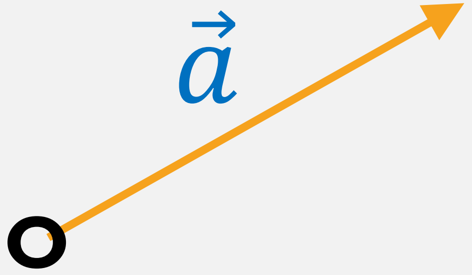
# KINEMATICS Pt. 1

- Vectors
- Position, displacement, velocity, and acceleration vectors
- Graphical relations: position–velocity and velocity–acceleration
- Examples

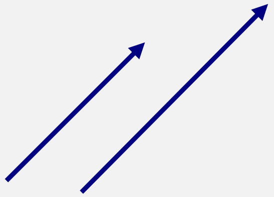
# VECTORS



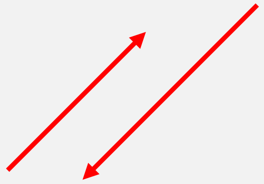

Vector $\vec{a}$ o $\boldsymbol{a}$

The magnitude of the vector is $\overrightarrow{|a|} = a$ and, graphically, it corresponds to the length of the arrow.

The origin of the vector is point **O**.



**CONCORDANT**
**(parallel)**
**same direction and same sense**

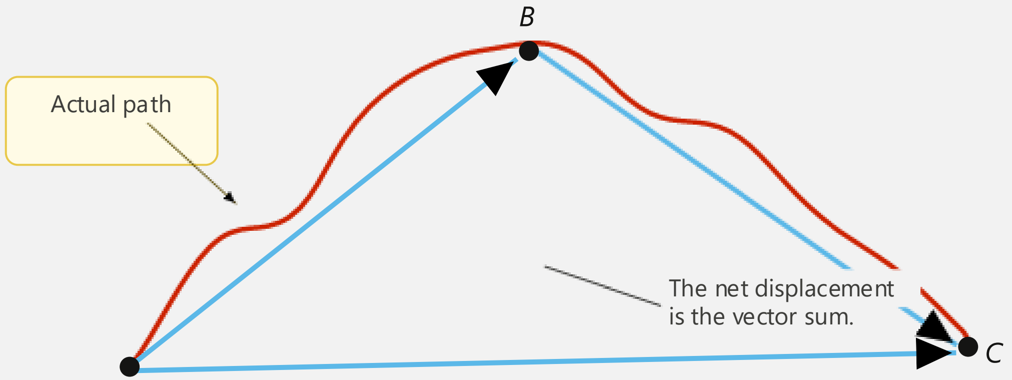


**DISCORDANT**
**(anti-parallel)**
**same direction but opposite sense**

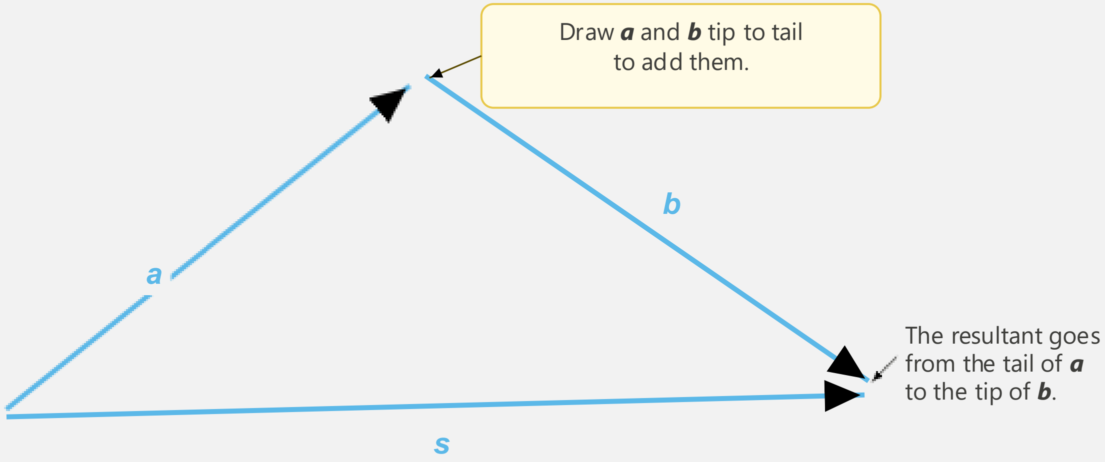
**EQUIPOLLENT**
**Equal in magnitude, sense, and direction**

