

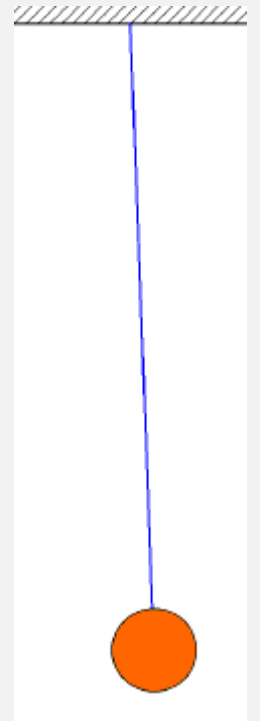
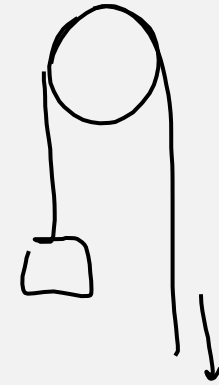
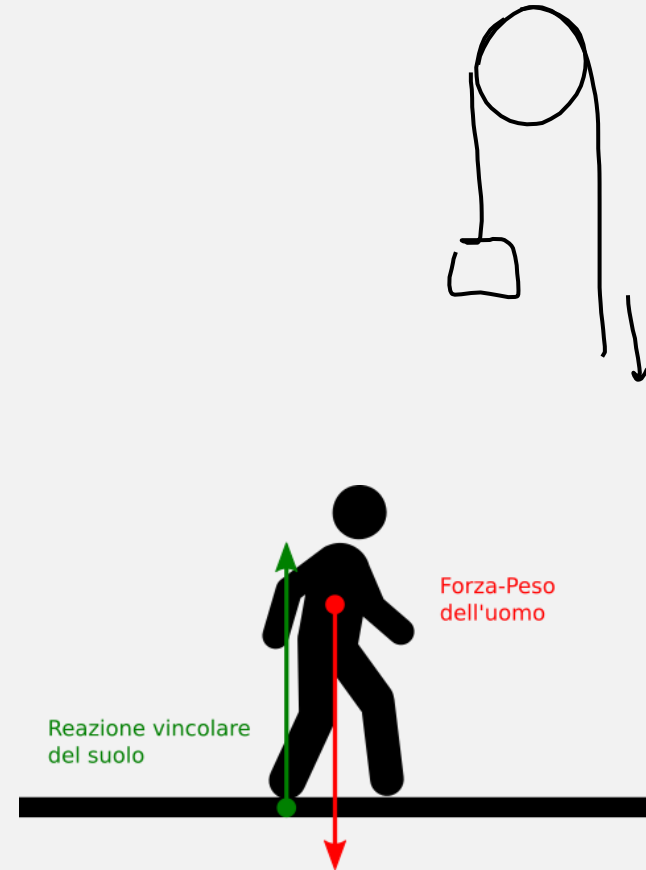
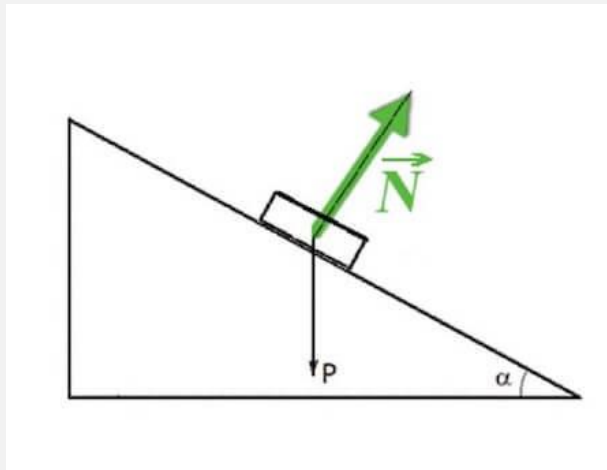
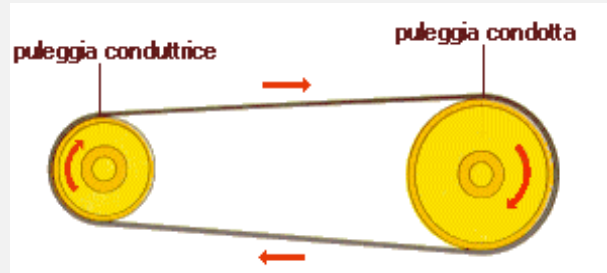
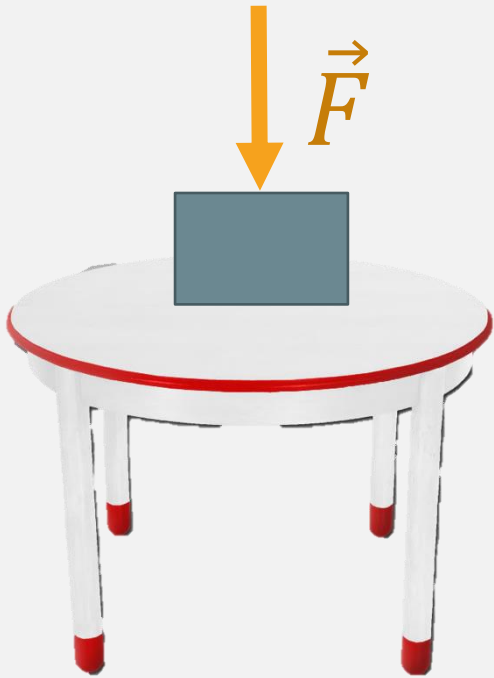
## **DYNAMICS pt.II**

### 1. Constraint forces

- a) Normal force
- b) Tension
- c) Friction

# CONTACT FORCES

## CONSTRAINT FORCE or CONSTRAINT REACTION

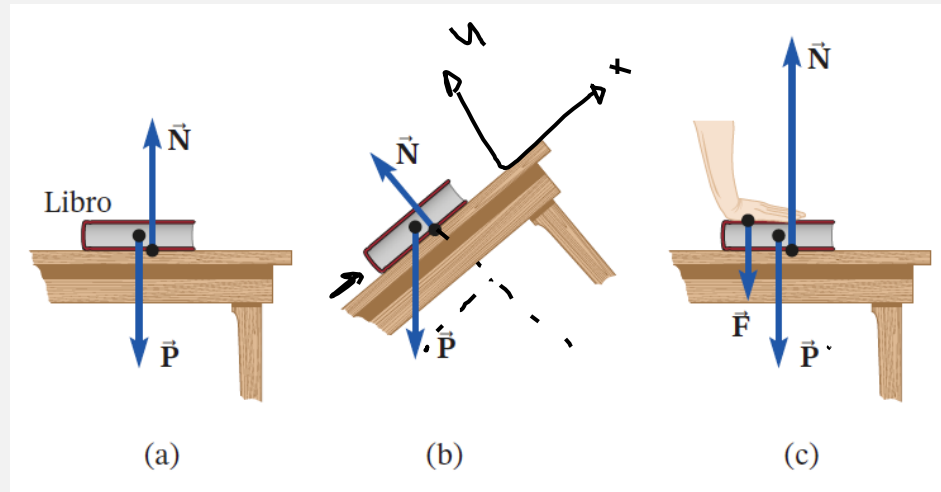


The constraints to which a body is subjected can themselves exert forces.  
The action of the constraint is represented by a force called constraint reaction.

