7 \leq 0$; $x^2 - 3x - 4 < 0$. $[x \leq 1 \vee x \geq 7; -1 < x < 4]$
- 24** $5x - 15x^2 > 0$; $x - 7x^2 - 2 < 0$. $[0 < x < \frac{1}{3}; \forall x \in \mathbb{R}]$
- 25** $x^2 - 8x + 15 < 0$; $x^2 + 13x + 36 \leq 0$. $[3 < x < 5; -9 \leq x \leq -4]$
- 26** $x^2 - 6\sqrt{2}x + 16 < 0$; $4x^2 - 4\sqrt{5}x + 1 < 0$. $[2\sqrt{2} < x < 4\sqrt{2}; \frac{\sqrt{5}-2}{2} < x < \frac{\sqrt{5}+2}{2}]$
- 27** $x^2 + \sqrt{7}x - 14 \geq 0$; $x^2 - 2\sqrt{3}x + 2 > 0$. $[x \leq -2\sqrt{7} \vee x \geq \sqrt{7}; x < \sqrt{3}-1 \vee x > \sqrt{3}+1]$
- 28** $\frac{x^2}{4} - x < \frac{21}{4}$; $4x^2 - \frac{1}{4} < 0$. $[-3 < x < 7; -\frac{1}{4} < x < \frac{1}{4}]$
- 29** $25(4x-1) + (x-5)^2 \geq 0$. $[x \leq -90 \vee x \geq 0]$

$$\mathbf{30} \quad 4x + 21 - x^2 > 0; \quad 5x^2 < 8x - \frac{16}{5}. \quad [-3 < x < 7; \text{nessun valore di } x]$$

$$\mathbf{31} \quad 4(x^2 - 1) < 4x - 1; \quad (3x + 2)^3 \geq (3x - 2)^3. \quad \left[-\frac{1}{2} < x < \frac{3}{2}; \forall x \in \mathbb{R}\right]$$

$$\mathbf{32} \quad \frac{3(x^2 - 1)}{4} > 3x^2 + \frac{5}{2}. \quad [\text{nessun valore di } x]$$

$$\mathbf{33} \quad \frac{x-1}{5} + \frac{1}{3} < \frac{(x-2)(x-3)}{6}. \quad \left[x < 1 \vee x > \frac{26}{5}\right]$$

$$\mathbf{34} \quad \frac{2x - (3 + x^2)}{2} - 1 > \frac{3 - x^2}{4}. \quad [\text{nessun valore di } x]$$

$$\mathbf{35} \quad (x+5)(x^2 - 5x + 25) < (x+5)^3. \quad [x < -5 \vee x > 0]$$

$$\mathbf{36} \quad (2x-1)(4x^2 + 2x + 1) \leq (2x-1)^3. \quad \left[0 \leq x \leq \frac{1}{2}\right]$$

$$\mathbf{37} \quad 3(x-1)(x+3) > 7 + 2(3x-2). \quad [x < -2 \vee x > 2]$$

$$\mathbf{38} \quad 4x(x-2) < 11 + (x-4)^2. \quad [-3 < x < 3]$$

$$\mathbf{39} \quad (2x-1)(x-3) - (x-1)[2(2x-1) + x] < 0. \quad \left[x < -\frac{\sqrt{3}}{3} \vee x > \frac{\sqrt{3}}{3}\right]$$

$$\mathbf{40} \quad x - (2x - \sqrt{5})(x + \sqrt{5}) + \sqrt{5} \leq 0. \quad \left[x \leq -\sqrt{5} \vee x \geq \frac{1 + \sqrt{5}}{2}\right]$$

$$\mathbf{41} \quad (x+5)^2 - (x-1)(2x+1) > 13(x+2). \quad [-2 < x < 0]$$

$$\mathbf{42} \quad [x - 4(1-x) + 2](1-x) < x - 2(1-2x). \quad \left[x < 0 \vee x > \frac{2}{5}\right]$$

