MANAGERIAL ECONOMICS

Corrado Pasquali

UNIVERSITÀ
DEGLI STUDI
DI TERAMO

4. QUICK MICROECONOMICS RECAP

→ We focus on how individuals make decisions.

→ In doing so, we will set a definition of rationality.

→ Our main character is the individual decision maker.

→ The decision maker does nothing but choosing an element from a given set.

→ We will prove that the decision maker's preferences can be represented by a utility function

→ We will prove that the decision maker chooses her preferred element in the choice set as if she were maximizing her utility function in the choice set.

WE START BY DEFINING THE SET THAT THE DECISION MAKER CHOOSES FROM

The Choice Set

The choice set is represented by X: an arbitrary set of objects such as a list of consumption bundles. Thus we could have:

$$X = \{\textit{Dyson hairdryer}, \textit{McQueen shoes}, \textit{Nike Tech tracksuit}, \textit{Subdued clothes}\}$$

or, more abstractly (and more cheaply)

$$X = \{a, x, y, w, z\}$$

WE THEN PROCEED TO STATE HOW (S)HE COMPARES ANY TWO GIVEN ELEMENTS IN THE CHOICE SET

Preferences

Preferences are represented by a preference relation, denoted by $\succeq$ (please, do note that $\succeq$ is not $\geq$).

This preference relation is what the decision maker uses to compare any pair of alternatives $x, y \in X$.

The expression

$$x \succeq y$$

is read as "x is at least as good as y" or "x is weakly preferred to y" for our decision maker.

"Weakly" means that either the decision maker likes x better than y or else she is indifferent between the two.

Preferences

Two further relations are then derived: the strict preference relation and the indifference relation.

The strict preference relation $\succ$ is so defined:

$$x \succ y \quad \text{iff} \quad x \succeq y \quad \text{but not} \quad y \succeq x.$$

On the other hand, the indifference relation is defined as:

$$x \sim y \quad \text{iff} \quad x \succeq y \quad \text{and} \quad y \succeq x.$$

WE THEN ASK TWO CONDITIONS TO BE MET FOR THE DECISION MAKER TO QUALIFY AS RATIONAL

Rationality

We identify "rationality" with two assumptions on the preference relation:

i) completeness

ii) transitivity.

Commpleteness

Definition

We will say that $\succeq$ is complete iff $\forall x, \forall y \in X$ you either have $x \succeq y$ or $y \succeq x$ or both.

So, completeness means that any and every two elements of the choice set can be compared.

Transitivity

Definition

We will say that $\succeq$ is transitive iff $\forall x, \forall y, \forall z \in X$, if $x \succeq y$ and $y \succeq z$ then $x \succeq z$.

Have a look at the "Money Pump" example posted on Unite E-Learn.

WE NOW LAY DOWN THE FIRST BRICK TO TRANSFORM A CHOICE INTO A MAXIMIZATION PROBLEM

Representing utility

The very fact that $\succeq$ is complete and transitive allows us to represent $\succeq$ by a utility function.

Utility functions

Definition

A function $u : X \to \mathbb{R}$ is a utility function representing $\succeq$ if $\forall x, \forall y \in X$ we have:

$$x \succeq y \Leftrightarrow u(x) \geq u(y).$$

Representation theorem

The following representation theorem is a cornerstone for economic analysis. We will limit ourselves to the statement (proving it is not that hard, though! Give it a try!)

Theorem

A preference relation $\succeq$ can be represented by a utility function iff it is complete and transitive.

Choice

A rational agent chooses then the element x of the choice set X to which the highest utility $u(x)$ is associated.

Formally, a rational agent choice problem is represented in these terms:

$$max_{x \in X}\ u(x).$$

LET NOW SOME CONSTRAINTS ENTER THE SCENE

Constraints

The very core of the matter is the decision maker being forced to face some constraints (we are talking "economics" after all!!).