# NORMAL FORCE

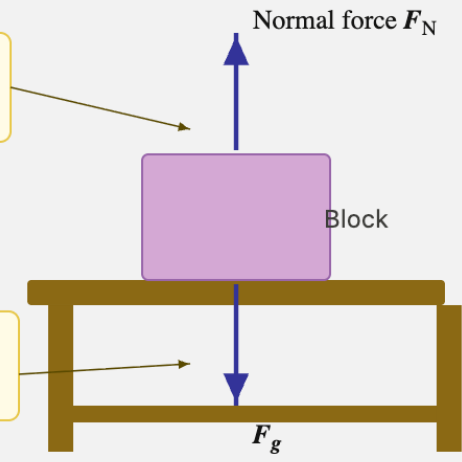
## SUPPORT CONSTRAINT FORCE

Force acting perpendicularly to the contact surface between two bodies

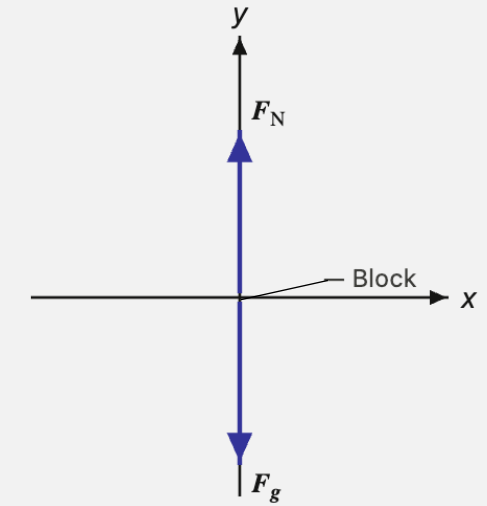


The normal force is the force exerted by the table on the block.

The gravitational force on the block is due to Earth's attraction.



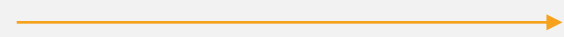
a)



The forces balance.

b)

The normal force is always directed perpendicularly to the contact surface and its magnitude can only be determined after analyzing all the other forces involved

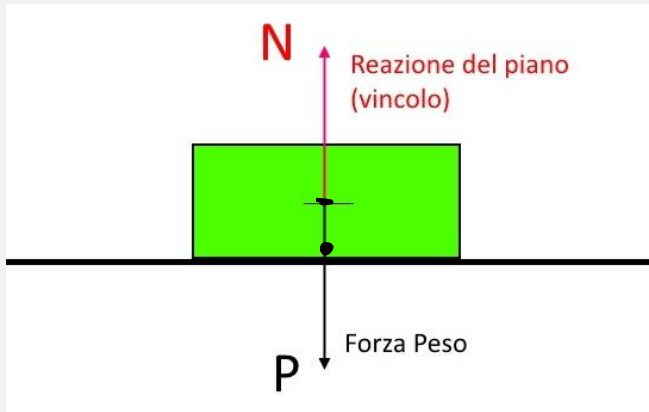


the constraint reaction is always orthogonal to the surface constituting the constraint and directed outward from it

# NORMAL FORCE

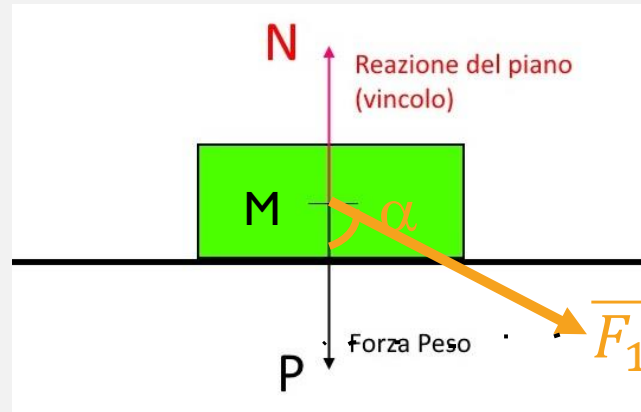
## SUPPORT CONSTRAINT FORCE

Force acting perpendicularly to the contact surface between two bodies



The body is in equilibrium, therefore the sum of the forces acting on it equals zero:

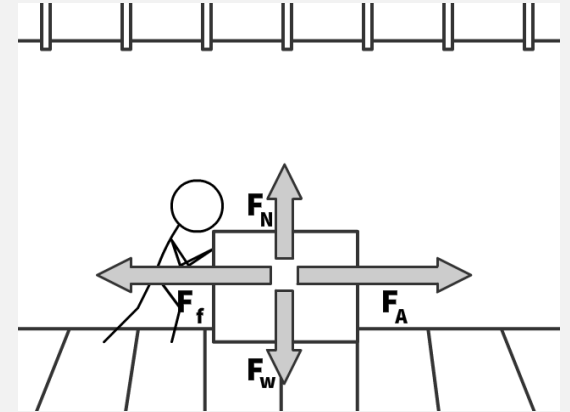
$$\begin{aligned}\vec{N} + \vec{W} &= 0 \\ \vec{N} &= -\vec{W} \\ \vec{N} &= -M\vec{g}\end{aligned}$$



In addition to the gravitational force, there is an external force  $\vec{F}_1$  which is oblique and forms an angle  $\alpha$  con l'asse verticale:

If the body is in equilibrium, the constraint reaction is given by the sum of the gravitational force and the vertical component of  $\vec{F}_1$ :

