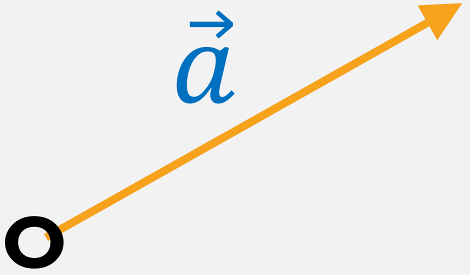
KINEMATICS Pt. I

- Vectors
- Position, displacement, velocity, and acceleration vectors
- Graphical relations: position–velocity and velocity–acceleration
- Examples

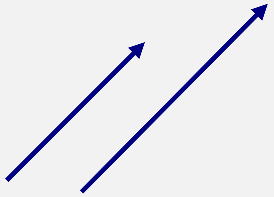
VECTORS



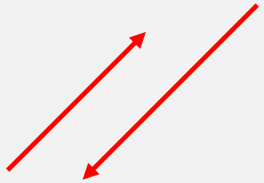
Vector $\vec{a}$ o $\boldsymbol{a}$

The magnitude of the vector is $\overrightarrow{|a|} = a$ and, graphically, it corresponds to the length of the arrow.

The origin of the vector is point **O**.



CONCORDANT
(parallel)
same direction and same sense

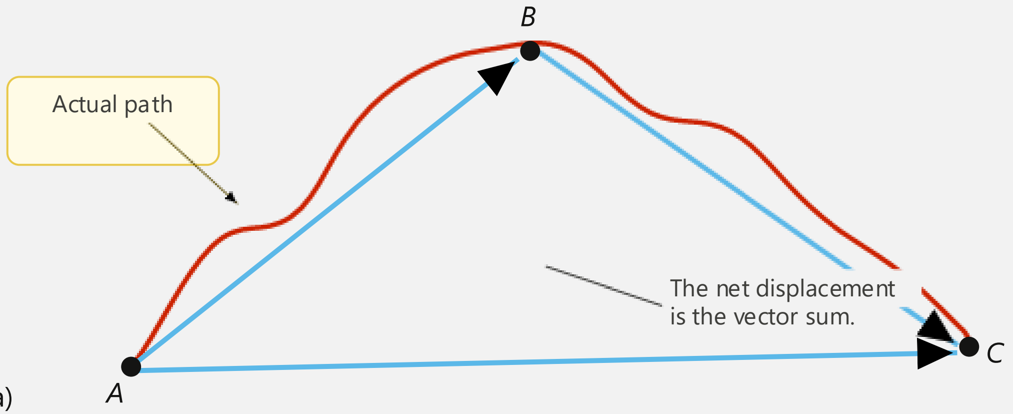


DISCORDANT
(anti-parallel)
same direction but opposite sense

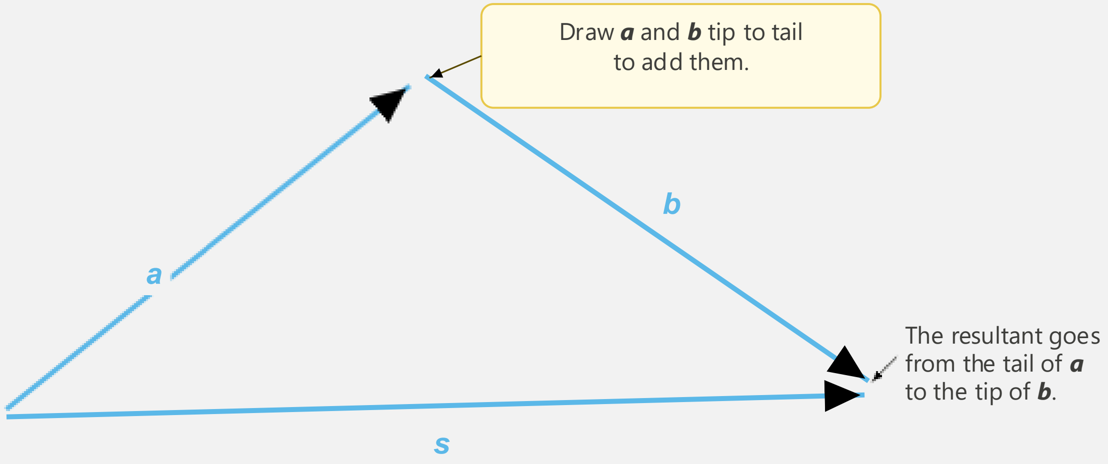
EQUIPOLLENT
Equal in magnitude, sense, and direction

VECTOR OPERATIONS

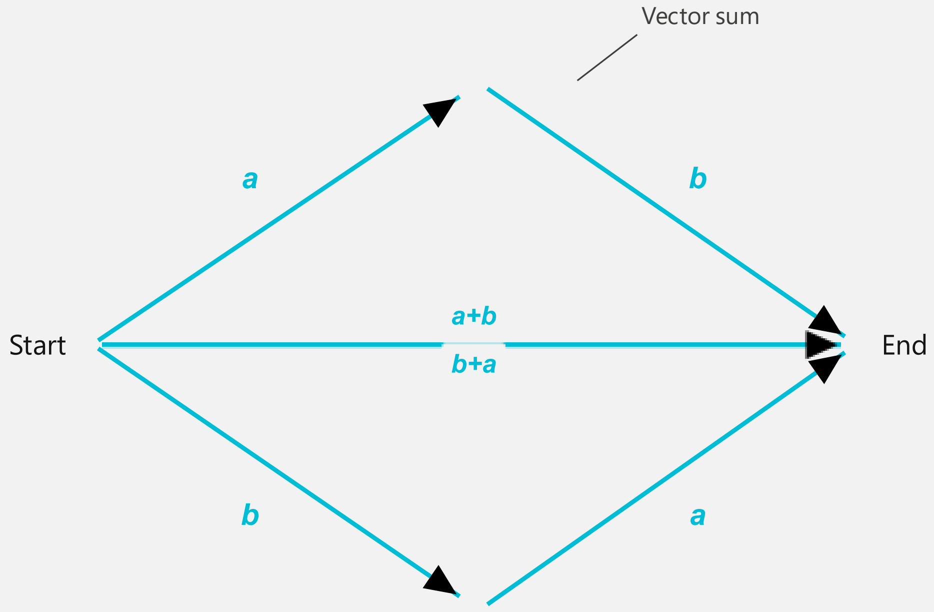
Vector Addition



a)



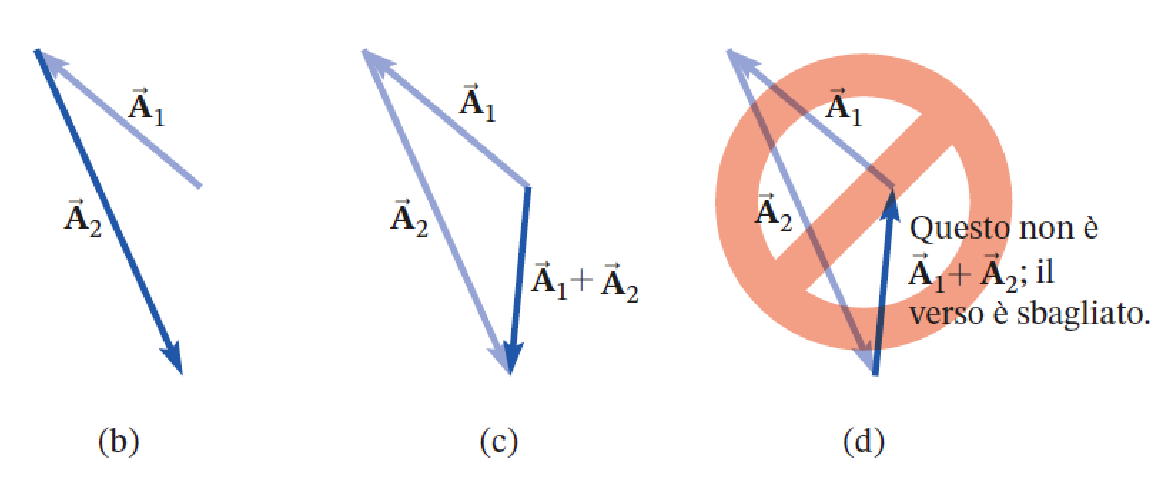
b)

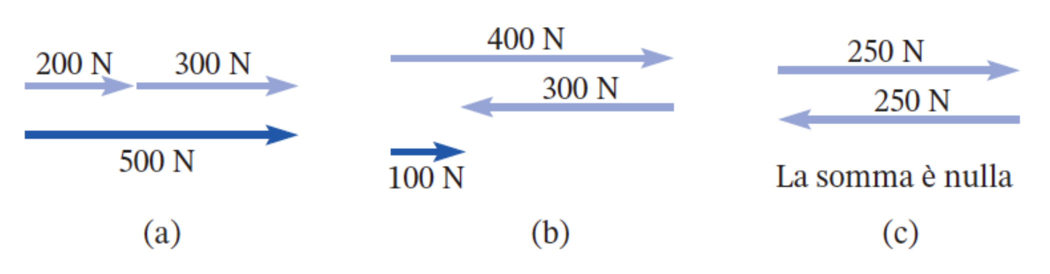


The resultant vector from the addition does not depend
on the order of the vectors being added.