Under this perspective, the consumer problem is the classic example.

Constraints

Le m be a fixed amount of money available for the consumer.

We then let $p = (p_1, \ldots, p_k)$ be a vector of prices for $1, \ldots, k$ goods.

Let then the set of affordable objects for the consumer be:

$$B = \{x \in X : p_1 x_1 + \ldots + p_k x_k \leq m\}$$

B is called the *budget set* and $p_1 x_1 + \ldots + p_k x_k \leq m$ is called the *budget constraint*.

Constraints

In vector notation, we can write $px \leq m$ and have the consumer problem written as:

$$choose\ x\ to\ maximize\ u(x)\ subject\ to\ x \in B$$

or equivalently:

$$choose\ x\ to\ maximize\ u(x)\ subject\ to\ x \in X\ and\ px \leq m$$

or as we usually do as:

$$max_{x \in B}\ u(x)$$

Constraints

Let us consider an economy with two goods: $X = \mathbb{R}^2_+$. We shall call (x_1, x_2) a consumption bundle of x_1 units of good 1 and x_2 units of good 2. Any such bundle is thus a point on a two-dimensional graph.

Prices of the goods p_1 and $_2$ are known and given, the consumer is a price taker. Let m be the consumer's income.

The consumer's budget constraint is thus: $p_1x_1 + p_2x_2 \leq m$.

The budget set

The *budget set* is the set of all bundles that are affordable at a given price and income:

$$B = \{x \in X : p_1 x_1 + p_2 x_2 \leq m\}.$$

The budget line is the set of bundles such that:

$$p_1 x_1 + p_2 x_2 = m$$

This can also be written as:

$$x_2 = \frac{p_1}{p_2} x_1 + \frac{m}{p_2}.$$

This is a linear function with vertical intercept m/p_2, horizontal intercept m/p_1 and a slope of $-p_1/p_2$.

The budget set

The slope of the budget line measures the rate at which the market is willing to exchange good 1 for good 2.

So, the slope measures the opportunity cost of consuming good 1: the consumer has an alternative opportunity to consuming 1 unit of good 1, consisting in selling the unit of good 1, for an amount equal to p_1, and then using the proceeds to buy good 2 – that is $\frac{p_1}{p_2}$ units of good 2.

We also know that when prices or income change, then the budget line moves. We won't expand on this.

AND NOW TO INDIFFERENCE

Indifference curves

Indifference curves are level curves, that is, they represent the set of $(x_1; x_2)$ for which the utility is constant at some level, say U as in the following figure:

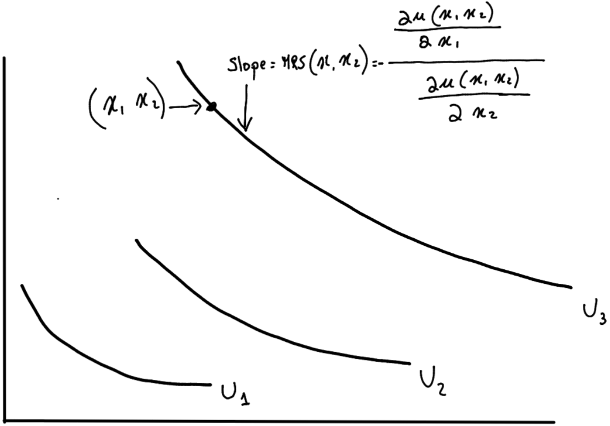


Figure: Indifference curves and the MRS

WE NOW NEED TWO ASSUMPTIONS TO BE SURE THAT INDIFFERENCE CURVES ARE REALLY CURVES AND THAT THEY ARE CONVEX

Two assumptions: monotonicity

The first is called *monotonicity* i.e. “more is better than less”.