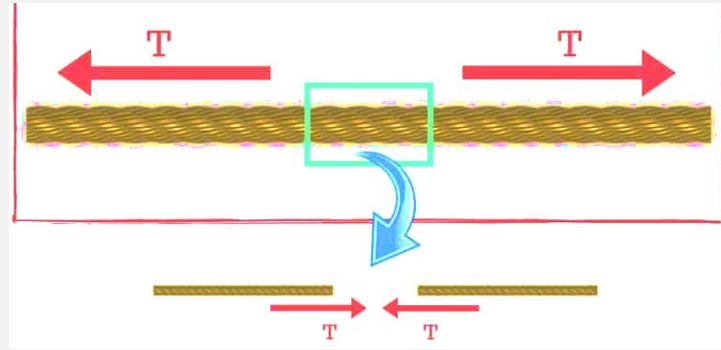
$$|\vec{N}| = F_1 \cos \alpha + W$$



# TENSION

## IDEAL ROPES:

- Negligible mass
- Inextensible
- Flexible



$$\sum_{\text{TOT}} \vec{F} = m_{\text{corda}} \vec{a} = \vec{F} - \vec{T}$$

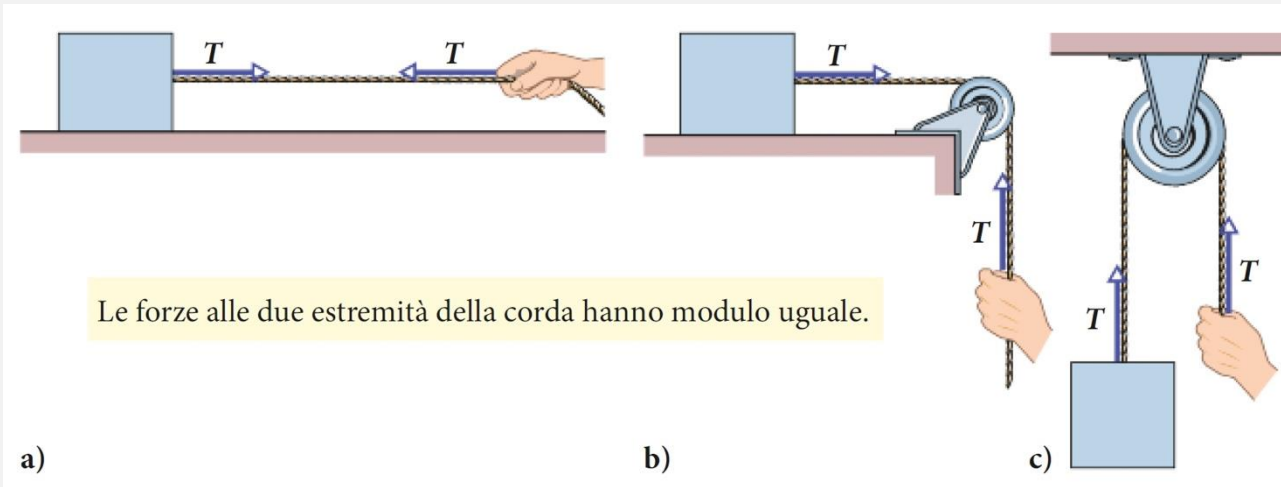
(Newton's second law:  $ma = \text{sum of applied forces}$ )

$$m_{\text{corda}} = 0$$

$$0\vec{a} = \vec{F} - \vec{T} \rightarrow \vec{F} = \vec{T}$$

If the rope is ideal, applying a force  $\vec{F}$  all'estremità della fune è esattamente equivalente all'applicazione della forza  $\vec{F}$  directly to the mass  $m$

# TENSION

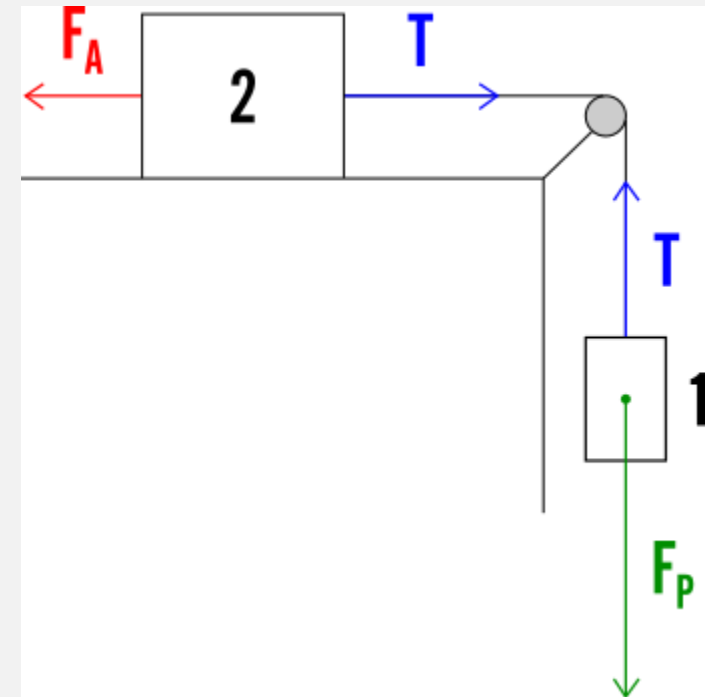


**Figura 5.9 a)** La corda, ben tesa, è in tensione. Se la sua massa è trascurabile, la corda tira il corpo e la mano con una forza  $T$ , anche se la corda scorre attorno a una carrucola priva di massa e di attrito come in **b)** e in **c)**.

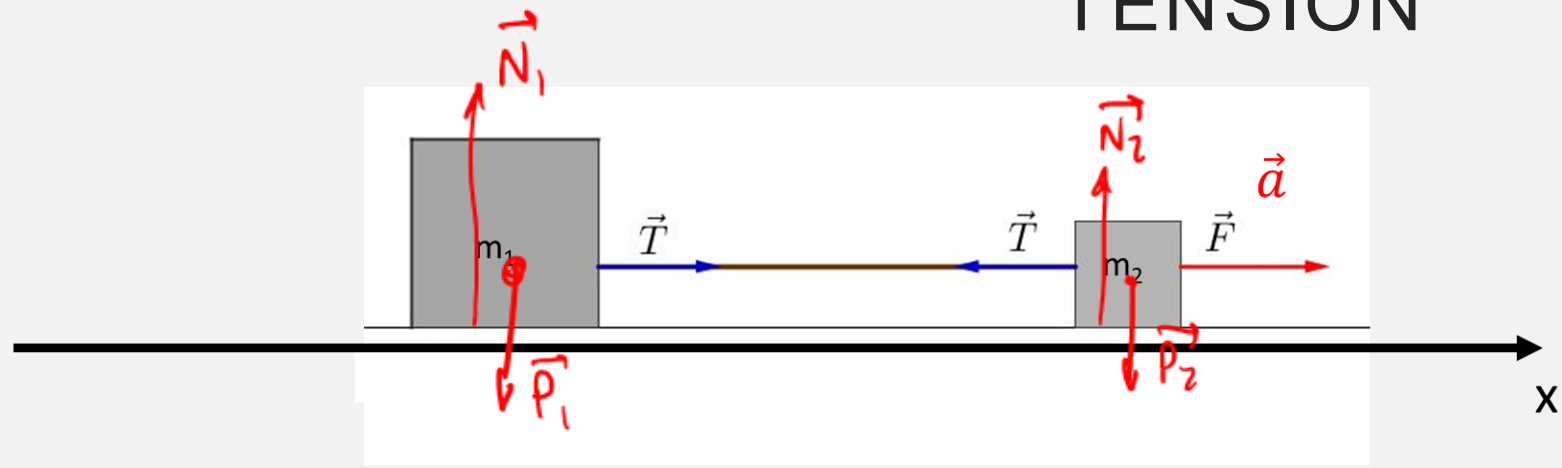
## PULLEY

- (b) A vertical force is applied to move a body horizontally
- (c) A body is lifted by applying a vertical force directed downward