VECTOR OPERATIONS

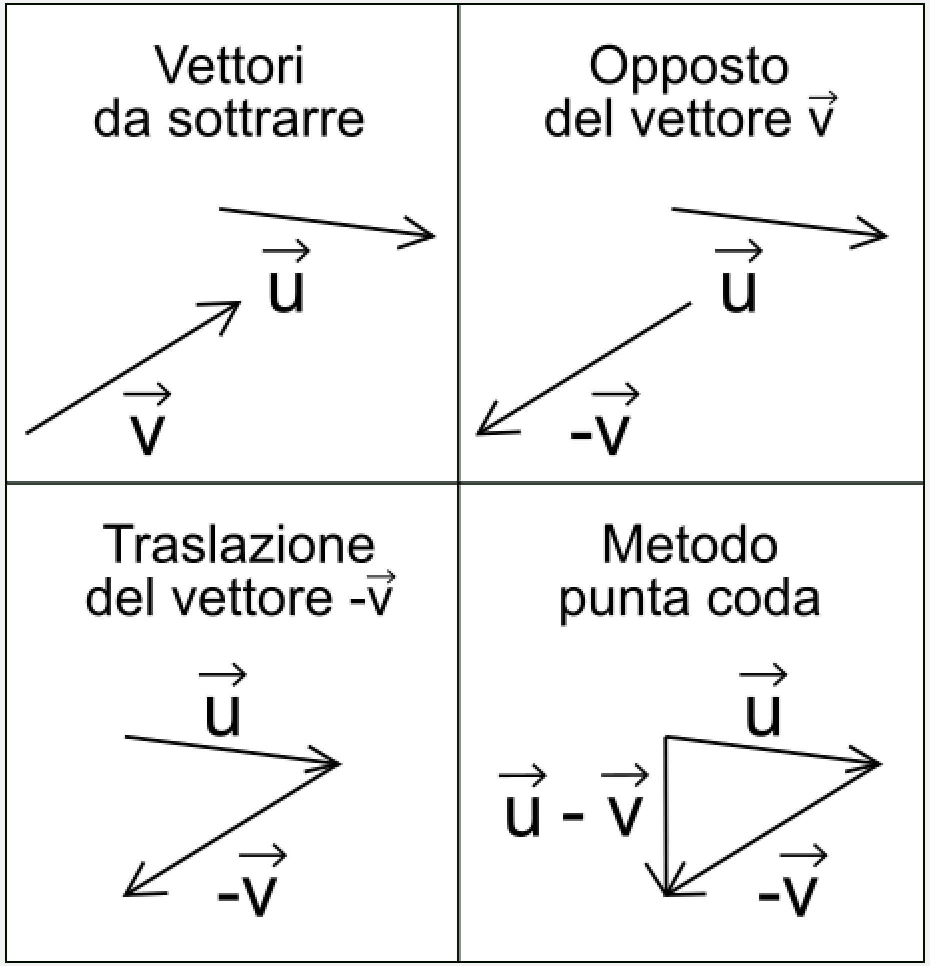
Vector Addition

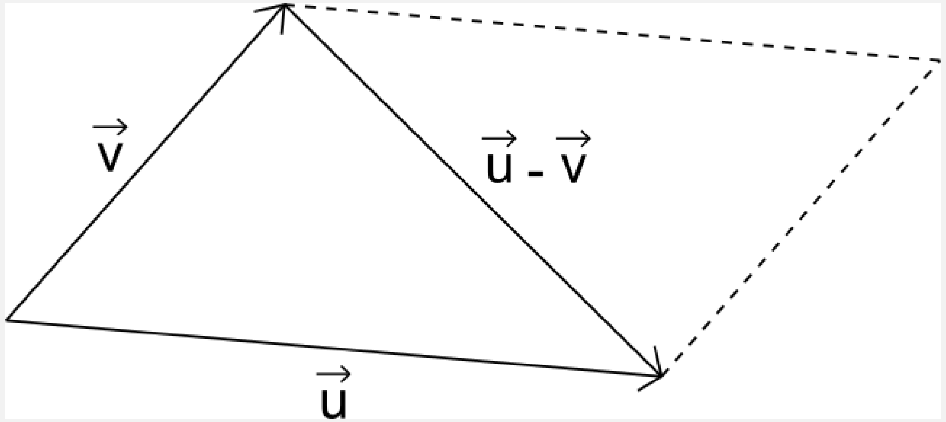


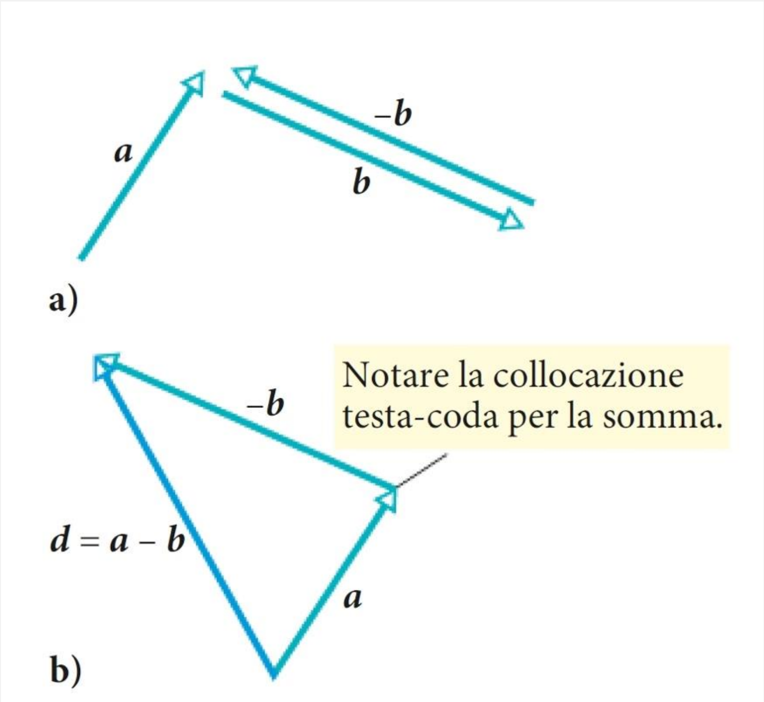


VECTOR OPERATIONS

Vector Subtraction

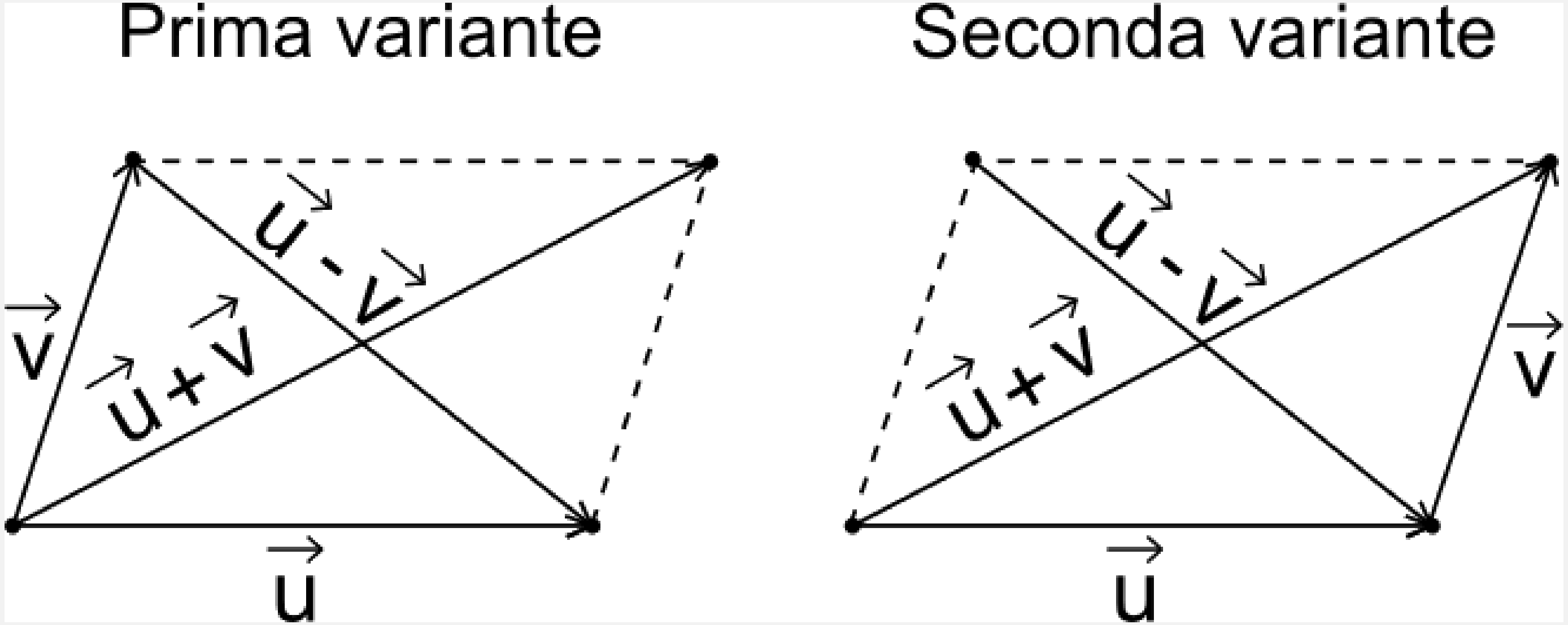




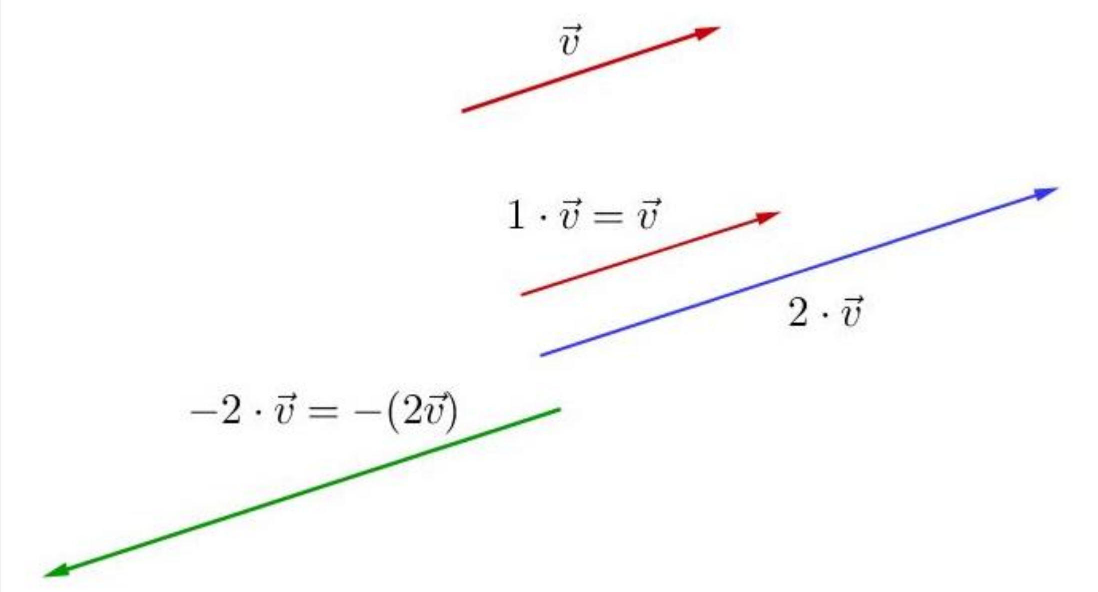


VECTOR OPERATIONS

Vector Subtraction