$$\mathbf{43} \quad \frac{1-3x}{5} - \frac{(2-x)(2+x)}{3} < x - \frac{6}{5} + \frac{1+x^2}{15}. \quad [0 < x < 6]$$

$$\mathbf{44} \quad \frac{5+3x^2}{6} \geq \frac{1}{4} \left(3 + \frac{1}{3} + 2x^2\right) - \frac{x^2-4}{3}. \quad [x \leq -2 \vee x \geq 2]$$

$$\mathbf{45} \quad [x^2 - (1-x)^2](1-2x) + 10 < 3(x-1) + x(1-x). \quad [x < -2 \vee x > 2]$$

Disequazioni frazionarie

Risolvere le seguenti disequazioni frazionarie numeriche:

- 1 $\frac{2x-4}{x^2+x+2} \geq 0;$ $\frac{x^2}{x^2-1} \leq 0.$ $[x \geq 2; -1 < x < 1]$
- 2 $\frac{3-2x}{2x^2+1} \geq 0;$ $\frac{8x-x^2-7}{9x^2-8x-1} > 0.$ $\left[x \leq \frac{3}{2}; -\frac{1}{9} < x < 7 \wedge x \neq 1\right]$
- 3 $\frac{x^2+5x+4}{x^2-5x-6} < 0;$ $\frac{x^2}{x^2-4} \geq 0.$ $[-4 < x < 6 \wedge x \neq -1; x < -2 \vee x = 0 \vee x > 2]$
- 4 $\frac{x^2-2x+1}{x-2} < 0;$ $\frac{x^2-3x}{x^2+2} > 0.$ $[x < 2 \wedge x \neq 1; x < 0 \vee x > 3]$
- 5 $\frac{x^2-3x+5}{x^2-1} \leq 0;$ $\frac{5x^2-4x}{x^2-6x+9} \leq 0.$ $\left[-1 < x < 1; 0 \leq x \leq \frac{4}{5}\right]$
- 6 $\frac{5x^2-3x-2}{9x^2+15x-6} > 0;$ $\frac{5x^2-4x}{x^2-6x+9} > 0.$ $\left[x < -2 \vee -\frac{2}{5} < x < \frac{1}{3} \vee x > 1; x < 0 \vee \frac{4}{5} < x < 3 \vee x > 3\right]$
- 7 $\frac{x^2-9}{x^2+8x+16} \geq 0;$ $\frac{x^2-5x+6}{x^2-3x-10} > 0.$ $[(x \leq -3 \wedge x \neq -4) \vee x \geq 3; x < -2 \vee 2 < x < 3 \vee x > 5]$
- 8 $\frac{x^2+10x-56}{x^2-2x-48} > 0;$ $\frac{2x^2+x-1}{x^2-2x} \geq 0.$ $\left[x < -14 \vee -6 < x < 4 \vee x > 8; x \leq -1 \vee 0 < x \leq \frac{1}{2} \vee x > 2\right]$
- 9 $\frac{4x^2+4x+1}{x^2+x+5} \leq 0;$ $\frac{2}{x^2+1} - 1 > 0.$ $\left[x = -\frac{1}{2}; -1 < x < 1\right]$
- 10 $\frac{3x^2-x-2}{6x^2-x-7} < 0;$ $x \geq \frac{1}{4x-3}.$ $\left[-1 < x < -\frac{2}{3} \vee 1 < x < \frac{7}{6}; -\frac{1}{4} \leq x < \frac{3}{4} \vee x \geq 1\right]$
- 11 $\frac{x-1}{x+1} + \frac{3}{x} < \frac{1}{4};$ $\frac{x-2}{2} > \frac{3}{x-1}.$ $[-1 < x < 0; -1 < x < 1 \vee x > 4]$
- 12 $1 - \frac{6}{1-4x^2} > \frac{2}{2x-1} - \frac{3}{2x+1}.$ $\left[x < -\frac{1}{2} \vee -\frac{1}{2} < x < 0 \vee x > \frac{1}{2}\right]$
- 13 $\frac{x}{x+2} < 5 - \frac{x}{3-x};$ $\frac{x}{6} - \frac{1}{x-6} > \frac{x-1}{6} - \frac{x-1}{6-x} + \frac{x}{6}.$ $[x < -\sqrt{6} \vee -2 < x < \sqrt{6} \vee x > 3; x < 6]$
- 14 $\frac{1}{x-2} - \frac{1}{x} > \frac{2}{15};$ $\frac{x}{2} > 1 - \frac{1}{2x}.$ $[-3 < x < 0 \vee 2 < x < 5; x > 0 \wedge x \neq 1]$
- 15 $\frac{x-5}{3} > \frac{3}{x-5};$ $\frac{x-1}{x+1} + \frac{3}{x} < \frac{1}{4}.$ $[2 < x < 5 \vee x > 8; -1 < x < 0]$