# VECTOR OPERATIONS

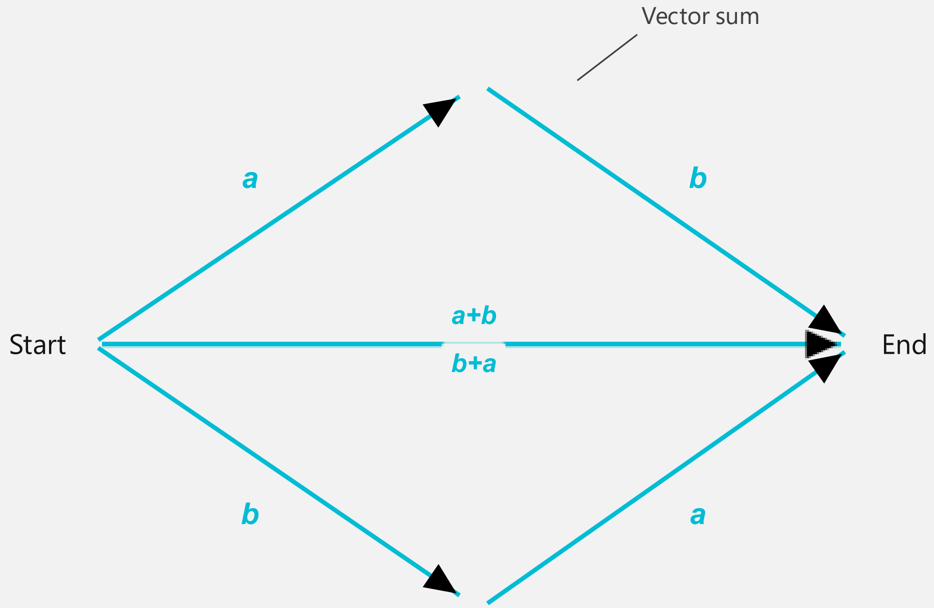
## Vector Addition




a)



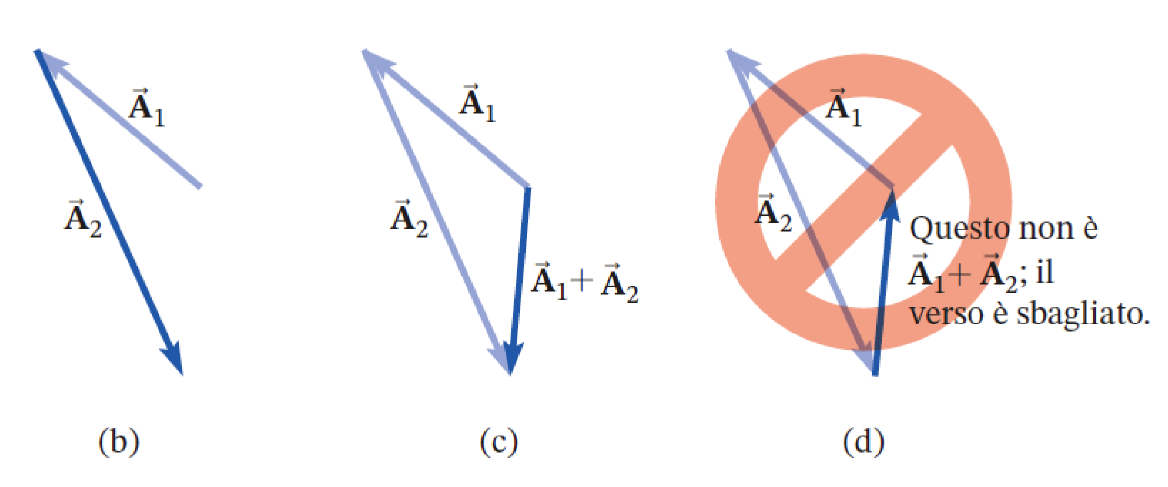

b)