Vector–Scalar Product

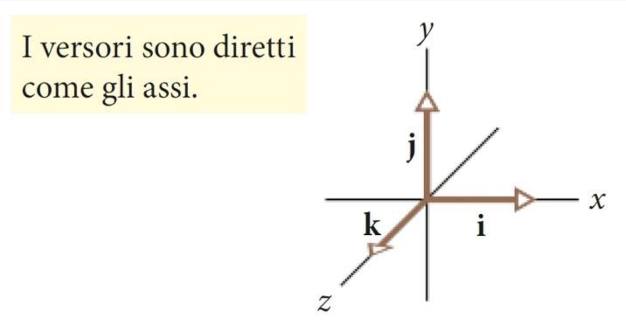


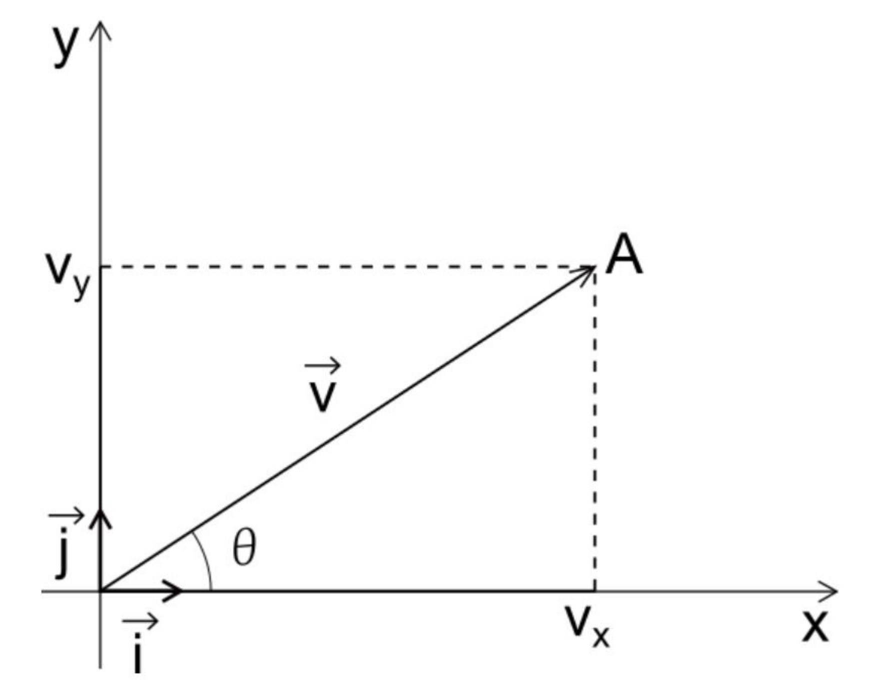
The vector $\vec{v}$ has

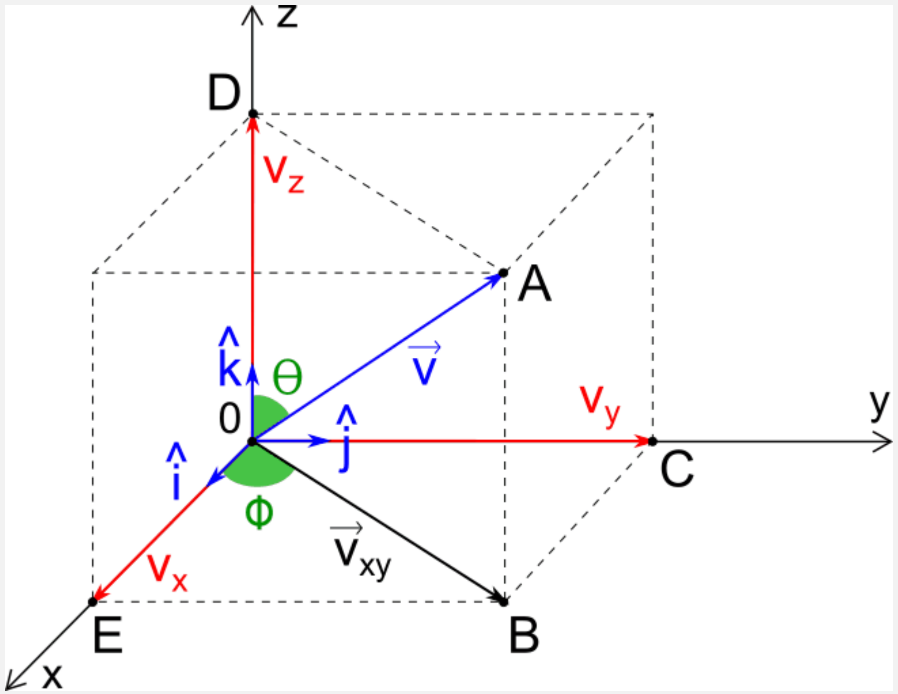
- the **same direction** as $\vec{u}$
- the **same sense** as $\vec{u}$ if $k > \mathbf{0}$, **opposite sense** if $k < \mathbf{0}$
- magnitude $k|\vec{u}| = ku$