If the system is in equilibrium, it means that body 1 is not heavy enough to move body 2 (friction)



# TENSION



- $m_1$  and  $m_2$ , connected by an ideal rope.
- On  $m_2$  a force  $\vec{F}$  directed to the right is applied.
- $m_1$  will have an acceleration  $\vec{a}_1$  and  $m_2$  will have an acceleration  $\vec{a}_2$
- The rope is ideal: it does not stretch/shorten: the system moves as a rigid body  $\rightarrow$  unique acceleration for the whole system  $a_1 = a_2 = a$

On  $m_1$  the gravitational force and constraint reaction act vertically, cancelling each other out  $\rightarrow$  the vertical net force is equal to zero

On  $m_1$  the only effective force is tension:

$$m_1 a = T$$

$$\Sigma F = m_1 a = T$$

On  $m_2$  the gravitational force and constraint reaction act vertically, cancelling each other out  $\rightarrow$  the vertical net force is equal to zero

On  $m_2$  act  $\vec{F}$  and  $\vec{T}$ , which have opposite directions:

$$m_2 a = F - T$$

$$\Sigma F = m_2 a$$

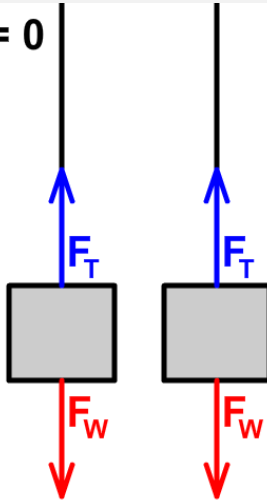
$$\begin{cases} m_1 a = T \\ m_2 a = F - T \end{cases}$$

$$a = \frac{F}{m_1 + m_2}$$

$$T = \frac{m_1 F}{m_1 + m_2}$$

Net Force = 0  
For Both

Static  
Equilibrium



Dynamic  
Equilibrium

Tension > Weight

Net Force Up

Acceleration Up

Not Equilibrium

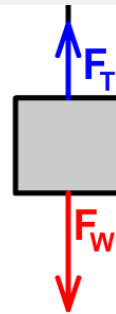


Tension < Weight

Net Force Down

Acceleration Down

Not Equilibrium

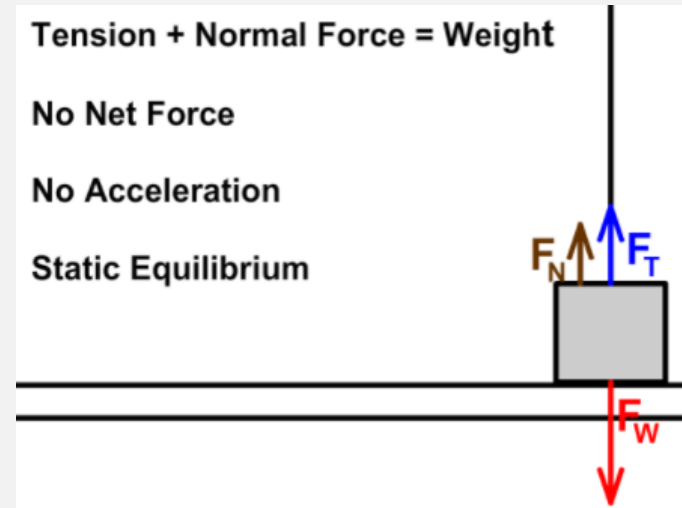


Tension + Normal Force = Weight

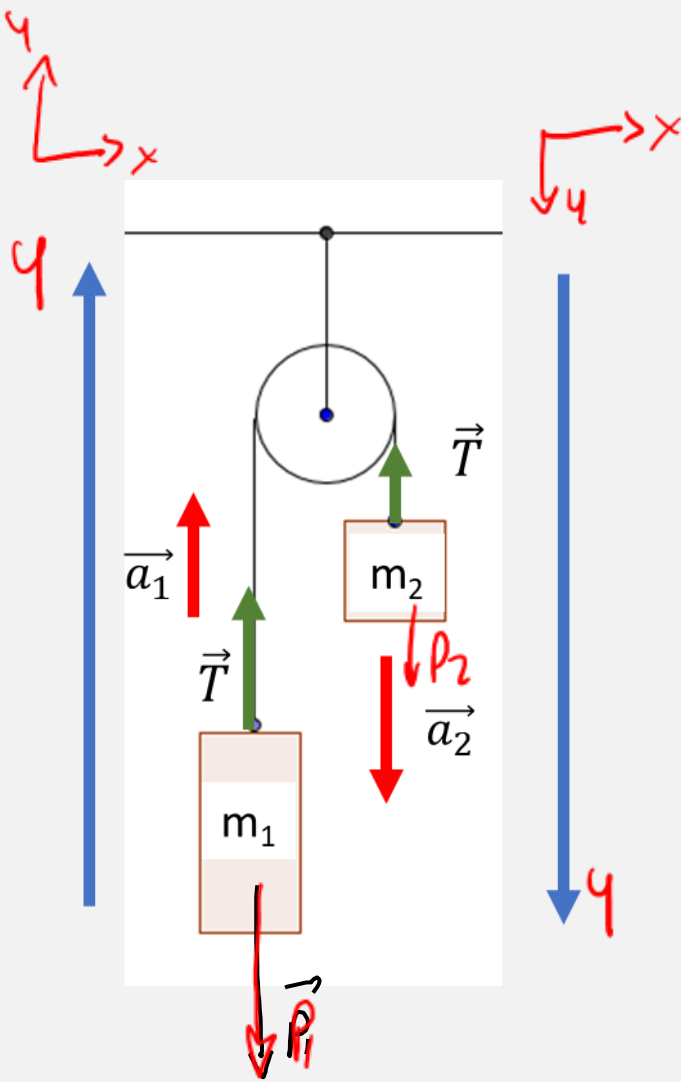
No Net Force

No Acceleration

Static Equilibrium



# TENSION – PULLEYS



Two masses are suspended from an ideal rope ( $m_1$  and  $m_2$ )