Definition

Consider (x_1, x_2) as a bundle of goods and let (x_1', x_2') be any other bundle with at least as much of both goods and more of one. That is, $x_1' \geq x_1$ and $x_2' \geq x_2$ with at least one strict inequality. (Strict) monotonicity of preferences requires that:

$$\text{if } (x_1', x_2') > (x_1, x_2) \text{ then } (x_1', x_2') \succ (x_1, x_2)$$

Two assumptions: monotonicity

Thus, (strict) monotonic preferences imply that more of (resp. less) of both goods is a better (resp. worse) bundle. Monotonicity of preferences implies that the utility function which represents preferences is monotonic increasing in both its arguments:

$$\frac{\partial u(x_1, x_2)}{\partial x_1}, \frac{\partial u(x_1, x_2)}{\partial x_2} > 0; \text{for any } (x_1, x_2) \in X$$

Strict monotonicity also implies that indifference curves have a negative slope:

$$-MRS(x_1, x_2) < 0$$

Two assumptions: convexity

Definition

A set is convex if with $x, y \in X$ and $\alpha \in [0,1]$ then $\alpha x + (1-\alpha)y \in X$.

That is a set is convex if it contains the entire segment joining any two of its elements.

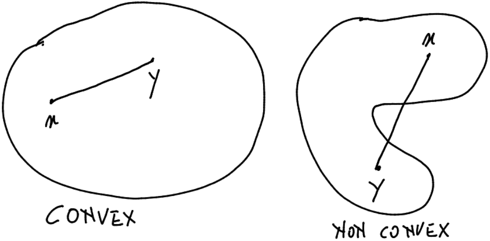


Figure: Convex and non convex sets

Two assumptions: convexity

Convexity of preferences requires that the set of weakly preferred allocations to any given allocation is convex.

Formally, for any $(x_1, x_2) \in X$; the set $\{(y_1, y_2) \in X : (y_1, y_2) \succeq (x_1, x_2)\}$ is convex.

Convexity of preferences implies that the utility function representing preferences is concave and that indifference curves are convex.

WE CAN NOW TURN THE CONSUMER PROBLEM TO A FULL BLOWN FORMALIZATION

The consumer problem I

$$\max_{x \in X} u((x_1, x_2)) \text{ s.t. } p_1 x_1 + p_2 x_2 = m$$

under the assumptions that the preference relation represented by $u(x_1, x_2)$ are monotonic and convex.

The consumer problem II

Painted with an extremely broad brush: a bundle (x_1^*, x_2^*) is an optimal choice for the consumer if the set of bundles that the consumer prefers to (x_1^*, x_2^*) (i.e. the set of bundles above the indifference curve through (x_1^*, x_2^*)) has an empty intersection with the bundles she can afford (i.e. the bundles beneath her budget line).

The consumer problem III

It then follows that at (x_1^*, x_2^*) the indifference curve is tangent to the budget line:

$$MRS(x_1^*, x_2^*) = \frac{\alpha x_2^*}{\beta x_1^*} = -\frac{p_1}{p_2}$$

and at (x_1^*, x_2^*) the budget constraint must be satisfied:

$$p_1 x_1^* + p_2 x_2^* = m$$

The consumer problem IV

Now that we have these two equations in two unknowns that can be solved for the optimal bundle, we can derive the demand function for good 1:

$$x_1^* = \frac{\alpha}{\alpha + \beta} \; \frac{m}{p_1}$$

and for good 2:

$$x_2^* = \frac{\alpha}{\alpha + \beta} \; \frac{m}{p_2}$$

AND NOW TO THE CORE OF THE MATTER: SOCIETY

Equilibrium and efficiency

We investigate the fundamental economic problem of allocation and price determination in a simple economy.

Our aim is to describe what outcomes might arise by giving individuals the opportunity to voluntarily exchange goods.

Equilibrium and efficiency

We will follow two simple principles:

i) Rationality - individuals choose the best patterns of consumption that are affordable for them;

ii) Equilibrium - prices adjust in such a way that the amount that people demand of some good is equal to the amount that is supplied. We will then determine equilibrium prices, equating demand and supply and show that these prices solve the allocation problem efficiently.