OPERATIONS BETWEEN VECTORS

Vector Components / Projections on x and y

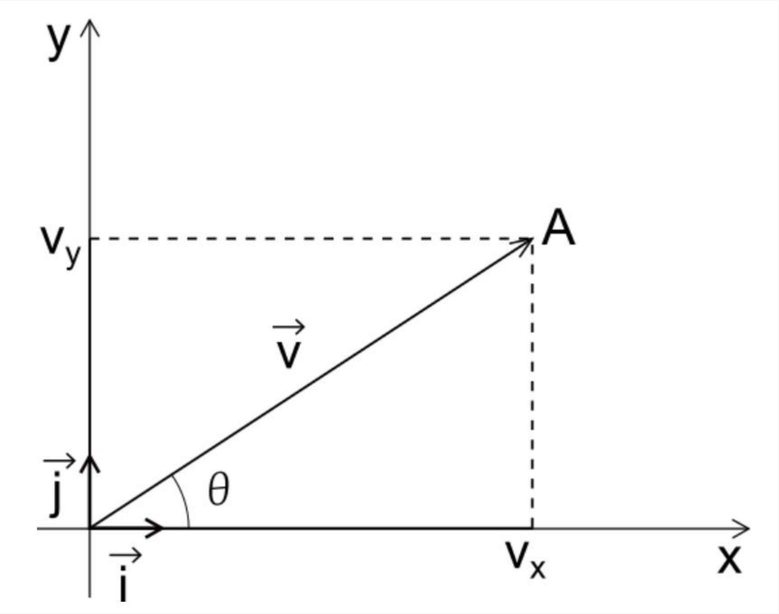


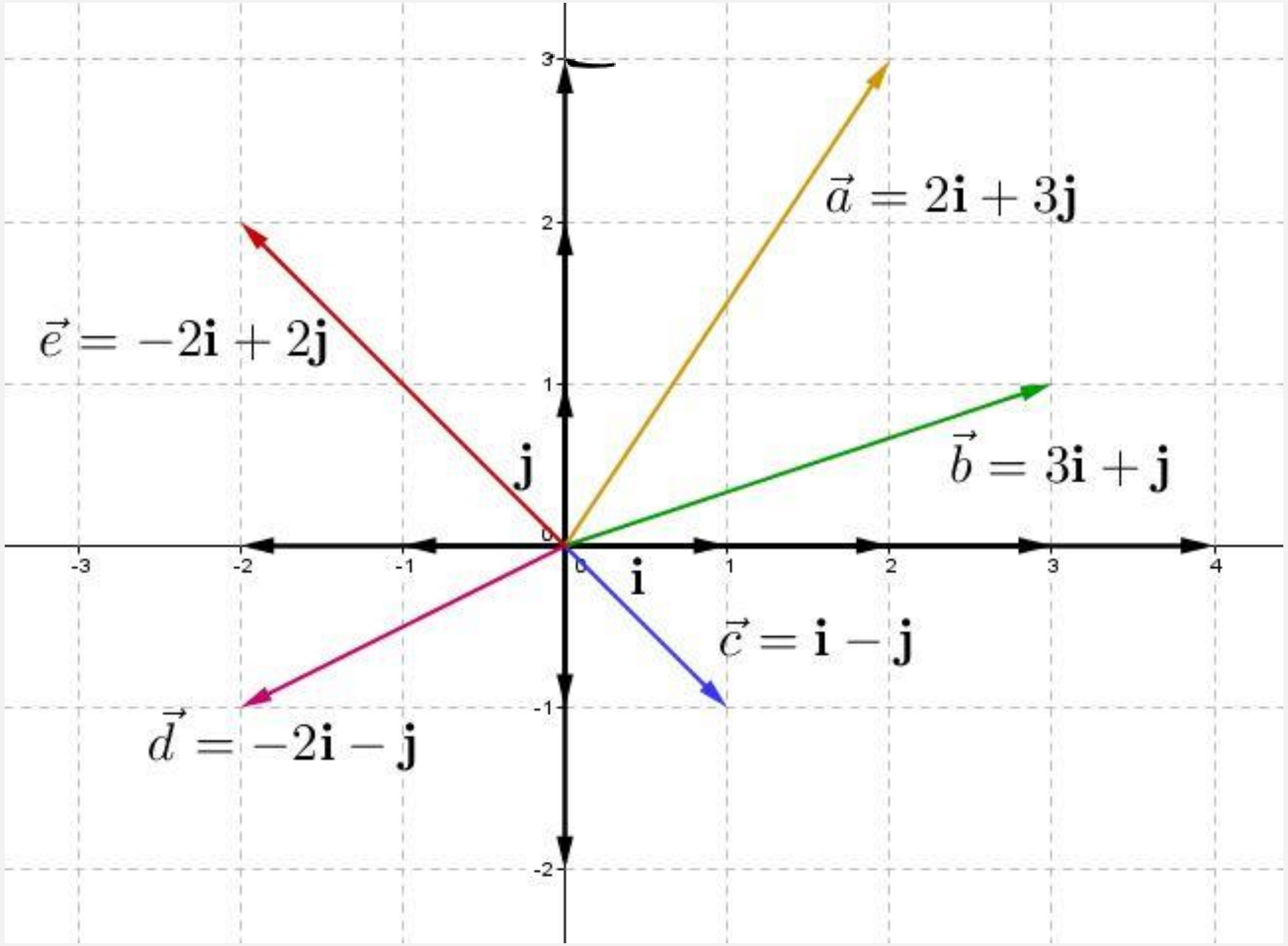




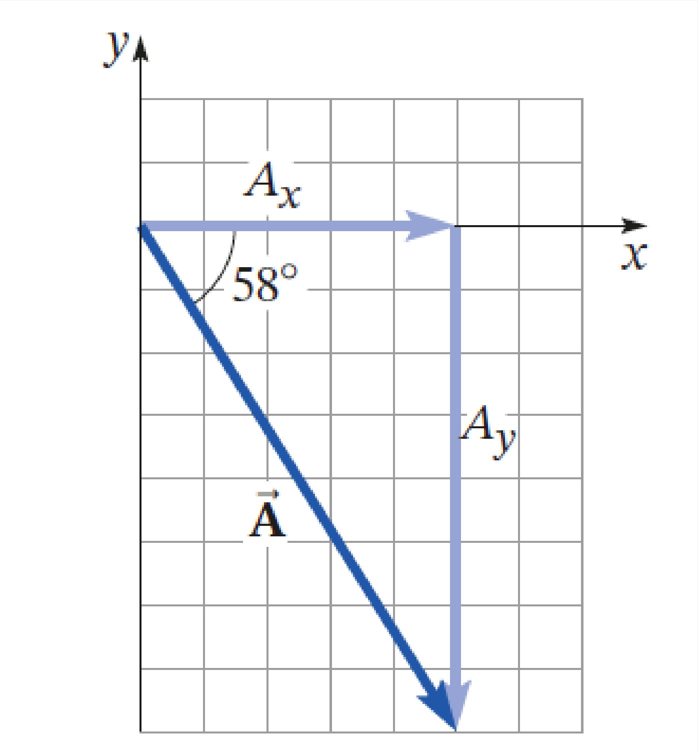
OPERATIONS BETWEEN VECTORS

Vector Components / Projections on x and y





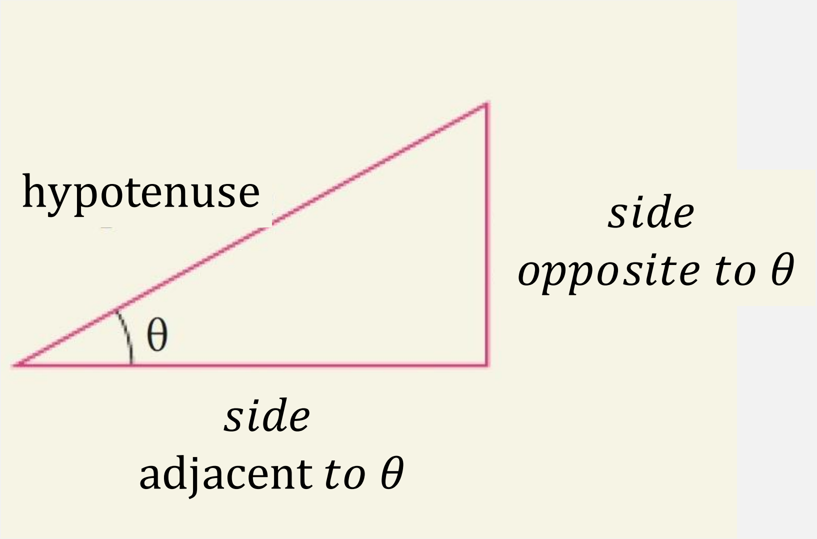
Vector Components / Projections on x and y



$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$



VECTOR OPERATIONS

Algebra with Components

$$\boldsymbol{r} = \boldsymbol{a} + \boldsymbol{b}$$

$$r_x = a_x + b_x$$

$$r_y = a_y + b_y$$

$$r_z = a_z + b_z$$

r is equal to the vector (***a***+***b***): if this is true, each component of ***r*** must coincide with the corresponding component of (***a***+***b***)

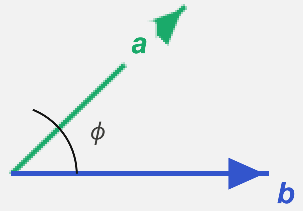
Two vectors are equal if their respective components are all equal to one another

$$\boldsymbol{d} = \boldsymbol{a} + (-\boldsymbol{b}) \rightarrow \quad d_x = a_x - b_x \qquad d_y = a_y - b_y \qquad d_z = a_z - b_z$$

where $\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$

VECTOR OPERATIONS

Dot Product

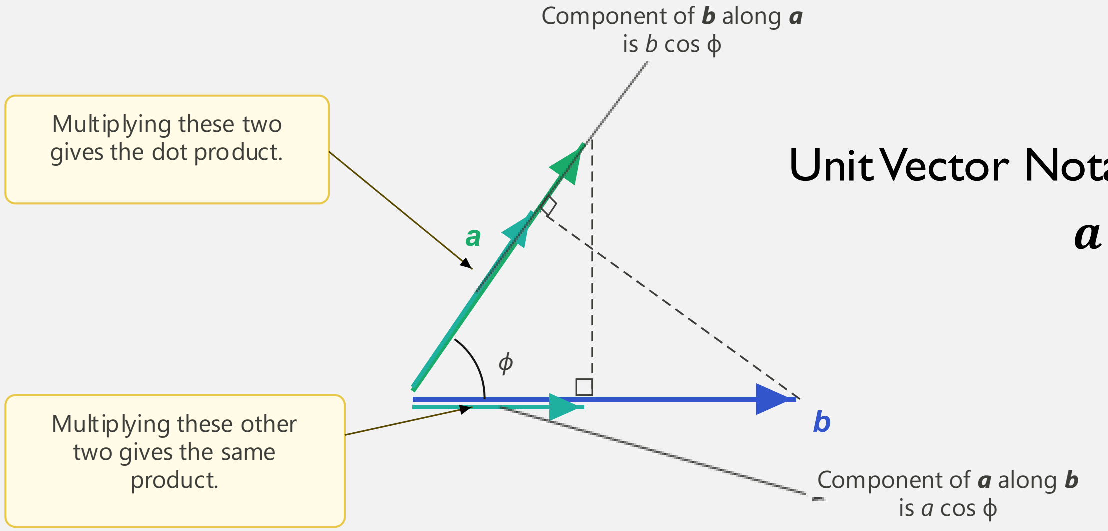


a)

Dot Product of Vectors ***a*** and ***b*** («***a*** dot ***b***»):

$$\boldsymbol{a} \cdot \boldsymbol{b} = ab\, cos\varphi$$

All the terms on the right-hand side are scalars; therefore, the product ***a·b*** is a scalar!



b)

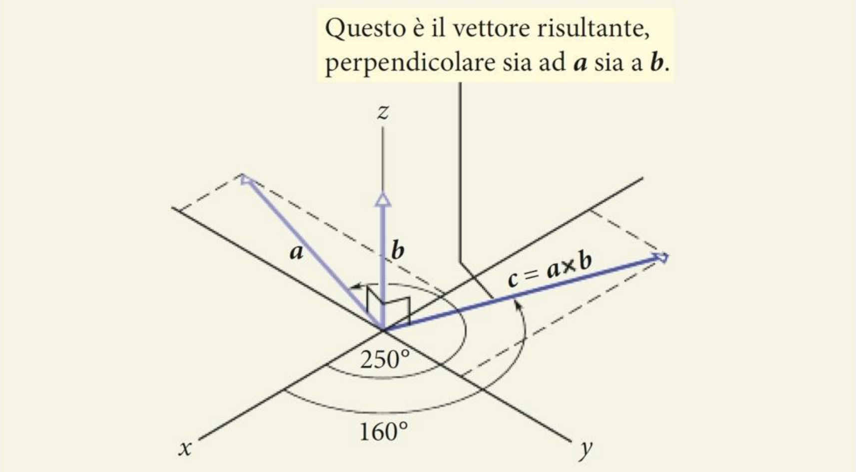
Unit Vector Notation:

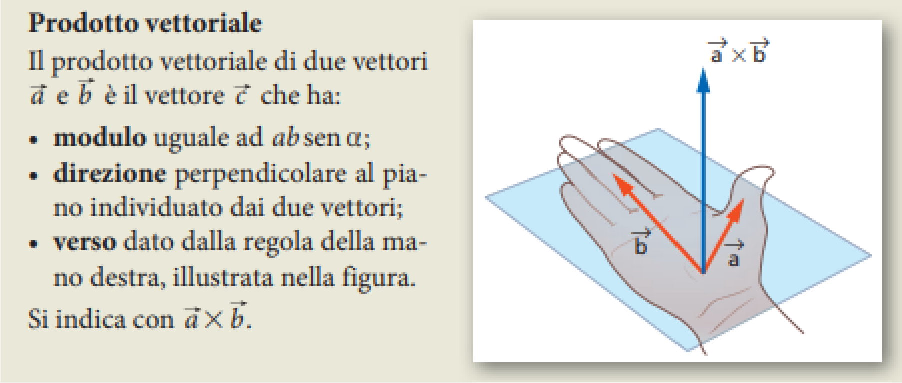
$$\boldsymbol{a} \cdot \boldsymbol{b} = (a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k}) \cdot (b_x \boldsymbol{i} + b_y \boldsymbol{j} + b_z \boldsymbol{k})$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$$ [Distributive Property]

VECTOR OPERATIONS

Vector Product





Prodotto vettoriale

Il prodotto vettoriale di due vettori $\vec{a}$ e $\vec{b}$ è il vettore $\vec{c}$ che ha:

- **modulo** uguale ad ab sen α;
- **direzione** perpendicolare al piano individuato dai due vettori;
- **verso** dato dalla regola della mano destra, illustrata nella figura.