$$\Sigma F_1 = m_1 a$$

$$\begin{cases} m_1 a_1 = T - m_1 g \\ m_2 a_2 = m_2 g - T \end{cases}$$

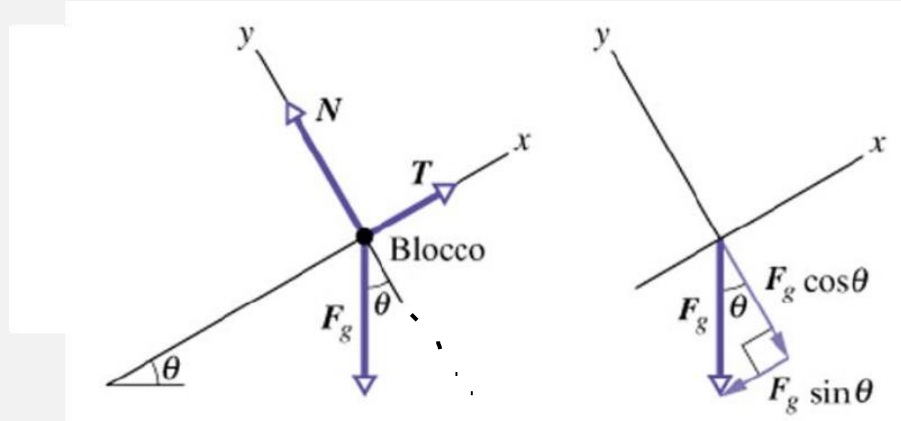
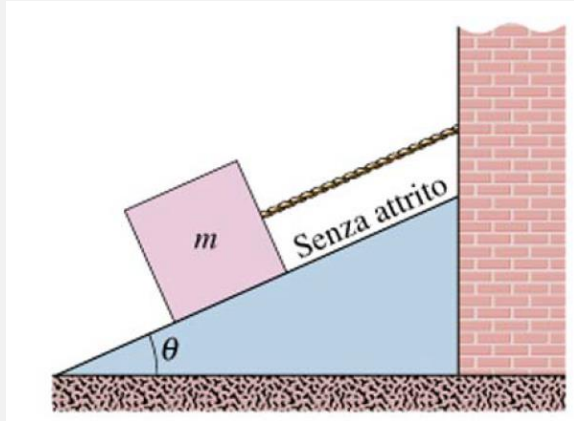
Since the acceleration is in fact unique,

$$a = \frac{m_2 - m_1}{m_1 + m_2} g$$

Three possible scenarios:

1.  $m_1 = m_2$ : system in equilibrium, because  $m_2 - m_1 = 0$ , hence  $a = 0$
2.  $m_1 < m_2$ : the pulley moves in the direction of  $m_2$
3.  $m_1 > m_2$ : it will be  $m_1$  that descends  $\rightarrow$  the acceleration will be  $< 0$

# TENSION



$$\theta = 30^\circ, m = 15\text{kg}$$

Determine the tension in the rope and the constraint reaction, knowing that the system is in equilibrium.

Equilibrium  $\rightarrow$  the resultant of the forces is equal to 0:  $\vec{T} + \vec{N} + \vec{F}_g = m\vec{a} = 0$

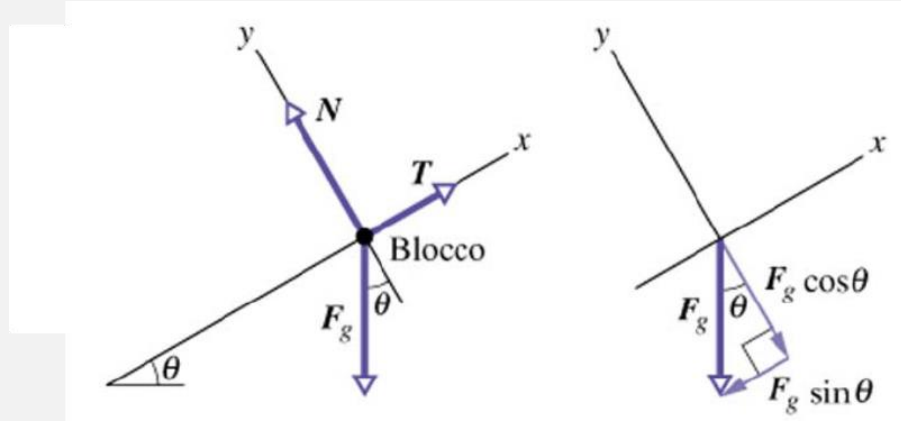
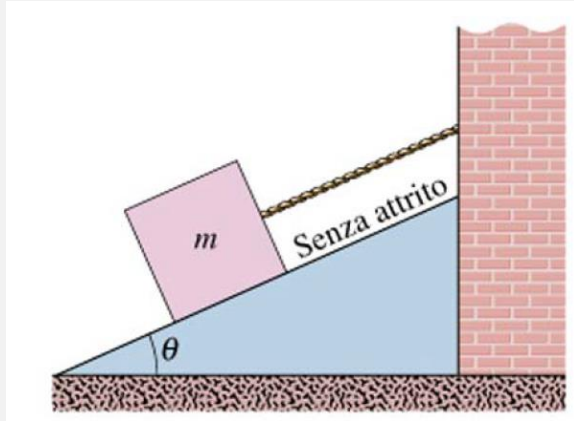
Along the  $x$ -axis: tension and the component of the gravitational force parallel to the inclined plane ( $F_g \sin \theta$ ):

$$T - mg \sin \theta = 0$$

Along the  $y$ -axis: constraint reaction and the component of the gravitational force perpendicular to the inclined plane ( $F_g \cos \theta$ ):

$$N - mg \cos \theta = 0$$

# TENSION



$$\theta = 30^\circ, m = 15\text{kg}$$

Determine the tension in the rope and the constraint reaction, knowing that the system is in equilibrium.

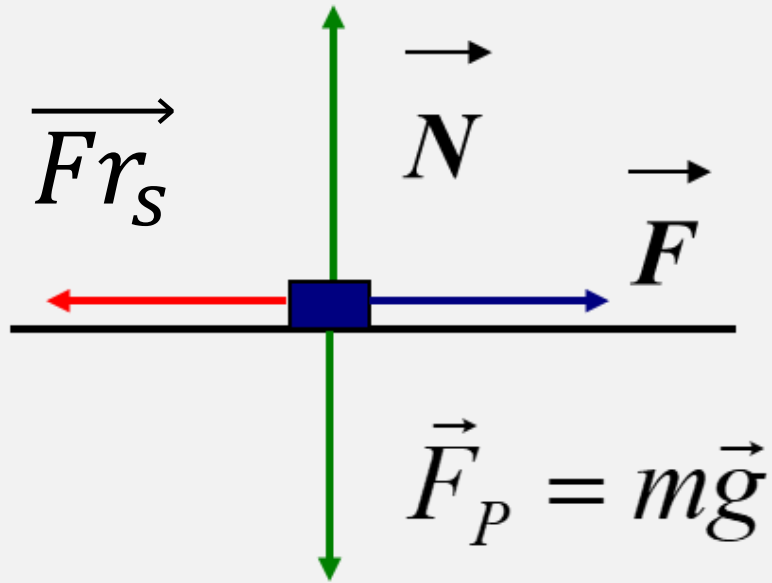
The rope breaks: what is the acceleration of the block?

