MANAGERIAL ECONOMICS

Corrado Pasquali

7. AGENCY THEORY SECOND PART

Introduction

- We will now try and examine the class of contractual implications of informational asymmetries.
- To that end, we will start from a somehow unrealistic assumption: let information be perfect and let A be a social maximizer of efficiency
- We will use the results from this totally unrealistic assumption as a benchmark for what follows.
- Not surprisingly, we will first examine what is called a "first best" solution for the P/A model.

First best

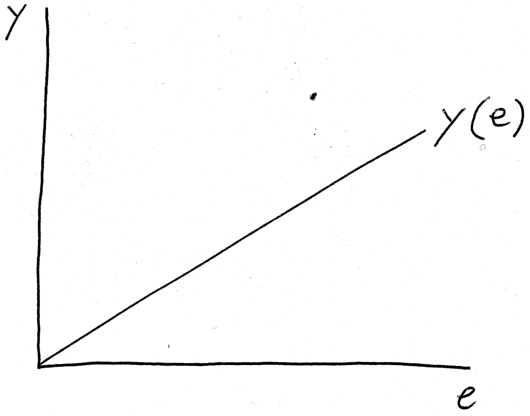
Building blocks and main assumptions

- Let there be a principal P and an agent A
- Let info be perfect
- Let effort e be perfectly observable from P
- A maximizes social surplus
- Once again: these assumptions are totally "heroic" and utterly unrealistic. They are only made to sketch what the optimal solution for an agency relation looks like.

First best

Building blocks and main assumptions

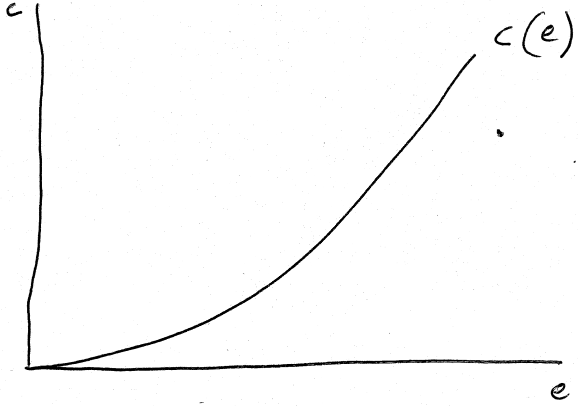
- A determines y thanks to his effort e.
- We thus have $y(e)$
- $y(e)$ is a linear increasing function
- $y'(e)$ is thus constant.


Y
Y(e)
e

First best

Building blocks and main assumptions

- e is a disutility for A
- let the monetary cost correspond to the cost function $c(e)$
- $c(e)$ is a convex, increasing function
- $c'(e)$ is thus increasing.


c
c(e)
e

First best

Building blocks and main assumptions

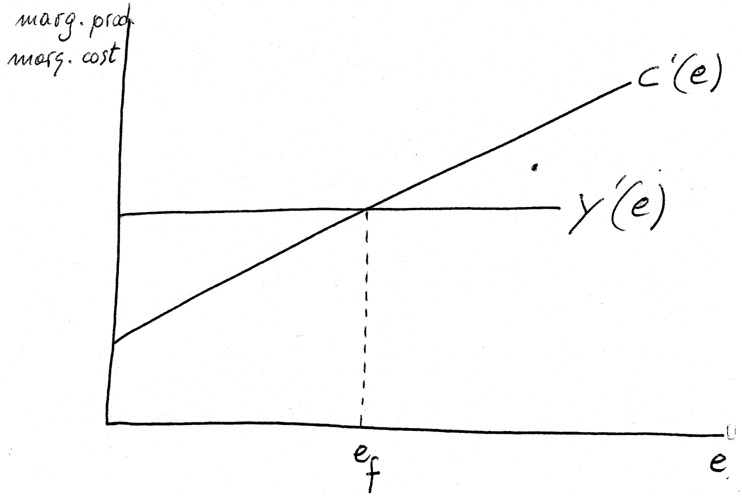
- The total surplus produced by the agency relation will be given by:

$$S = y(e) - c(e)$$

- Now, what is the optimal level of effort e_f?
- We have an easy answer: the optimal level of effort will be the one for which it holds true that:

$$y'(e_f) = c'(e_f)$$

- i.e. marginal product equals marginal cost


marg. prod.
marg. cost
c'(e)
y'(e)
e_f
e

First best

Building blocks and main assumptions

- Under this conditions, what will the contract that selects e_f look like?
- That is: what will the contract that maximizes social surplus look like?

First best

Optimal contract

- To get e_f, it will be enough that the contract forces P to compensate A with a positive wage if A chooses e_f and with $w = 0$ otherwise.
- The only other conditions to be met is that w be at least equal to A's reservation wage (i.e. that the participation constraint be met)

Second best

Main assumptions

- Let us try to be more realistic and assume that e be not observable
- While $y(e)$ is observable and measurable

Second Best

Agent's utility

- A has the following expected utility function

$$u = E[\sqrt{w} - e]$$

Second Best

Reserve utility

- Let A's reserve utilitity be:

$$u_0 = 3$$

Second Best

P's expected utility

- P's expected utility is given by:

$$\Pi = E[y - w]$$

Second Best

Effort levels

- Effort e can take up two levels:

$$e_l = 0$$

$$e_h = 4$$

Second Best

Contribution levels

- Contribution can take up two levels:

$$y_A = 200$$

$$y_B = 0$$

Second best best

Expected results

With $e_L = 0$ we have: $p(y_b) = 0.8$ and $p(y_a) = 0.2$
With $e_H = 4$ we have: $p(y_b) = 0.3$ and $p(y_a) = 0.7$

Expected results will thus be:

$0.8(0) + 0.2(200) = 40$
$0.3(0) + 0.7(200) = 140$

Note well: we will keep using these values in what follows.