Si indica con $\vec{a} \times \vec{b}$.

Vector product of vectors ***a*** and ***b*** ("***a*** vector ***b***") → vector ***c***, whose magnitude is:

$$\boldsymbol{c} = ab\ sin\varphi$$

No Commutative Property: $\boldsymbol{a} \times \boldsymbol{b} \neq \boldsymbol{b} \times \boldsymbol{a}$

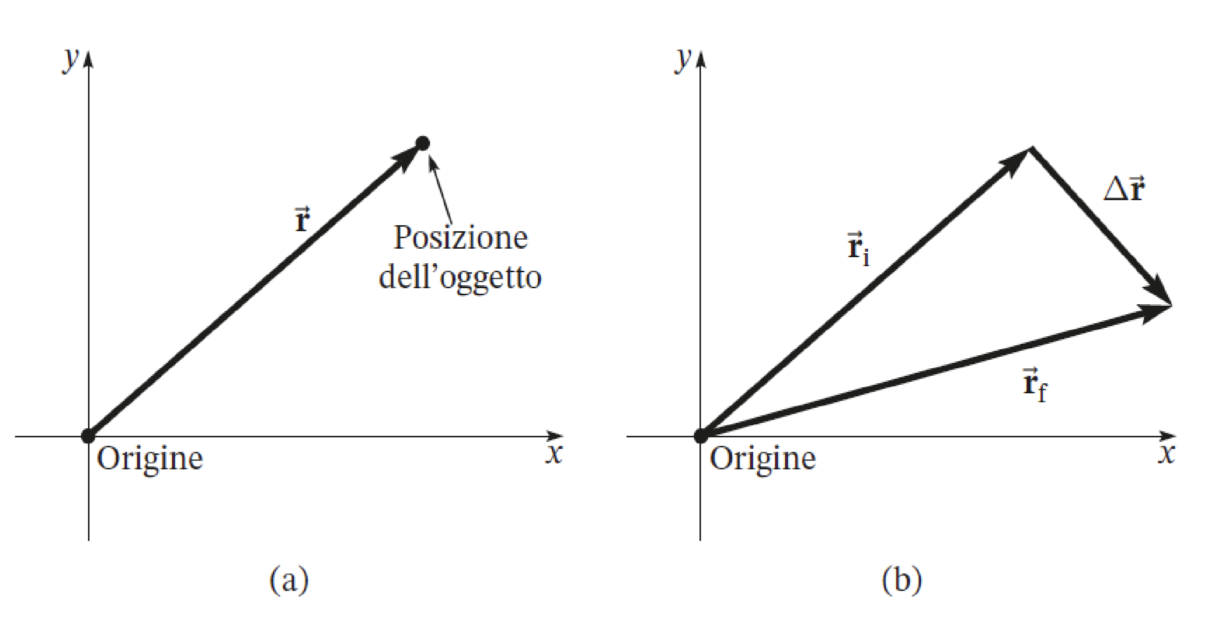
Yes Distributive Property: $\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c}$

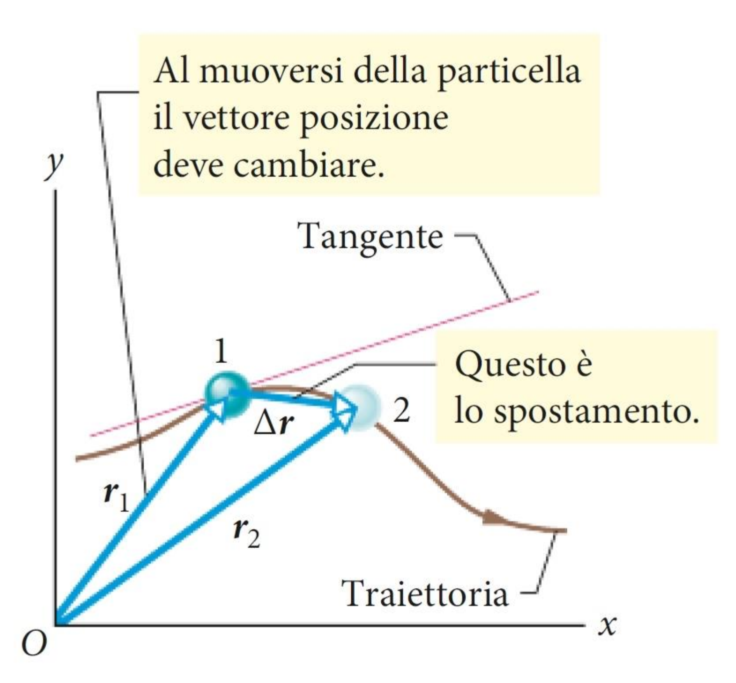
With ***a*** // ***b*** → $\boldsymbol{a} \times \boldsymbol{b} = 0$

With $\boldsymbol{a} \perp \boldsymbol{b}$ → $|\boldsymbol{a} \times \boldsymbol{b}|$ = max

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

Position Vector – Displacement Vector





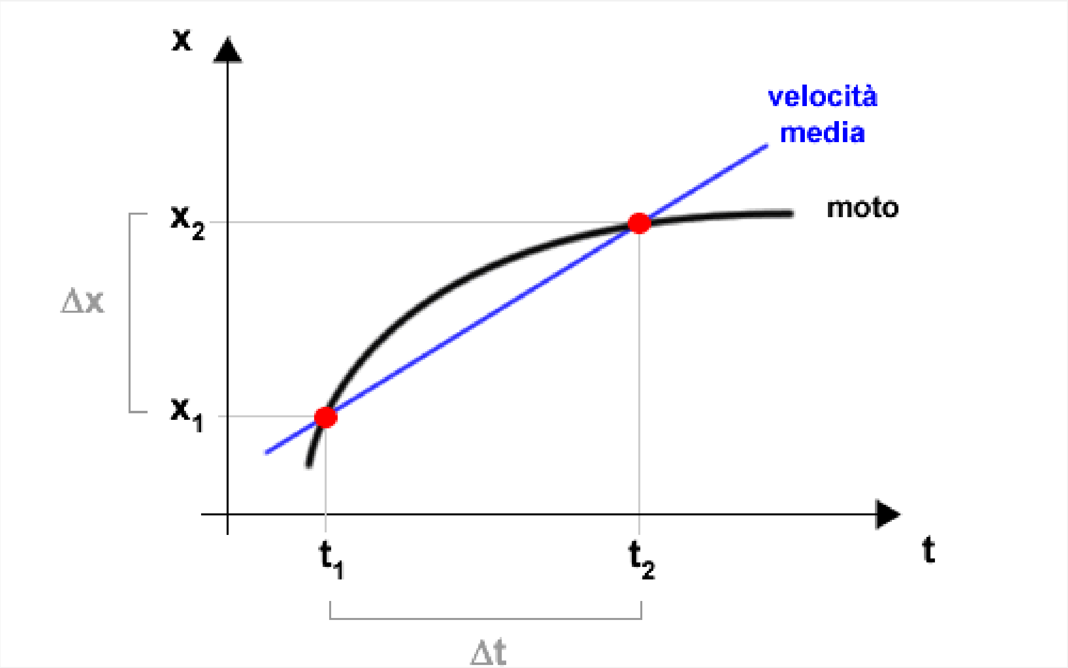
$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$ Position Vector

$$\Delta\vec{r} = (x_f - x_i)\cdot\hat{\imath} + (y_f - y_i)\cdot\hat{\jmath} = \Delta x\cdot\hat{\imath} + \Delta y\cdot\hat{\jmath}$$

Displacement Vector

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

Average Velocity Vector



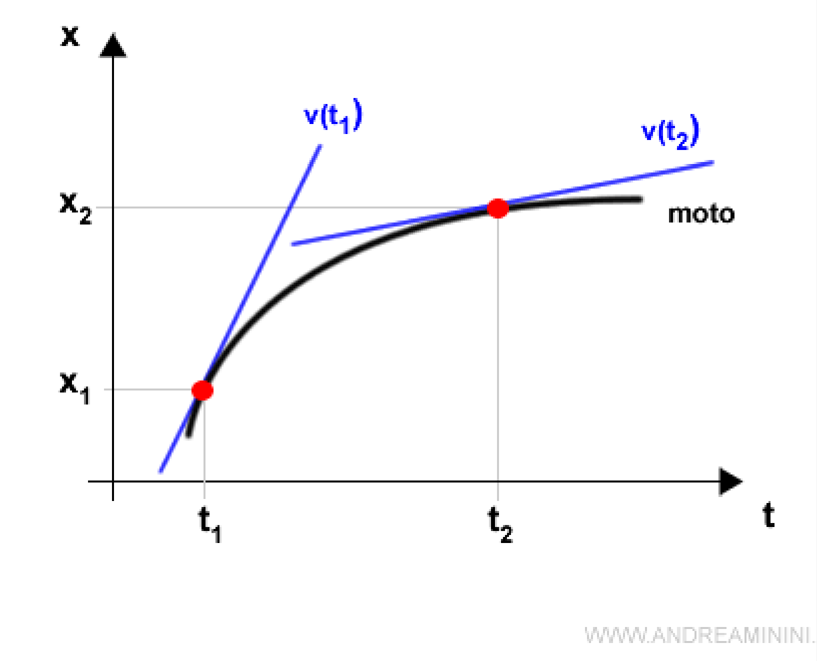
$$\vec{v}_m = \frac{\Delta\vec{r}}{\Delta \mathrm{t}} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \qquad [L]/[t] = [Lt^{-1}]$$