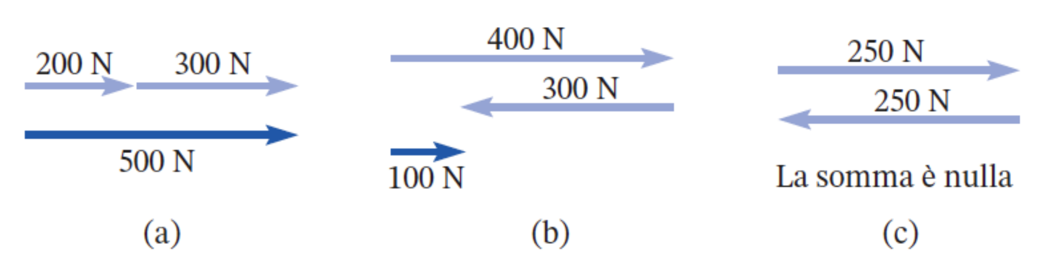



The resultant vector from the addition does not depend on the order of the vectors being added.

# VECTOR OPERATIONS

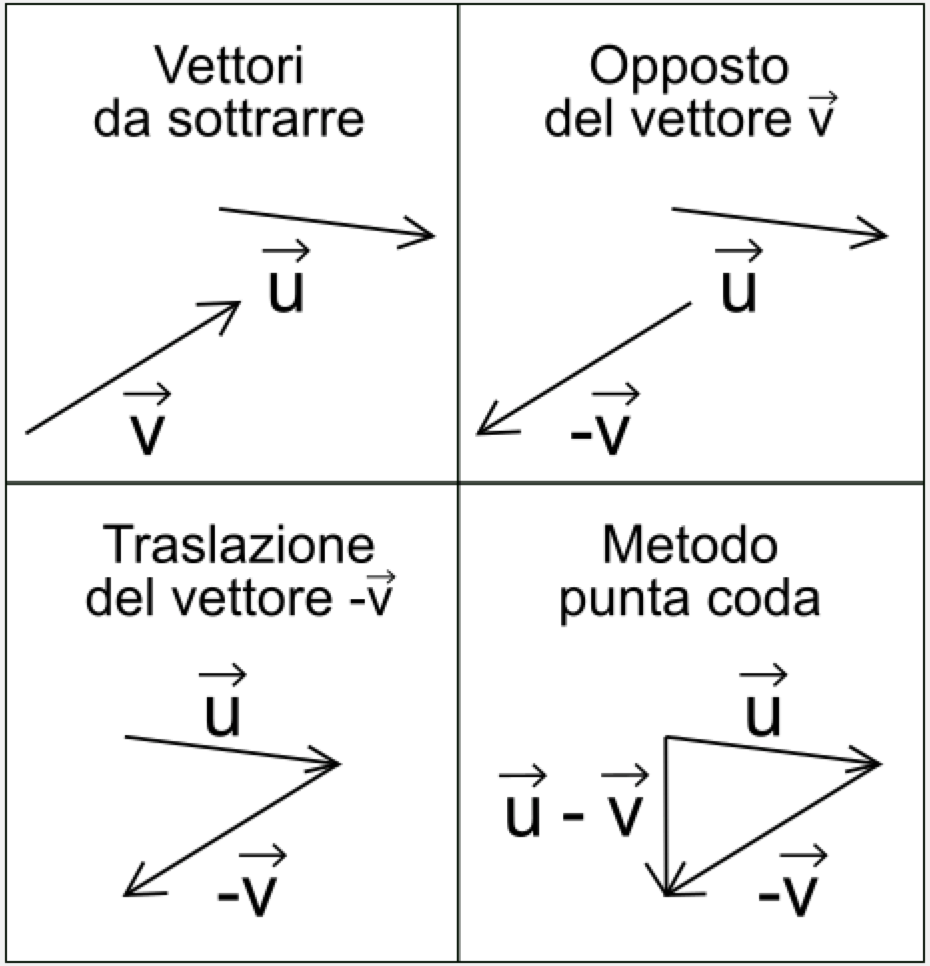
## Vector Addition

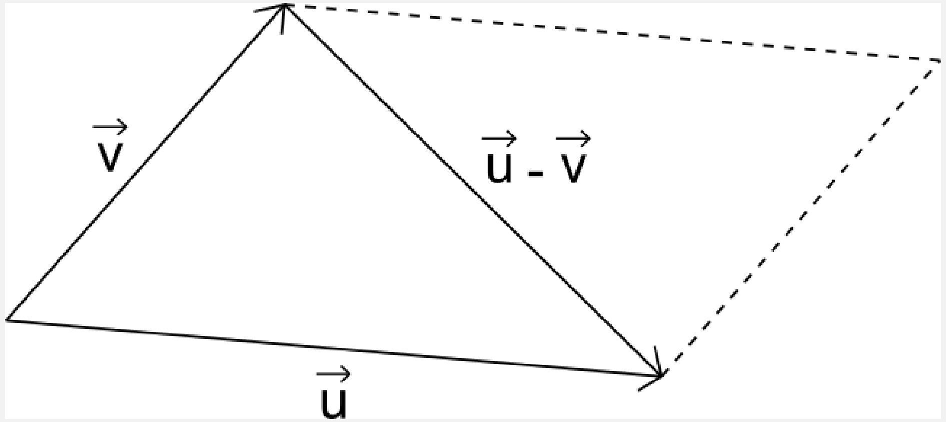


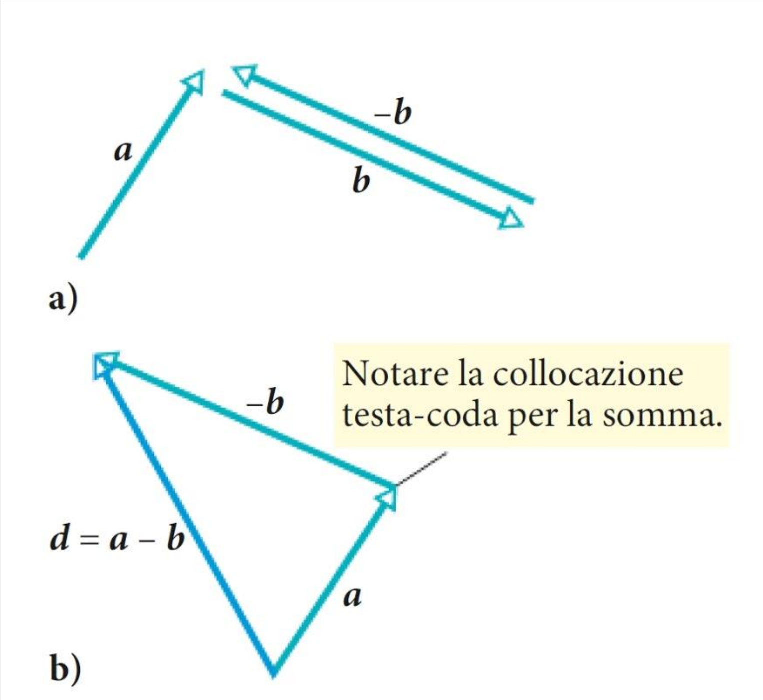