Along the  $y$ -axis the resultant is always equal to zero:  $N - mg \cos \theta = 0$

Along the  $x$ -axis, the tension disappears: the only component effective in making the body move is  $mg \sin \theta$

Value of the acceleration

# STATIC FRICTION



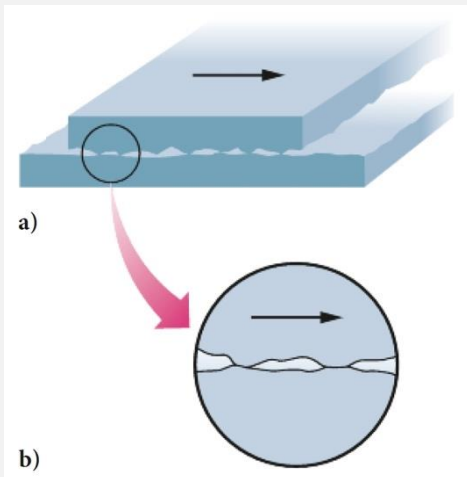
Body on a horizontal plane subjected a force  $\vec{F}$  but does not move:  $\vec{F} = \vec{F}r_s$

$$\vec{F} \leq \vec{F}r_{s \max} = \mu_s \vec{F}_P = \mu_s \vec{N}$$

Horizontal support plane:

$$\vec{F}_P = -\vec{N} \quad |\vec{F}_P| = |\vec{N}| = mg$$

$$Fr_{s \max} = \mu_s mg$$



# STATIC FRICTION

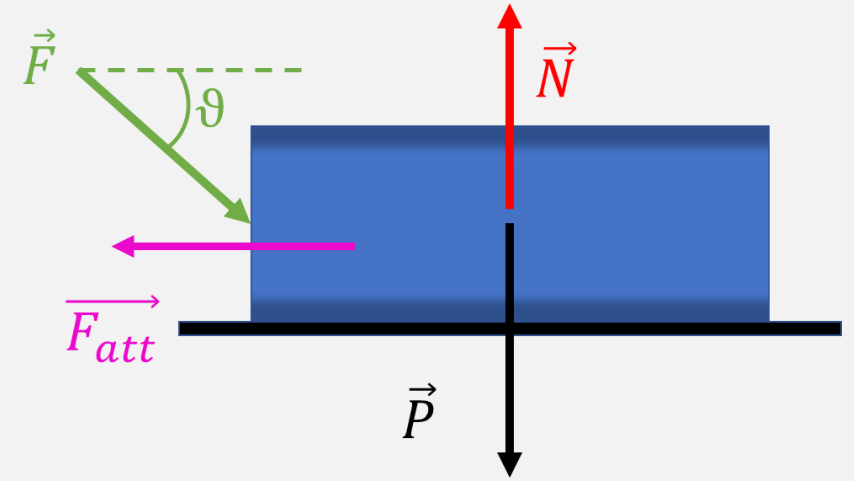
Equilibrium  $\rightarrow$  the net force is zero

Along the x-axis: horizontal component of the applied force ( $F \cos \theta$ ) and friction force ( $\vec{F}_r$ )

$$F \cos \theta - F_r = 0$$

Along the y-axis: constraint reaction and vertical component of the force ( $F \sin \theta$ ):

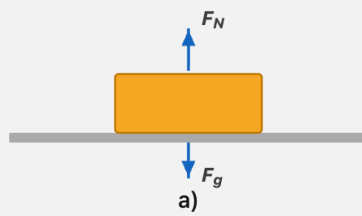
$$mg + F \sin \theta - N = 0 \rightarrow F \sin \theta + mg = N$$



$$\mu_s = \frac{F_r}{N} \quad \mu_s = \frac{F \cos \theta}{F \sin \theta + mg}$$

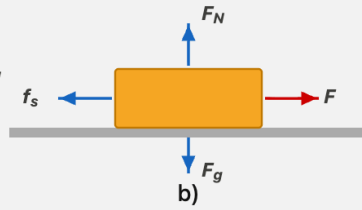
# KINETIC FRICTION

No attempt to slide the block.  
No friction force.



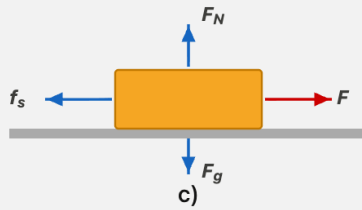
Friction force magnitude = 0

A force  $F$  tries to slide the block,  
balanced by friction. No motion.



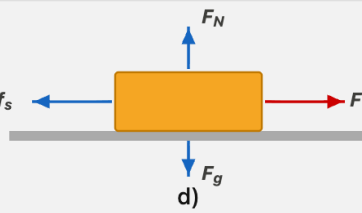
Friction force magnitude =  $F$

$F$  is larger, still balanced  
by friction. No motion.



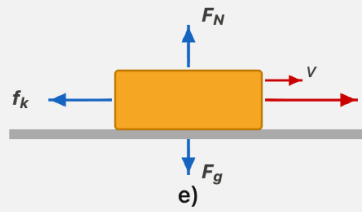
Friction force magnitude =  $F$

$F$  is even larger, still  
balanced by friction. No motion.



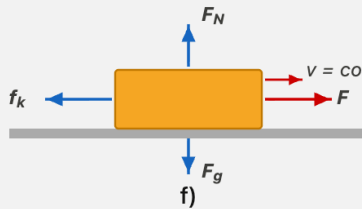
Friction force magnitude  $\approx F$

Finally, applied force exceeds  
static friction. Block slides  
and accelerates.

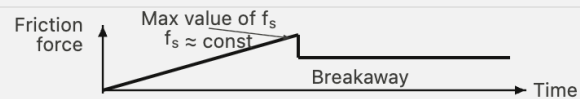


Weak dynamic  
friction force.

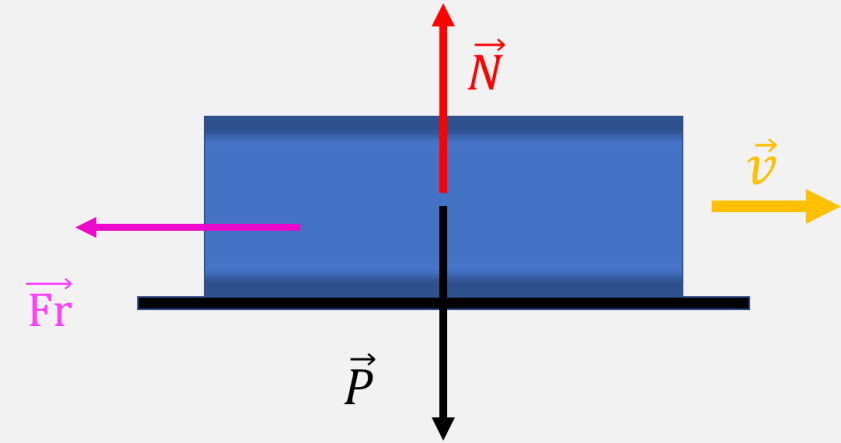
To maintain constant speed,  
reduce  $F$  to match kinetic  
friction (now weaker).