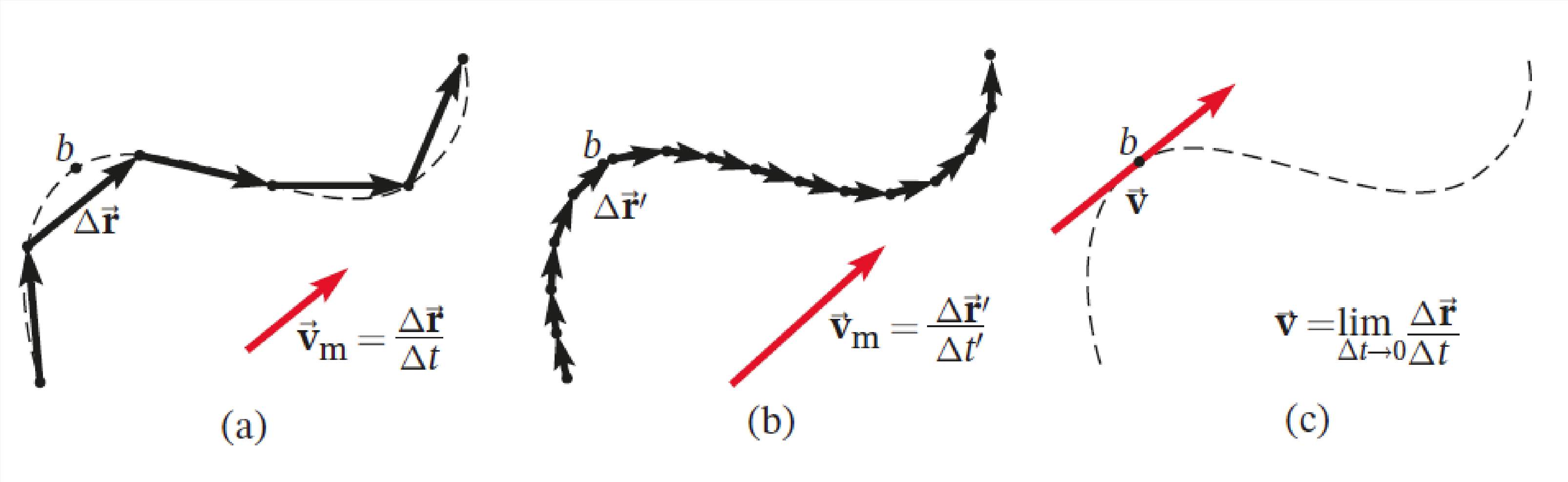
$$\vec{v}_m = \frac{\Delta x\boldsymbol{i} + \Delta y\boldsymbol{j} + \Delta z\boldsymbol{k}}{\Delta \mathrm{t}} = \frac{\Delta x}{\Delta t}\boldsymbol{i} + \frac{\Delta y}{\Delta t}\boldsymbol{j} + \frac{\Delta z}{\Delta t}\boldsymbol{k}$$

$$v_{m,x} = \frac{\Delta x}{\Delta t}\ , \qquad v_{m,y} = \frac{\Delta y}{\Delta t}$$

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

Instantaneous Velocity Vector





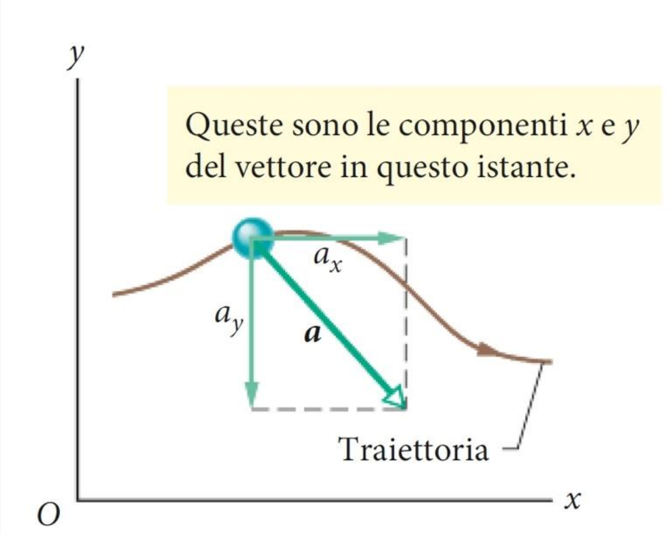
In Scalar Form:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \quad , \quad v_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

Same direction and same sense as the displacement at that instant

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

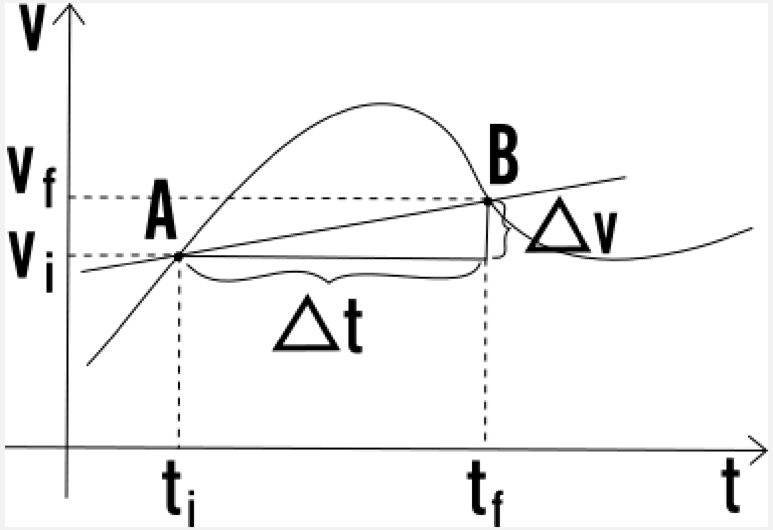


Average Acceleration Vector

$$\vec{a}_m = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t}$$

$$a_{m,x} = \frac{\Delta v_x}{\Delta t}, a_{m,y} = \frac{\Delta v_y}{\Delta t}$$

Same direction and same sense as $\Delta\vec{v}$ $[Lt^{-1}]/[t] = [Lt^{-2}]$

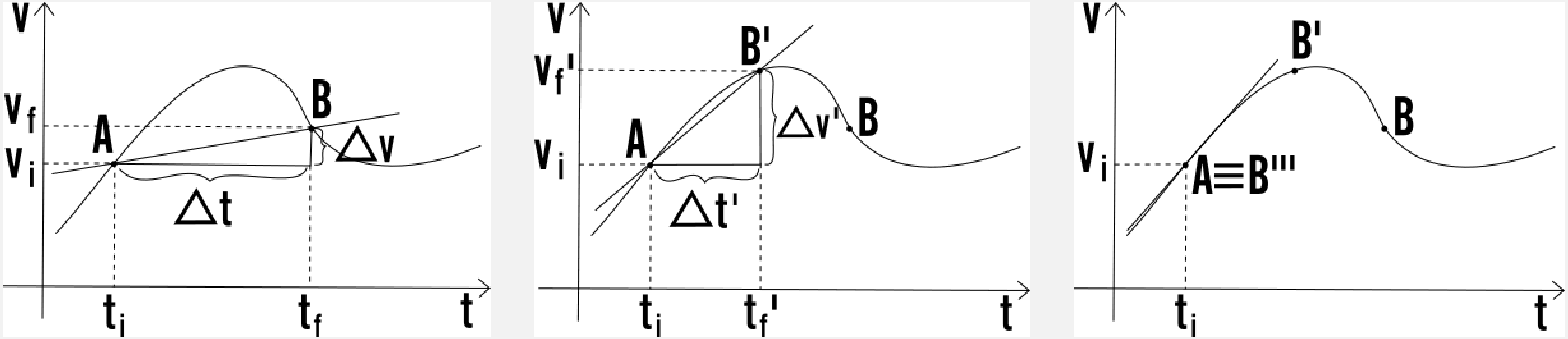


POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

Instantaneous Acceleration Vector

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

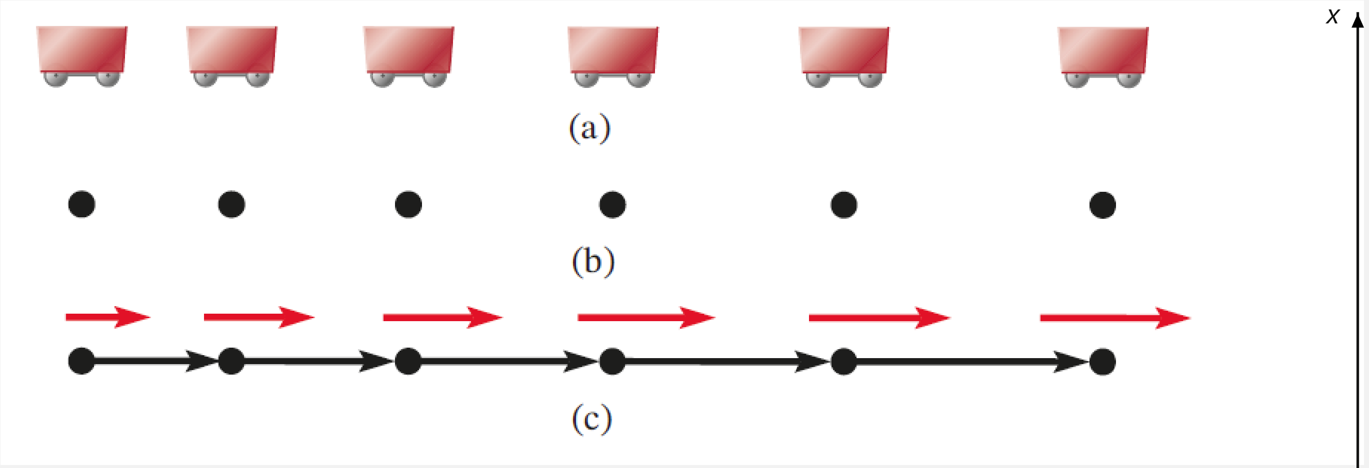
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad , \qquad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