# VECTOR OPERATIONS

## Vector Subtraction

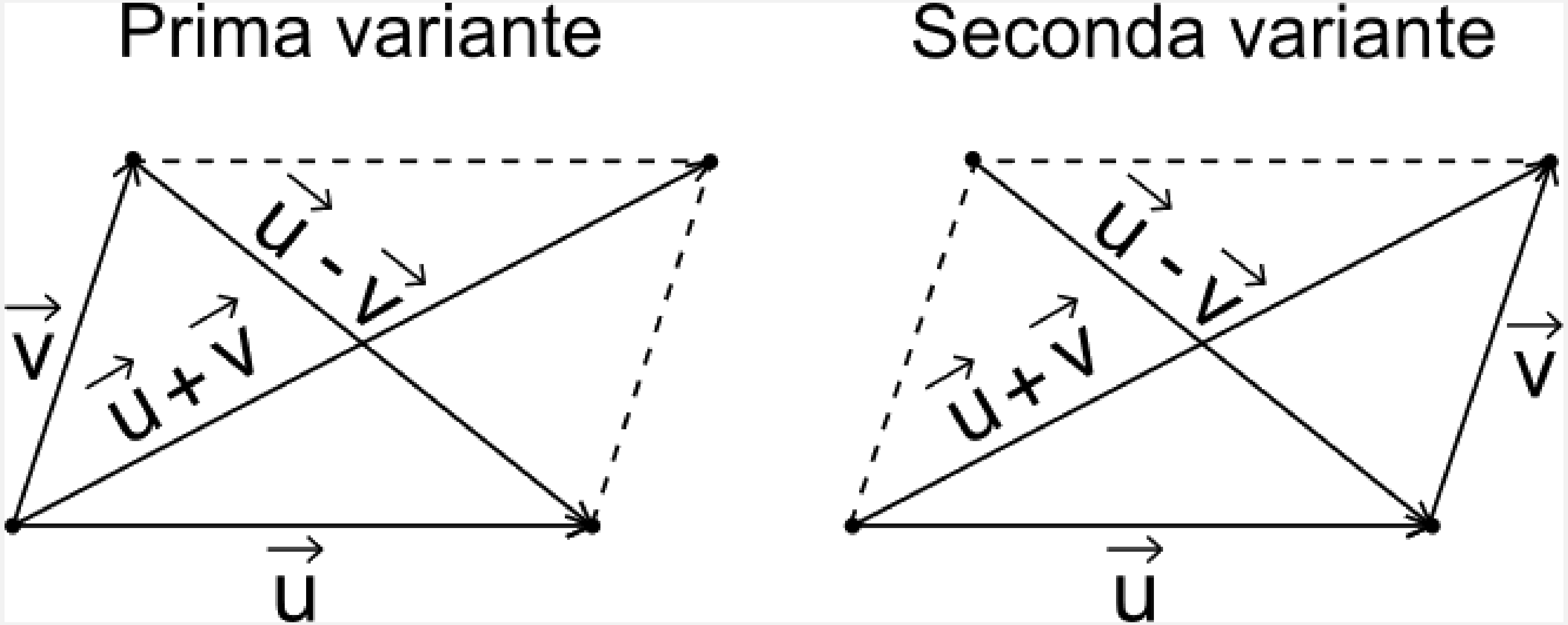


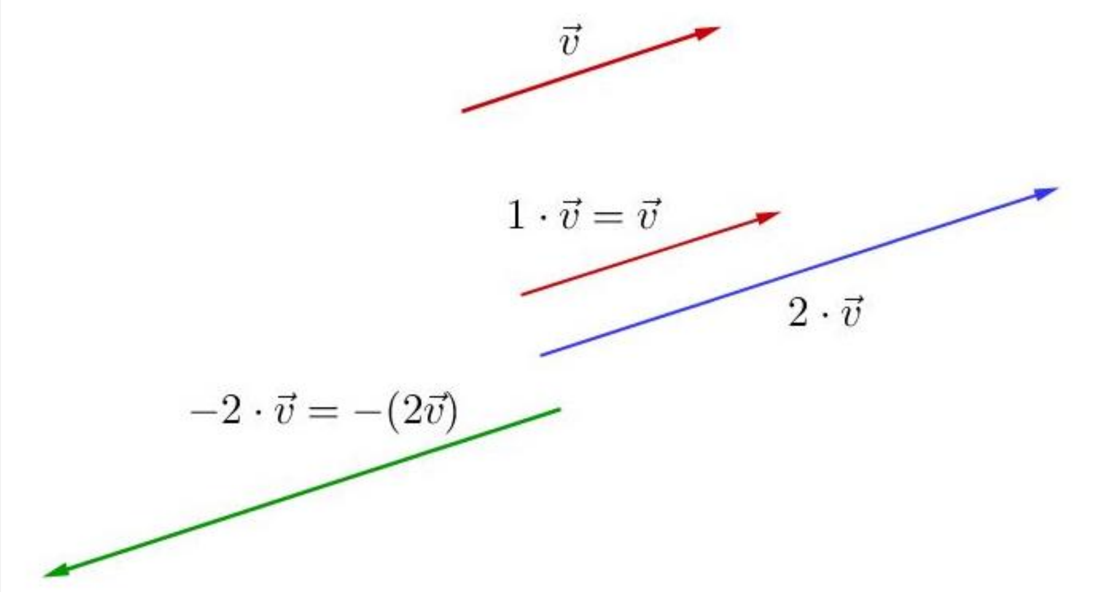



# VECTOR OPERATIONS

## Vector