A pure exchange economy

We will consider a pure exchange 2×2 economy (i.e. two consumers and two goods).

Let $\{1, 2\}$ denote the set of consumption goods.

Let $\{A, B\}$ denote the set of consumers.

A pure exchange economy

Consumer A and B initial endowments are denoted by:

$$w^A = (w_1^A, w_2^A)$$
$$w^B = (w_1^B, w_2^B)$$

Consumer A's and B's consumption bundles are denoted by:

$$x^A = (x_1^A, x_2^A)$$
$$x^B = (x_1^B, x_2^B)$$

Allocations

Definition

An allocation is a pair of bundles x^A and x^B.

An allocation is feasible if:

$$x_1^A + x_1^B = w_1^A + w_1^B$$

and

$$x_2^A + x_2^B = w_2^A + w_2^B$$

WE NOW NEED A TOOL TO (PAIRWISE) COMPARE DIFFERENT SOCIETAL ALLOCATIONS

Pareto efficiency

Definition

A feasible allocation $x^A + x^B$ is Pareto efficient if there is no other feasible allocation $y^A + y^B$ such that $y^A \succeq x^A$ and $y^B \succeq x^B$ with at least one $\succ$.

Pareto efficiency

That is, an allocation is Pareto efficient if it is feasible and there is no other feasible allocation for which one consumer is at least as well off and the other consumer is strictly better off.

This implies that for Pareto efficient allocation

i) there is no way to make both consumers strictly better or,

ii) all of the gains from trade have been exploited, that is, there are no mutually advantageous trades to be made.

The Pareto frontier

This can geometrically be easily seen in a graphical representation of the Pareto Frontier. (use the blackboard here)

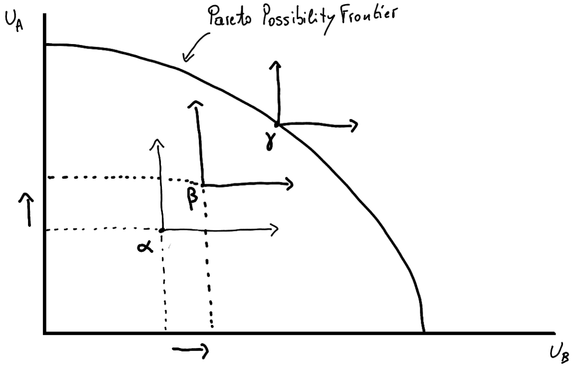


Figure: The Pareto Possibility Frontier

MEET THE EDGEWORTH BOX

The Edgeworth Box

The Edgeworth Box is a simple way to illustrate some key aspects of a pure exchange economy.

Let us consider a 2x2 economy, let the decision makers be $\{Adam, Eve\}$ and finally let $\{Beer, Wine\}$ be the set of goods.

Adam

We first represent a cartesian space for Adam (henceforth A), let its origin be O_L.

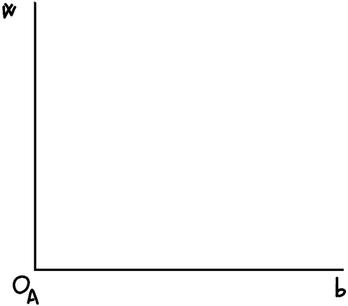


Figure: Adam's coordinates

Eve

We then represent Eve's space in the very same way.