Same weak dynamic  
friction force.

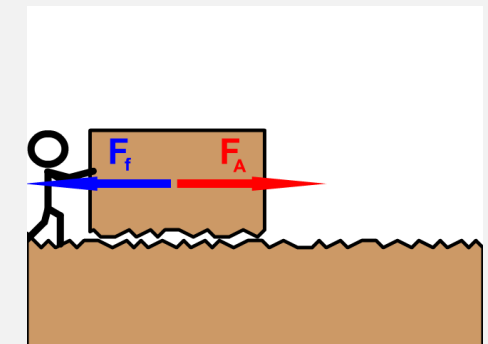
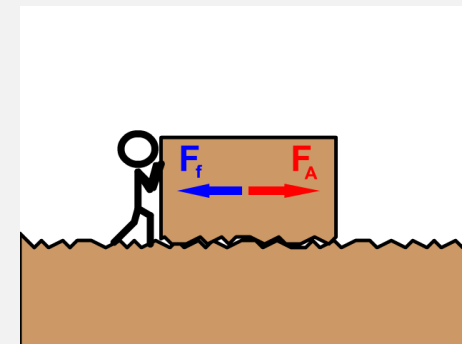


Kinetic friction has  
one value (cannot  
match applied force).



$$Fr = \mu_D N$$

$$\mu_S > \mu_D$$



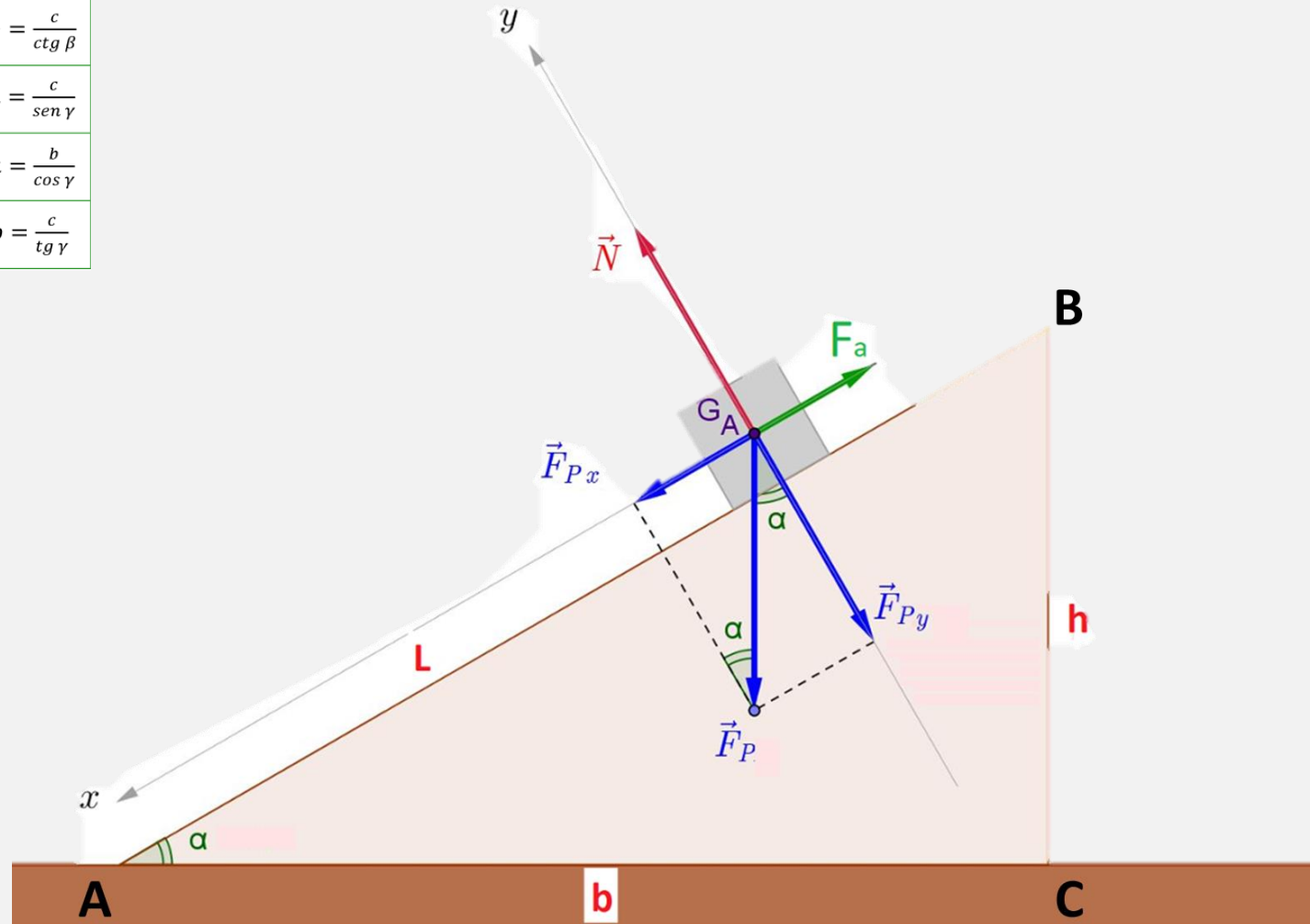
# INCLINED PLANE

	$\text{sen } \beta = \frac{b}{a} \rightarrow b = a \cdot \text{sen } \beta$	e	$a = \frac{b}{\text{sen } \beta}$
	$\text{cos } \beta = \frac{c}{a} \rightarrow c = a \cdot \text{cos } \beta$	e	$a = \frac{c}{\text{cos } \beta}$
	$\text{tg } \beta = \frac{b}{c} \rightarrow b = c \cdot \text{tg } \beta$	e	$c = \frac{b}{\text{tg } \beta}$
	$\text{ctg } \beta = \frac{c}{b} \rightarrow c = b \cdot \text{ctg } \beta$	e	$b = \frac{c}{\text{ctg } \beta}$
	$\text{sen } \gamma = \frac{c}{a} \rightarrow c = a \cdot \text{sen } \gamma$	e	$a = \frac{c}{\text{sen } \gamma}$
	$\text{cos } \gamma = \frac{b}{a} \rightarrow b = a \cdot \text{cos } \gamma$	e	$a = \frac{b}{\text{cos } \gamma}$
	$\text{tg } \gamma = \frac{c}{b} \rightarrow c = b \cdot \text{tg } \gamma$	e	$b = \frac{c}{\text{tg } \gamma}$