$$16 \quad \frac{x^2 - 4x + 4}{3x^2 - 5x + 2} > 0;$$

$$\frac{x-5}{x+3} \leq \frac{8-x}{3-x}.$$

$$\left[x < \frac{2}{3} \vee (x > 1 \wedge x \neq 2); -3 < x < 3 \vee x \geq 13 \right]$$

$$17 \quad \frac{x^2 - 4}{3x} - \frac{1}{x} \leq \frac{1}{2} - \frac{2}{x}.$$

$$\left[x \leq -\frac{1}{2} \vee 0 < x \leq 2 \right]$$

$$18 \quad \frac{2}{5} \left(\frac{1}{x-2} - 1 \right) < \frac{1}{5(x^2 + x - 6)} - \frac{2+x}{x+3}.$$

$$[-3 < x < -1 \vee 1 < x < 2]$$

$$19 \quad \frac{1}{x+1} \left(\frac{6}{x-1} + 3 \right) < 1 + \frac{2}{x-1}.$$

$$[(x < 1 \wedge x \neq -1) \vee x > 2]$$

$$20 \quad (x-1) \left(1 + \frac{2}{x} \right) + x < 3 - \frac{2}{x}.$$

$$[x < 1 \wedge x \neq 0]$$

$$21 \quad \frac{x+1}{x^2-4} - \frac{1}{x+2} - \frac{2-x}{x^2+4x+4} \leq 0.$$

$$[x < 2 \wedge x \neq -2]$$

$$22 \quad \frac{3}{x^2-5x+6} + \frac{4-x}{3-x} > \frac{6-x}{2-x}; \quad \frac{x}{\sqrt{3}} - 1 > 1 - \frac{\sqrt{3}}{x}. \quad \left[2 < x < \frac{7}{3} \vee x > 3; x > 0 \wedge x \neq \sqrt{3} \right]$$

$$23 \quad (\sqrt{2} + 1)x > 2 - \frac{1}{x}; \quad \frac{8}{x+2} \geq \frac{x+2}{x}. \quad [x > 0; -2 < x < 0 \vee x = 2]$$

$$24 \quad 9 + \frac{367}{x^2-x} + \frac{3}{x-1} < 0; \quad \frac{x}{x+3} - \frac{1}{4} < \frac{x-1}{2x}. \quad [0 < x < 1; -3 < x < 0 \vee 1 < x < 6]$$

$$25 \quad x - 2\sqrt{3} \leq \frac{1-4\sqrt{3}}{x+2\sqrt{3}}. \quad [x < -2\sqrt{3} \vee -2\sqrt{3} + 1 \leq x \leq 2\sqrt{3} - 1]$$

$$26 \quad \frac{x+2}{x-2} - \frac{2-x}{x+3} > \frac{10}{x(x+1)-6}. \quad \left[x < -3 \vee -\frac{1}{2} < x < 0 \vee x > 2 \right]$$