Second best

Let us now take two different paths corresponding to two different assumptions:

- e is observable
- e is not observable

Second best

Assume e is observable

- In a sense, we already know the solution: P pays a positive wage if e_H has been chosen and a null wage otherwise.
- So, there isn't anything interesting hhere: everything works as it did in the First Best scenario.
- As a simple exercise, let us try and determine a wage such that e_H will be exerted.

Second best

participation constraint

- First we check that the participation constraint be satisfied
- Reminder: the agent's utility function is: $u = E[\sqrt{w} - e]$
- Thus the participation constraint is satisfied for $w = 49$, that is:

$$\sqrt{49} - 4 \leq 3$$

Second best

Results

We will thus have:

- P pays $w = 49$
- A subscribes to the contract
- A chooses e_H
- A's expected utility will be equal to 3
- Note: A is not bearing any risk

Second best

Results

- As to P, we will have:

$$\Pi(e_H = 0.3(0 - 49) + 0.7(200 - 49) = 91$$

Second best

Results

What if we had e_L?

- A would choose $e_L = 0$ if $\sqrt{w} - 0 \leq 3$ i.e. $w = 9$
- This being the case, expected utility for P would be given by:
  $\Pi(e_L = 0.8(0 - 9) + 0.2(200 - 9) = 31$

Second best

summing up

- In the first case:
  i. $w = 49$
  ii. A chooses $e_H = 4$
  iii. A's expected utility is 3
  iv. P's expected utility is 91
- In the second case:
  i. $w = 9$
  ii. A chooses $e_L = 0$
  iii. A's expected utility is 3
  iv. P's expected utility is 31

A choice of e_H thus maximizes total surplus and, at the same time, no risk is however bore by A

Second best

Assume e is not observable

- As e is now assumed to be not observable, nececessity has it to incentivize A
- This in turn means that w must be bound to y
- As we will see, this implies a loss in efficiency

Second best

the contract

- A contract will set two wage levels: w_a and w_b
- This time wages will however be linked to y_A and $y - b$
- We thus ask: what are the conditions under which A chooses e_H rather than e_L?

Second best

participation constraint

- This will be given by:

$$u(e_H) = 0.3\sqrt{w_B} + 0.7\sqrt{w_A} - 4 \leq 3$$

Second best

Incentive compatibility constraint

- This will be given by:

$$0.3\sqrt{w_B} + 0.7\sqrt{w_A} - 4 \geq 0.8\sqrt{w_B} + 0.2\sqrt{w_A} - 0$$

Note: the left member is expected utility relative to low effort while the right member is expected utility relative to low effort.

Second best

Wage levels

- We now determine two wage levels w_A and w_B in such a way that profit is maximized while both participation and incentive compatibility constraint are are satisfied.
- Let these be w_A and w_B

Second best

Wage levels

- Skipping every calculations, by solving participation and incentive compatibility constraints we have:
- $\sqrt{w_a} = 9.4$ and $\sqrt{w_b} = 1.4$
- so: $w_a = 88.36$ and $w_b = 1.96$
- We thus conclude that for $y = 200$, A receives $w = 88.36$ and $w = 1.96$ with $y = 0$

Second best

Utility levels

- A's expected utility is thus $u(e_H) = 0.3\sqrt{1.96} + 0.7\sqrt{88.36} - 4 = 3$
- P's expected utility is thus
  $\Pi(e_H = 0.3(0 - 1.96) + 0.7(200 - 88.36) = 77.56$

Second best

Surplus reduction

Let us compare the results just obtained with the first best contract.

Under first best, we had:

- A's expected utility 3
- P's expected utility 91

Under conditions of asymmetric info we had:

- A's expected utility 3
- P's expected utility 77.56

We thus face a reduction in total surplus. What does it stem from?

Second best

Surplus' loss causes I

- This loss of social surplus is due to an inefficient risk allocation.
- In order to have A choosing e_H P had been forced to allocate a good degree of risk to A
- What rkind of rrisk are we talking about?

Second best

Surplus' loss causes II

- Even if A chooses e_H, in the 30% of cases he just get $w = 1.96$
- Total surplus drops as wage costs for P to the end of giving A the right incentives become higher (A's utility stays constant, though)
- So: for P (risk neutral) those extra costs are a reduction of profit while for A (risk averse) the higher wage level is merely sufficient to protect him from risk.

Incentives' intensity

As a matter of fact, b^* (the optimal level of incentives) will be higher:

1. the smaller is uncertainty in production. It is noteworthy that as uncertainty gets smaller accuracy in performance measurement increases and a strict correlation of wages to performance is way more convenient (this happens as risks on agent will be very small);

Incentives' intensity

As a matter of fact, b^* will be higher:

1. the smaller is the agent's risk aversion. If bearing risk is not costly for the agent strong incentives are a good idea because compensating the agent for risk becomes relatively cheaper;

Incentives' intensity

As a matter of fact, b^* will be higher:

1. the smaller is the marginal cost of effort. That is: incentives tend to be stronger the slower the disutility of effort grows as agent chooses a higher level of effort;

Incentives' intensity

As a matter of fact, b^* will be higher:

1. the larger is effort's marginal productivity. That is: it is optimal to give strong incentives whenever one gets large increases in output as effort increases.