MANAGERIAL ECONOMICS

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Bertrand's Model: price determination in duopoly

Introduction

Bertrand (1883) argues that price, rather than output, is the key decision variable for most firms.

In Cournot's model, the firms decide their output levels and then allow the market price to adjust accordingly.

In the Bertrand model each firm sets its own price, and then sells as much output as it can at the chosen price.

Introduction

Bertrand uses a zero conjectural variation assumption concerning prices:
each firm assumes its rival will stick to the rival's current price.

The model rests on implicit assumptions that the output of the two firms
is identical and there are no transaction or search costs.

Therefore, customers flow effortlessly to the firm that is currently offering
the lowest price.

Round 1

To locate the equilibrium in the Bertrand model, assume, as in the Cournot model, that the firms take their price decisions sequentially and that both firms face a horizontal marginal cost function $MC_A = MC_B$.

In Round 1, firm A sets its price initially at the monopoly level, PM, and earns the monopoly profit.

Then firm B arrives.

How should B react to A's initial price decision?

Round 1

By setting its price fractionally below PM, say at PM - ϵ where ϵ is a very small amount, B undercuts A and gains all of A's customers.

By doing so, B earns a profit fractionally below the monopoly profit.

Round 2

In Round 2, how should A react to B's intervention in Round 1?

Using the same reasoning, by setting its price fractionally below PM - ϵ, say at PM - 2 ϵ, A undercuts B and gains all of B's customers. Firm A earns a profit a little further below the monopoly profit.

Then, by setting its price at PM - 3, B undercuts A again and regains all of A's customers. B's profit is now a little further still below the monopoly profit.

Round 3

Similar reasoning also applies in Round 3 and in subsequent rounds, when further price-cutting takes place.

Is there ever an end to the price-cutting sequence?

The answer to this question is yes.

When price has fallen to the perfectly competitive level PC = MC, there is no incentive for either firm to cut price any further.

Bertrand duopoly

Although by so doing, either firm could still gain all of the other's customers, this would not be worthwhile if it required setting a price below marginal cost, at which normal profit would not be earned.

If firm A is the first to reach PC, at the next decision point firm B simply follows firm A, and also charges PC. Because consumers are indifferent between the two firms at this price, it is assumed each firm captures a 50 per cent share of the market at PC.

Bertrand duopoly

At the equilibrium price PC = MC, both firms produce output levels of
$q_A = q_B = 1/2Q_c$
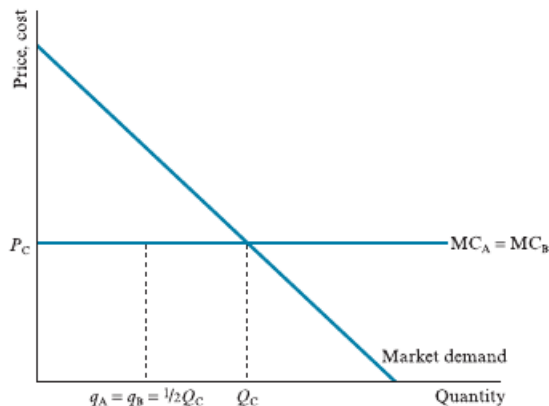
Price war in chainstores
FT January 15, 2016

Just before Christmas, one supermarket chain — Aldi — decided to cut vegetable prices dramatically: a bag of parsnips was sold for only 39 pence.

This move triggered a chain reaction:

1. Morrisons responded by selling 4 kg of vegetables for £1

2. Tesco and Sainsbury's followed

3. Lidl went even further and cut the price to 29 pence.

A price war began.

What happened to the companies

Customers were happy because prices fell. But for the supermarkets it was dangerous:

1. they sold more
2. but earned much less on each product

In fact:

1. Aldi's margins fell from 5.3% to about 3%
2. and were expected to drop to 1–2

Growth started to slow

At first Aldi grew very fast, but now analysts noticed that:

1. almost all growth came from opening new stores
2. existing stores were not selling more

So the price war was not creating new demand — it was simply shifting customers from one supermarket to another.

Lidl changed strategy

While Aldi kept focusing on low prices, Lidl changed approach:

1. it invested in better quality and presentation
2. improved its Christmas product range
3. made stores more attractive

As a result:

1. Lidl's sales grew faster
2. customers spent more
3. Lidl's margins held up better

The lesson of the story

At first, each company thought: "If I cut prices, I will win."

In the end, they realized: "If everyone cuts prices, everyone loses."

Lidl understood earlier that competing only on price is not enough and started winning by changing the game.

Analysis

UK supermarkets form an oligopoly: a few big firms, each one's decisions strongly affect the others.

The story shows **two different competitive logics** at work.

Cournot logic: capacity & presence

Before the price war, competition looked more like Cournot:

1) Firms expanded mainly by opening new stores (capacity decisions).

2) Total sales growth was driven by how many stores each firm had.

3) Prices were relatively stable.

This is classic Cournot thinking: firms compete by choosing quantities / capacity, price is the market outcome.

Aldi's growth mostly came from new stores $\rightarrow$ Cournot-style competition.

Bertrand logic: price war

The moment Aldi cut prices before Christmas, competition switched to Bertrand:

1. Firms began competing directly on price.
2. One price cut triggered immediate reactions.
3. Prices collapsed.
4. Margins shrank for everyone.