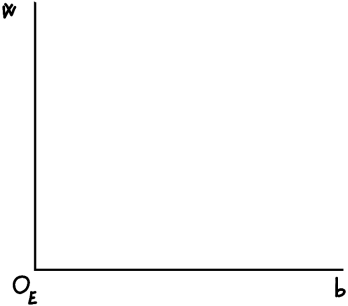


Figure: Eve's coordinates

Adam and Eve

Finally, we flip Eve's coordinates so that their origin lies orthogonally w.r.t. Adam's and we get:

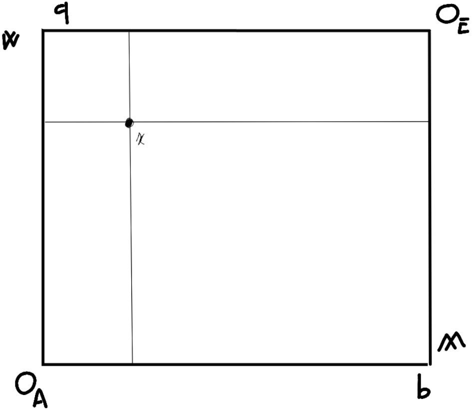


Figure: The Edgeworth box.

Indifference curves for A&E

We can then have some indifference curves drawn for the two

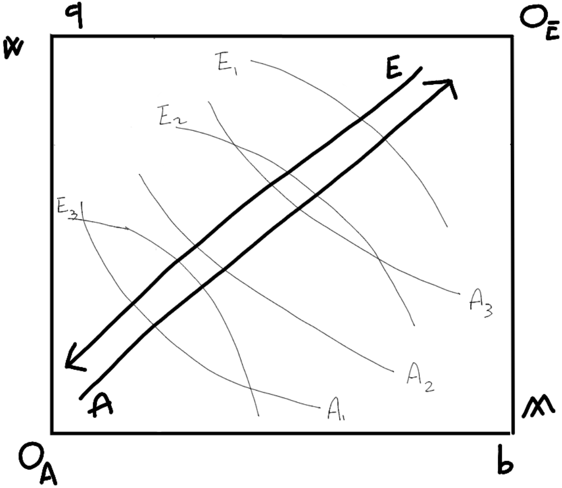
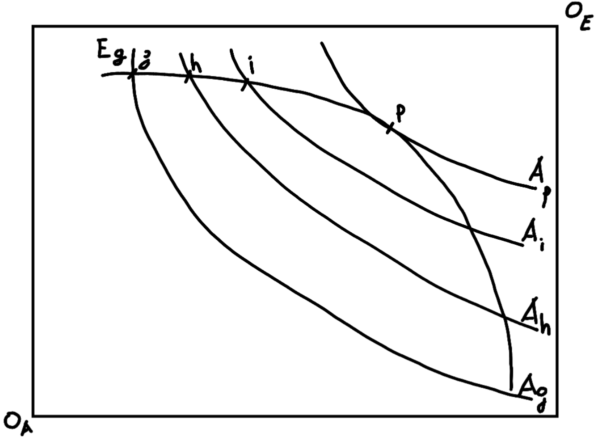


Figure: Indifference curves in the Edgeworth box.

Making Adam better off



Question

Can we imagine a simultaneous improvement for both agents at the same time?

Have a look at the following graph:

Making both better off

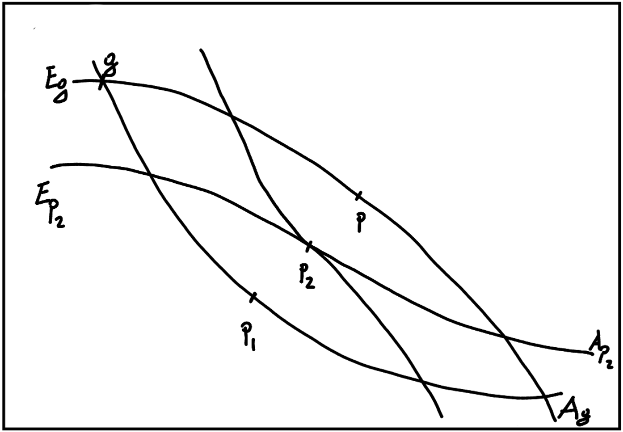


Figure: Improving both agents