$$27 \quad \frac{x+1}{x^2-4} - \frac{1}{x+2} - \frac{2-x}{x^2+4x+4} \leq 0; \quad \frac{2x-1}{2x^2+x+1} + \frac{x-1}{x+1} > 1. \quad [x < 2 \wedge x \neq -2; x < -1]$$

$$28 \quad \frac{5x}{1-x} < \frac{8}{x-2}. \quad \left[x < \frac{1-\sqrt{41}}{5} \vee 1 < x < \frac{1+\sqrt{41}}{5} \vee x > 2 \right]$$

$$29 \quad \frac{x^2-2}{x^3-2x^2+4x-8} + \frac{x+2}{x^2+4} > \frac{1}{x-2}. \quad [-\sqrt{10} < x < 2 \vee x > \sqrt{10}]$$

$$30 \quad \frac{1}{x^2-5x+6} - \frac{x+2}{x-2} \geq \frac{x+3}{x-3}. \quad \left[-\sqrt{\frac{13}{2}} \leq x < 2 \vee \sqrt{\frac{13}{2}} \leq x < 3 \right]$$

$$31 \quad \frac{2x}{3x-1} - \frac{3-x^2}{9x^2-6x+1} < 1. \quad \left[x \neq \frac{1}{3} \right]$$

Sistemi di disequazioni di grado superiore al primo

Risolvere i seguenti sistemi:

$$1 \quad \begin{cases} x^2 - 4x + 3 \leq 0 \\ x^2 - 4 > 0 \end{cases} ; \quad \begin{cases} x^2 - 7x - 8 \geq 0 \\ \frac{2x+5}{x-4} < \frac{x+3}{8-2x} \end{cases} \quad \left[2 < x \leq 3; -\frac{13}{5} < x \leq -1 \right]$$

$$2 \quad \begin{cases} 3x^2 - 4x - 7 < 0 \\ \frac{4x-6}{3} + x < x+1 \end{cases} ; \quad \begin{cases} 3x^2 - x + 5 < 0 \\ \frac{x+2}{3} - x > \frac{x-1}{4} \end{cases} \quad \left[-1 < x < \frac{9}{4}; \text{impossibile} \right]$$

$$3 \quad \begin{cases} x^2 + 3x > 4 \\ x(x+1) - 6x + 6 > 0 \end{cases} \quad [x < -4 \vee 1 < x < 2 \vee x > 3]$$

$$4 \quad \begin{cases} 3x^2 - x - 2 > 0 \\ 6x^2 - x - 7 > 0 \end{cases} ; \quad \begin{cases} x^2 - 5x + 6 < 0 \\ x^2 + 3x - 4 > 0 \end{cases} \quad \left[x < -1 \vee x > \frac{7}{6}; 2 < x < 3 \right]$$

$$5 \quad \begin{cases} x^2 - 4x + 4 > 0 \\ x^2 + 3x + 5 > 0 \end{cases} ; \quad \begin{cases} 6x^2 + 7x + 2 < 0 \\ 4x^2 - 3x + 15 > 0 \end{cases} \quad \left[x \neq 2; -\frac{2}{3} < x < -\frac{1}{2} \right]$$

$$6 \quad \begin{cases} (3x-1)^2 > (x+5)^2 + 2 \\ x^2 - 2x + 1 < 0 \end{cases} \quad [\text{nessuna soluzione}]$$

$$7 \quad \begin{cases} (x-\sqrt{3})(x+\sqrt{3}) > 0 \\ x^2 - 6x < 0 \end{cases} \quad [\sqrt{3} < x < 6]$$

$$8 \quad \begin{cases} x^2 - 4x + 3 \geq 0 \\ 3x - x^2 \geq 0 \end{cases} ; \quad \begin{cases} 3x^2 - 2x - 3 > 0 \\ 11x > \frac{16}{5} \end{cases} \quad \left[0 \leq x \leq 1 \vee x = 3; x > \frac{1+\sqrt{10}}{3} \right]$$