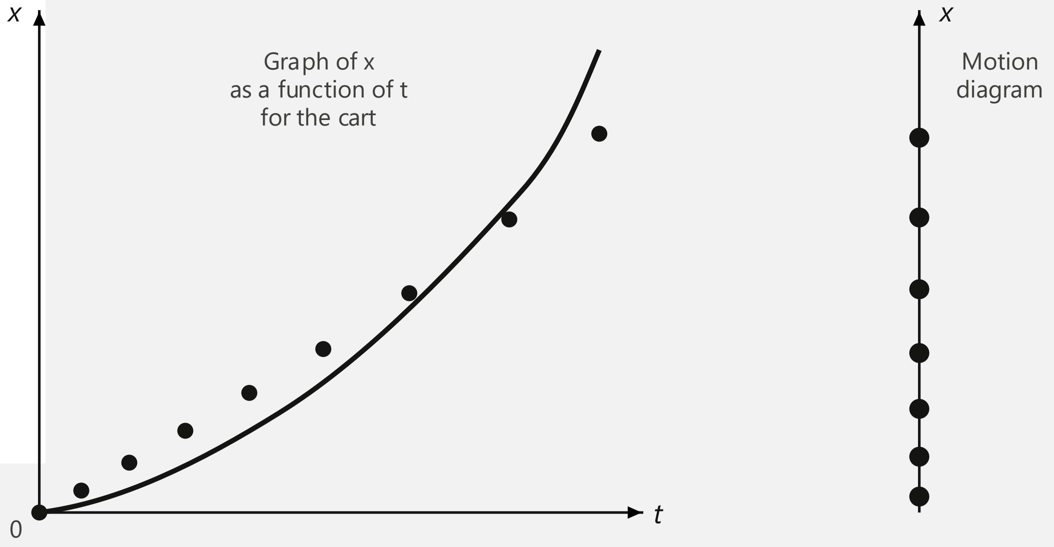


POSITION, DISPLACEMENT, VELOCITY AND ACCELERATION VECTORS

Motion Diagram

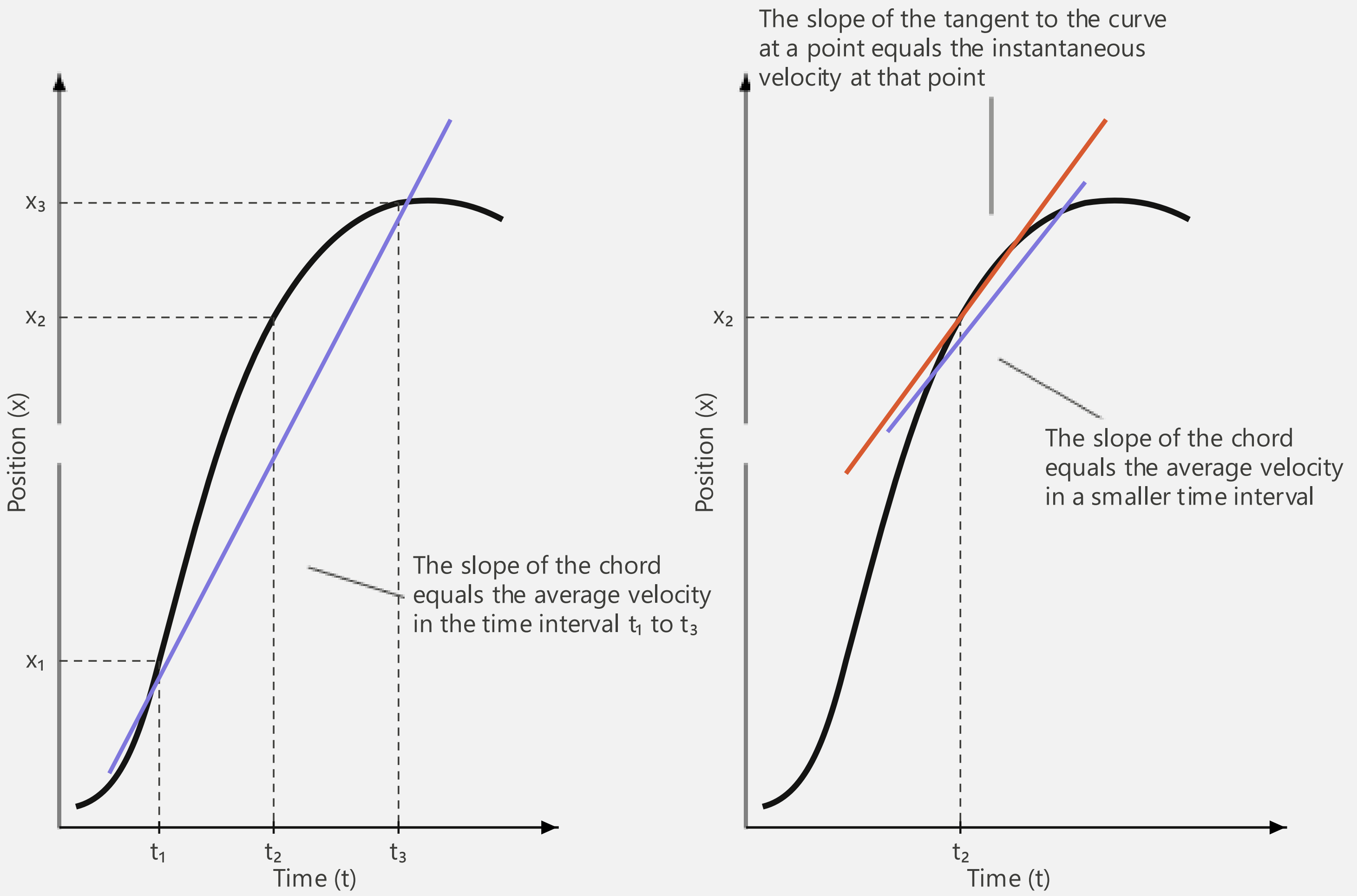
A motion diagram shows an object's position as a function of time





POSITION, DISPLACEMENT, VELOCITY AND ACCELERATION VECTORS

Graphical Relationship Between Position and Velocity

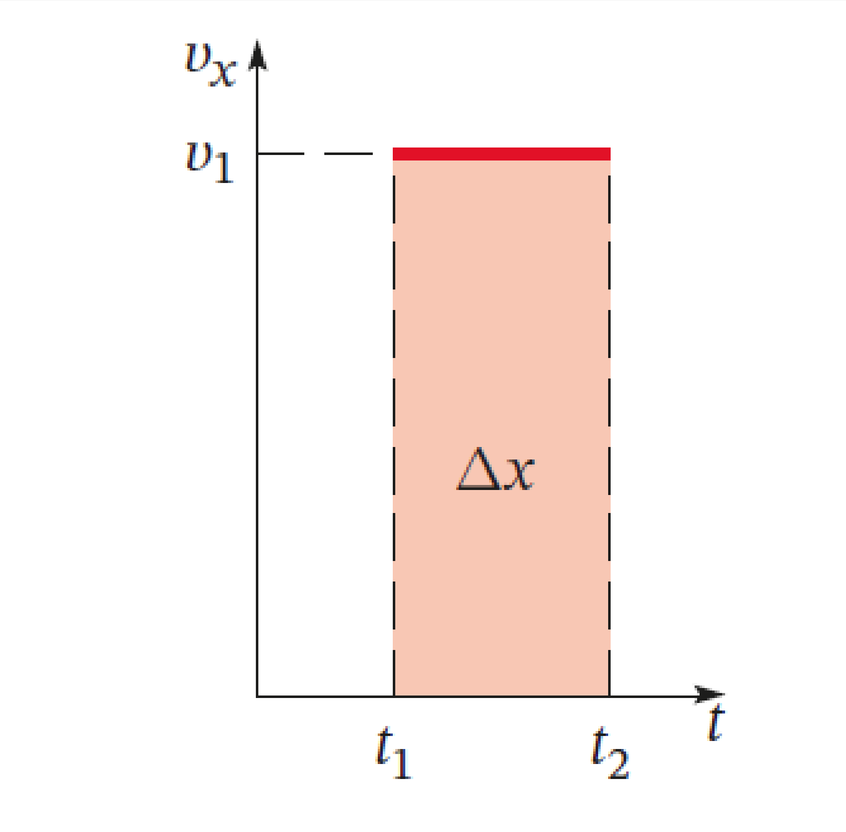


(a)

(b)