This is textbook Bertrand competition: when firms compete on price, equilibrium prices fall toward marginal cost.

Thus:

1. Falling margins
2. Profits under pressure
3. No real growth in total demand

Why the outcome is bad for everyone

Each firm reasons: "If I don't match the price cut, I lose customers."

So all firms cut prices.

This leads to:

1. lower profits for all
2. no lasting competitive advantage

That's the Bertrand prisoner's dilemma.

Why Lidl does better

Lidl partially escapes Bertrand by differentiating:

1. improves product quality & image

2. adds premium products

3. weakens pure price comparison

This moves competition away from pure Bertrand toward monopolistic competition, where prices and margins can recover.

Analysis

Table: Phases of Competition in the UK Supermarket Industry

Phase	Competition Type	Firms' Choice	Outcome
Before price war	Cournot	Store expansion (quantity)	Stable prices, growth
During price war	Bertrand	Price	Falling margins
Lidl's response	Differentiation	Product and image	Better performance

Game-theoretic explanation of the supermarket war

Players: Major UK supermarkets: Aldi, Lidl, Tesco, Sainsbury's, Morrisons, Asda.

Strategies: each firm chooses between two main strategies:

1) High price / cooperate (keep margins)
2) Low price / undercut (steal customers)

Table: Strategic Interaction in the UK Supermarket Industry

	Rival's Strategy		
Firm's Strategy	High Price	Low Price	Differentiation
High Price	(10 , 10)	(0 , 12)	(8 , 9)
Low Price	(12 , 0)	(2 , 2)	(6 , 7)
Differentiation	(9 , 8)	(7 , 6)	(11 , 11)

Strategic logic

Each firm thinks:

– If others keep prices high, I gain a lot by undercutting.

–If others cut prices, I must cut prices too or I lose customers.

So cutting prices becomes the dominant strategy.

That makes the outcome:

Everyone cuts prices $\rightarrow$ everyone earns less.

This is the classic Prisoner's Dilemma.

Payoff Matrix (Simplified Bertrand Price Game)

Each firm chooses:

H = High price

L = Low price

Payoffs are profits (bigger number = better).

Table: Payoff Matrix: Simplified Bertrand Price Game

	Rival: High Price (H)	**Rival: Low Price (L)**
Firm: High Price (H)	(10, 10)	(0, 12)
Firm: Low Price (L)	(12, 0)	(2, 2)

How firms reason

For each firm:

If the rival chooses H → choosing L gives 12 instead of 10.

If the rival chooses L → choosing L gives 2 instead of 0.

So L strictly dominates H.

Therefore the unique Nash equilibrium is: (L, L)

Thus: price war & low margins for everyone.

Repeated Games and Tacit Collusion in Oligopoly

In a one–shot Bertrand price game, the dominant strategy for each firm is to cut price, leading to the Nash equilibrium characterized by low prices and low profits.

However, firms in many markets interact repeatedly over time. This repeated interaction allows for the possibility of *tacit collusion*, where firms sustain higher prices without explicit agreement.

Let the per–period profits/payoffs be:

π_C (cooperation: high prices), (i.e. profit per period under "high price")

π_D (deviation: undercutting), (i.e. one-period profit from undercutting)

π_P (punishment: price war). (i.e. profit per period in the price-war punishment phase)

A grim–trigger strategy

Consider a grim–trigger strategy: each firm sets high prices as long as the rival does the same; if a deviation is ever observed, both firms revert permanently to the price–war outcome.

The present value of cooperating forever is:

$$V_C = \frac{\pi_C}{1 - \delta},$$

while the present value of deviating once and then being punished forever is:

$$V_D = \pi_D + \frac{\delta \pi_P}{1 - \delta}.$$

A grim–trigger strategy

Tacit collusion is sustainable if:

$$V_C \geq V_D,$$

which is equivalent to:

$$\delta \geq \frac{\pi_D - \pi_C}{\pi_D - \pi_P}.$$

What this means in the supermarket story

1) Why a price war can start suddenly
Aldi cuts prices → that's like deviating from the "quiet" high-price outcome.

Others match quickly because:
– If they keep prices high while Aldi cuts, they suffer the worst payoff (lose customers).
– Matching is an immediate best response.

So the industry can shift from a tacitly cooperative path to punishment-like behavior (a price war).

2) Why price wars can be temporary
A prolonged price war is costly (low margins). If firms are patient and can monitor each other, they may drift back toward higher prices—especially after the high-stakes Christmas period ends.

Why tacit collusion is hard here (and why Lidl's differentiation matters)

Tacit collusion is easier when:

1. firms are similar,
2. products are comparable,
3. prices are transparent,
4. demand is stable,
5. and cheating is detectable.

In grocery retail, it's harder because:

1. promotions change weekly,
2. products are many and varied,
3. it's easy to "cheat" via hidden discounts,
4. and demand is seasonal (Christmas).

Differentiation

Differentiation (Lidl's "quality/premium" move) changes the game:

1. It makes pure price comparison weaker,

2. reduces the gain from undercutting on a single item,

3. and can support higher margins without needing industry-wide coordination.