$$9 \quad \begin{cases} \frac{1}{6}x^2 - \frac{5}{6}x + 1 > 0 \\ x + \frac{3}{2} > \frac{1}{2}x + 1 \end{cases} \quad [-1 < x < 2 \vee x > 3]$$

$$10 \quad \begin{cases} (5x+1)^2 > 10x+1 \\ x^2 + x\sqrt{2} \leq 2(x+\sqrt{2}) \end{cases} \quad [-\sqrt{2} \leq x \leq 2 \wedge x \neq 0]$$

$$11 \quad \begin{cases} x^2 - 5x + 6 \geq 0 \\ 2(2x-9) < x \\ x^2 + 2x - 15 \leq 0 \end{cases} ; \quad \begin{cases} 2x^2 - x - 6 \geq 0 \\ \frac{1}{3} + \frac{x-2}{6} \leq \frac{1}{2} \\ (2x-1)(1+x) \leq 5+2x \end{cases} \quad \left[-5 \leq x \leq 2 \vee x = 3; x = -\frac{3}{2} \vee x = 2 \right]$$

Valori assoluti

Equazioni, disequazioni e sistemi in cui compaiono valori assoluti di espressioni contenenti l'incognita

- | | | |
|-----------|---|--|
| 1 | $\left \frac{x^2 - 4}{x^2 - x + 3} \right = 1.$ | $\left[-\frac{1}{2}; 1; 7 \right]$ |
| 2 | $ x^2 - 6 = x.$ | $[2; 3]$ |
| 3 | $ x^2 + 3x = 2(x + 6).$ | $[-4; 3]$ |
| 4 | $ -x^2 + 4x + 1 = 2x + 2.$ | $[1; 3 + 2\sqrt{3}]$ |
| 5 | $ x^2 - 1 = x + 2.$ | $\left[\frac{1 \pm \sqrt{13}}{2} \right]$ |
| 6 | $ -2x^2 + 3x = x.$ | $[0; 2; 1]$ |
| 7 | $\left \frac{1}{3}x - x^2 + 1 \right = 2x + 1.$ | $[0; 3]$ |
| 8 | $ x^2 + 4x + 4 - 3x - 1 = 4x + 1.$ | $[-2; -1]$ |
| 9 | $3 < x^2 - 1 .$ | $[x < -2 \vee x > 2]$ |
| 10 | $ x^2 - 4 < 5.$ | $[-3 < x < 3]$ |

Risoluzione di equazioni irrazionali contenenti radicali quadratici

Risolvere le seguenti equazioni irrazionali numeriche:

- | | | |
|--|--------------------------|--------------------------------|
| 1 $\sqrt{x+2} = 4;$ | $\sqrt{2-x} = 1.$ | [14; 1] |
| 2 $\sqrt{4-3x} = 1;$ | $\sqrt{4+2x} = 6.$ | [1; 16] |
| 3 $\sqrt{6x-14} - 2 = 0;$ | $\sqrt{5+4x} = 11.$ | [3; 29] |
| 4 $\sqrt{3x+4} = \sqrt{7};$ | $\sqrt{9x-5} = 2.$ | [1; 1] |
| 5 $\sqrt{x+3} = x+1;$ | $\sqrt{x^2+4x} = x.$ | [1; 0] |
| 6 $\sqrt{x^2+21} = x+3;$ | $\sqrt{3x+7} = x-1.$ | [2; 6] |
| 7 $\sqrt{x^2-11} + 1 = x;$ | $\sqrt{x^2+3x+1} = x-1.$ | [6; impossibile] |
| 8 $\sqrt{x-3} = 3x-11;$ | $\sqrt{3-x} = 1-x.$ | [4; -1] |
| 9 $\sqrt{x-3} = 11-3x;$ | $\sqrt{5+2x} = 3x-3.$ | $\left[\frac{31}{9}; 2\right]$ |
| 10 $\sqrt{4x-3} = \sqrt{6x-1};$ | $\sqrt{3x-8} + 2 = x.$ | [impossibile; 3 e 4] |