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

Graphical Relationship Between Position, Time, and Velocity

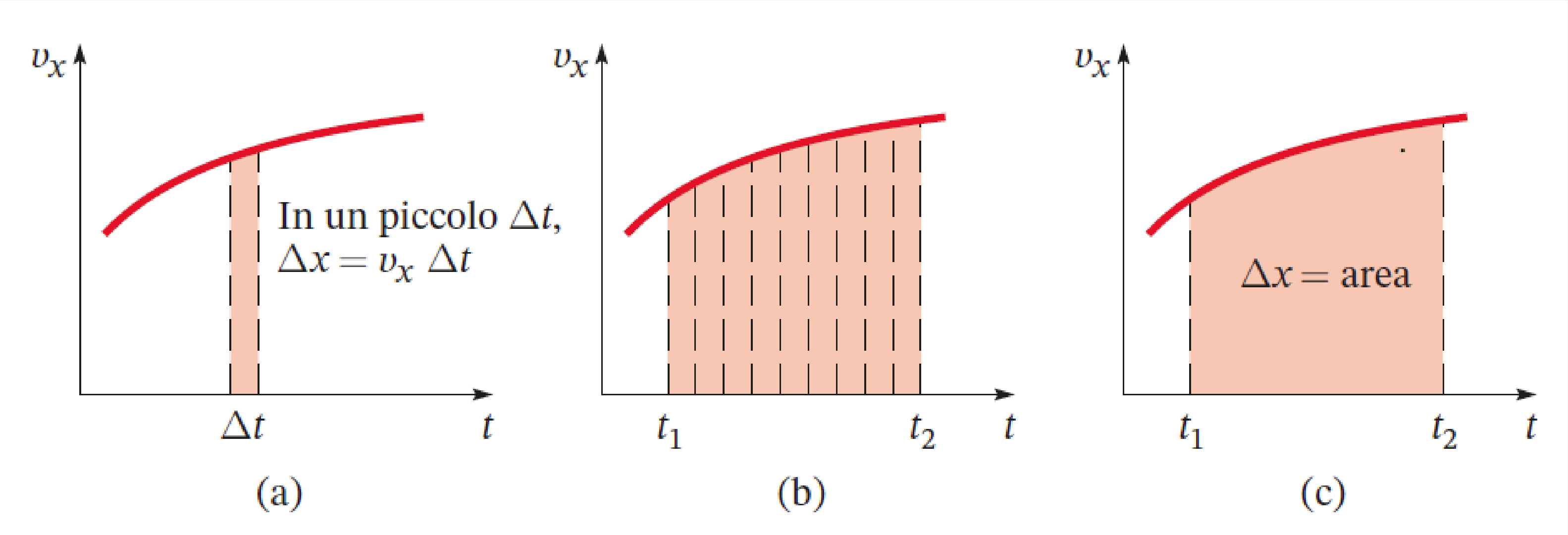


$$v_x = v_{m,x} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x \Delta t \text{ (for constant } v_x\text{)}$$

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

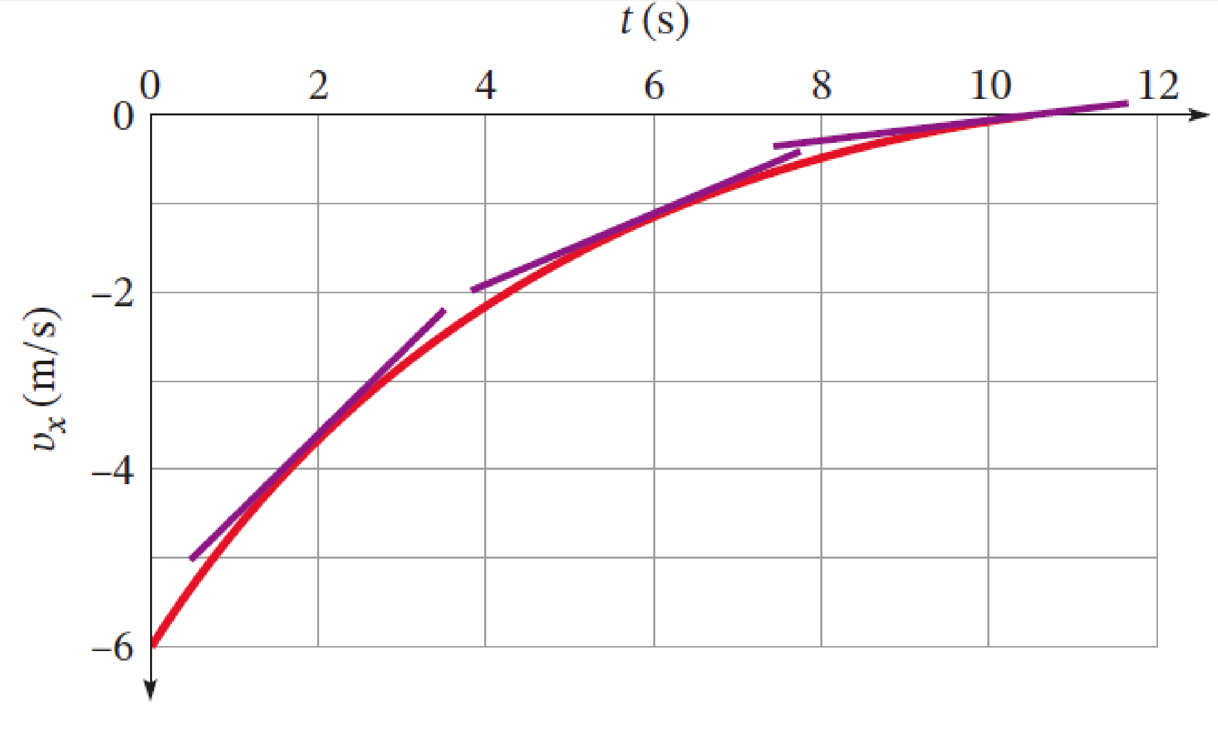
Graphical Relationship Between Position, Time and Velocity



$\text{for non} - \text{constant } v_x$

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

Graphical Relationship Between Velocity and Acceleration



Same graphical relationship as for displacement and velocity:

- a_x is the slope of the tangent at a given point on the curve $v_x(t)$
- Δv_x is the area under the curve $a_x(t)$ over a given time interval

POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

Example

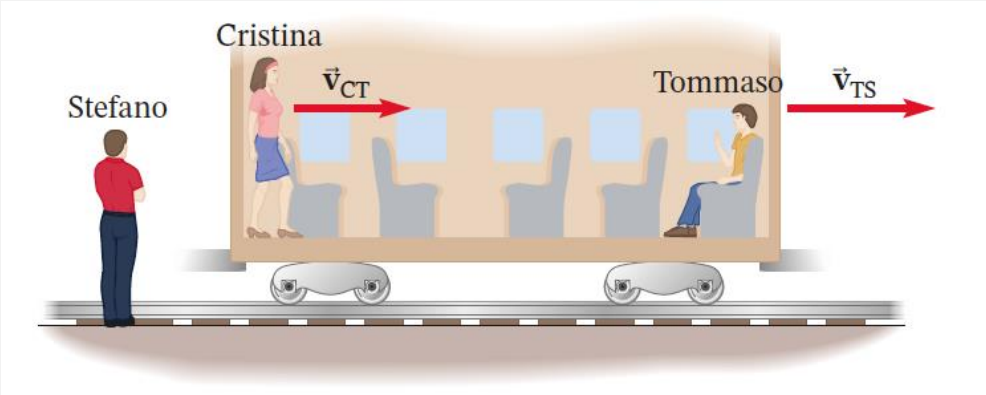
A skater is moving along a level road with a speed of 8.94 m/s; after 120.0 s the road begins to incline (at an angle of 15°) and the skater maintains a speed of 7.15 m/s.

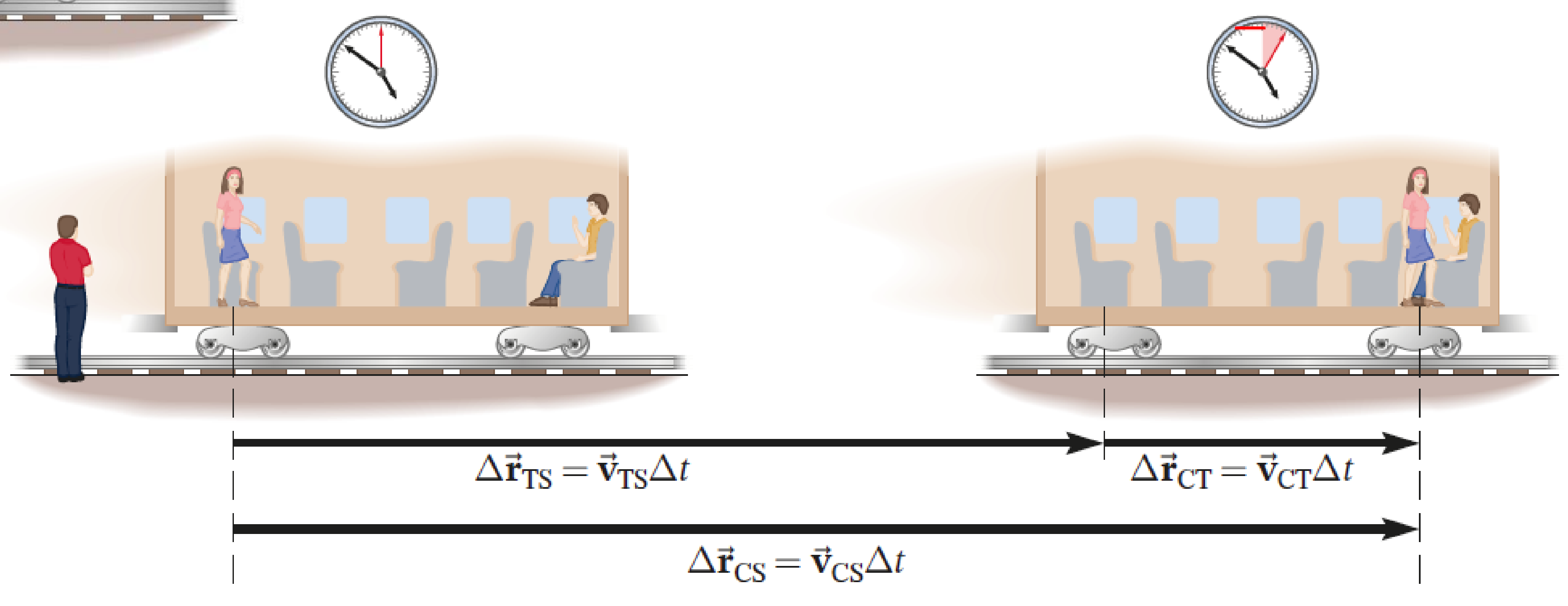
(a) What is the change in the skater's velocity?

(b) What is the skater's average acceleration over the 120 s time interval?

POSITION AND DISPLACEMENT VECTORS, VELOCITY AND ACCELERATION

Relative Velocity and Reference Frames





$$\Delta\vec{r}_{CS} = \Delta\vec{r}_{CT} + \Delta\vec{r}_{TS}$$

$$\vec{v}_{CS} = \vec{v}_{CT} + \vec{v}_{TS}$$