$$F_{Px} = F_P \sin \alpha$$

$$F_{Py} = F_P \cos \alpha = N$$

$$F_a = \mu_S N = \mu_S F_P \cos \alpha = \mu_S m g \cos \alpha$$



# INCLINED PLANE

## CASE WITHOUT FRICTION

$$F_{Px} = F_P \sin \alpha$$

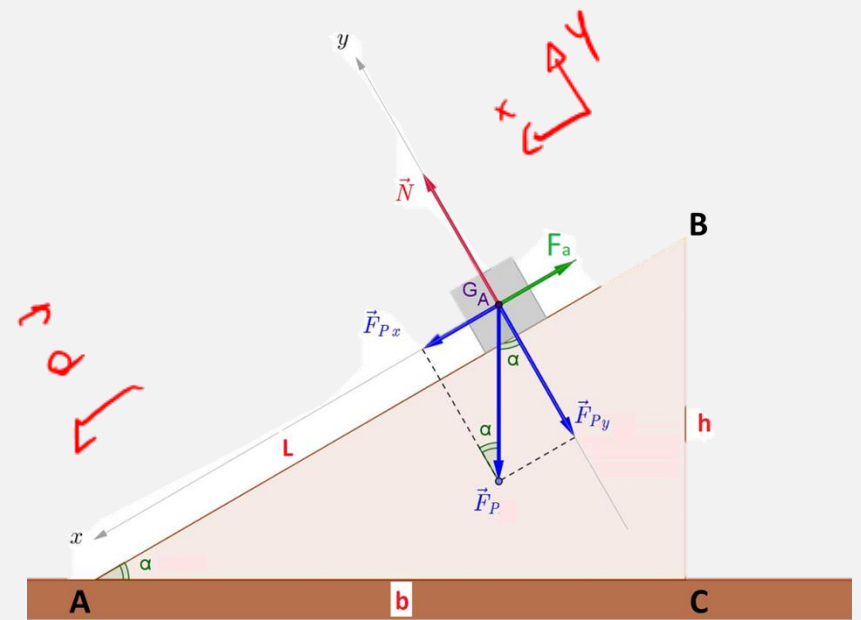
$$ma = F_{Px} = mg \sin \alpha \rightarrow a = g \sin \alpha$$

## CASE WITH FRICTION

$$F_a = \mu_D N = \mu_D F_P \cos \alpha = \mu_D mg \cos \alpha$$

$$ma = F_{Px} - F_a = mg \sin \alpha - \mu_D mg \cos \alpha$$

$$a = g(\sin \alpha - \mu_D \cos \alpha)$$



## EQUILIBRIUM CASE

$$\mu_s mg \cos \alpha = mg \sin \alpha$$

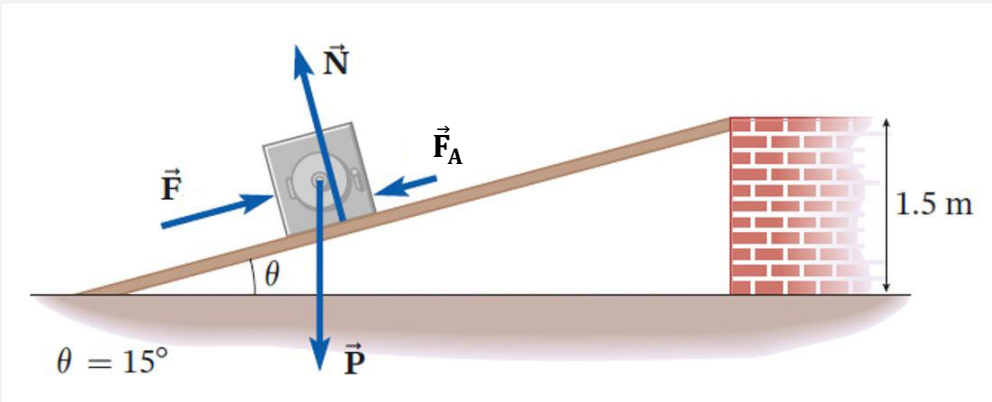
$$\sin \alpha = \mu_s \cos \alpha \rightarrow \mu_s = \tan \alpha$$

$$\frac{\sin \alpha}{\cos \alpha}$$



## Example

A safe must be moved along an inclined plane whose upper end is 1.5m above the ground. The mass of the safe is 510kg. The coefficient of static friction along the surface of the inclined plane is 0.42, while the coefficient of kinetic friction is 0.33. The plane makes an angle of  $15^\circ$  with the horizontal plane. A) Assuming the pushing force is parallel to the inclined plane, what magnitude will be needed to set the safe in motion on the ramp? B) Once the safe has been set in motion, what force must be applied to move it at constant velocity?





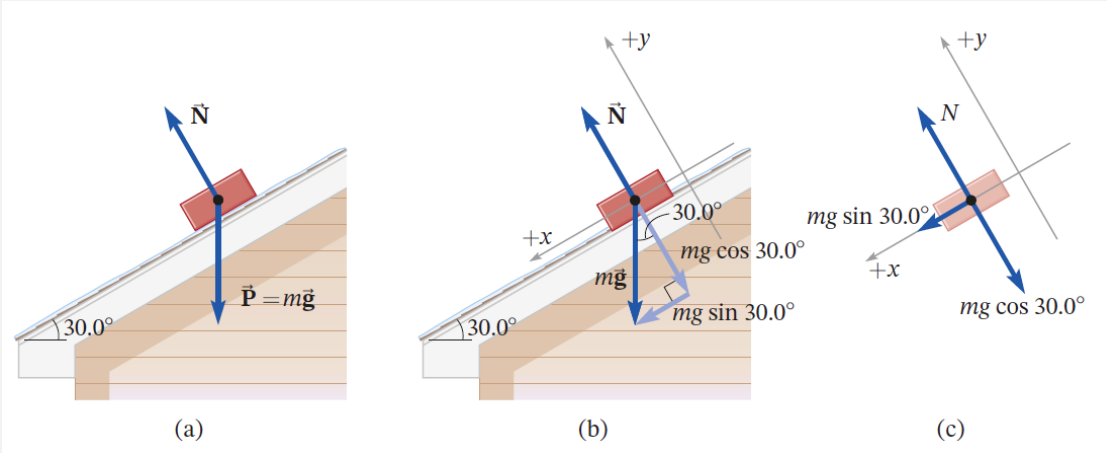
## Example

Given an ideal pulley with an ideal rope and two masses,  $m_1=26\text{kg}$  and  $m_2=42\text{kg}$ . Calculate the motion of the system.



## Example

A brick with a mass of 1 kg slides on the icy surface of a roof, inclined at  $30^\circ$ . If the brick starts from rest, what velocity will it have after 0.9 s, i.e. when it reaches the edge of the roof? Neglect friction.





## Example

Two boxes are connected by an ideal rope running over a pulley. The coefficient of kinetic friction between the surface and box 1 is 0.2. Find the acceleration of the system.