Subtraction



## Vector–Scalar Product

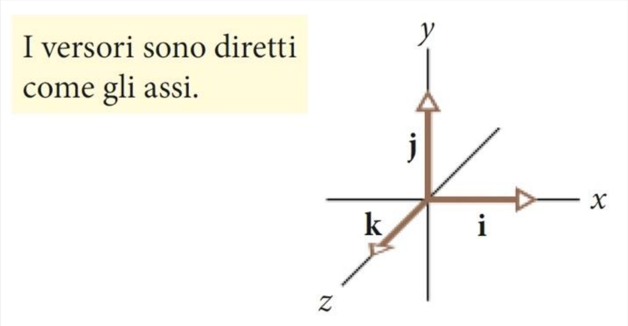


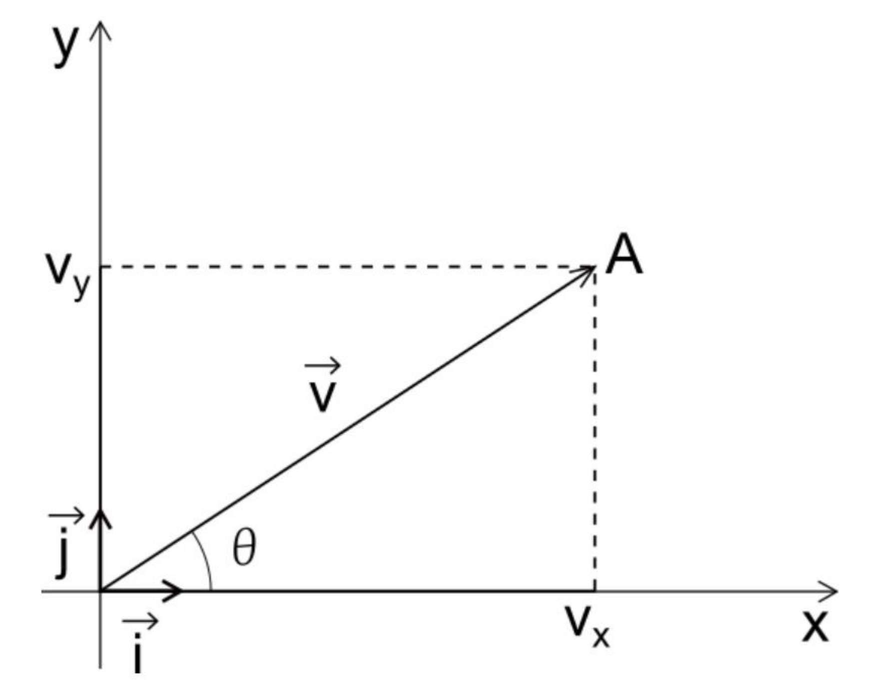

The vector $\vec{v}$ has

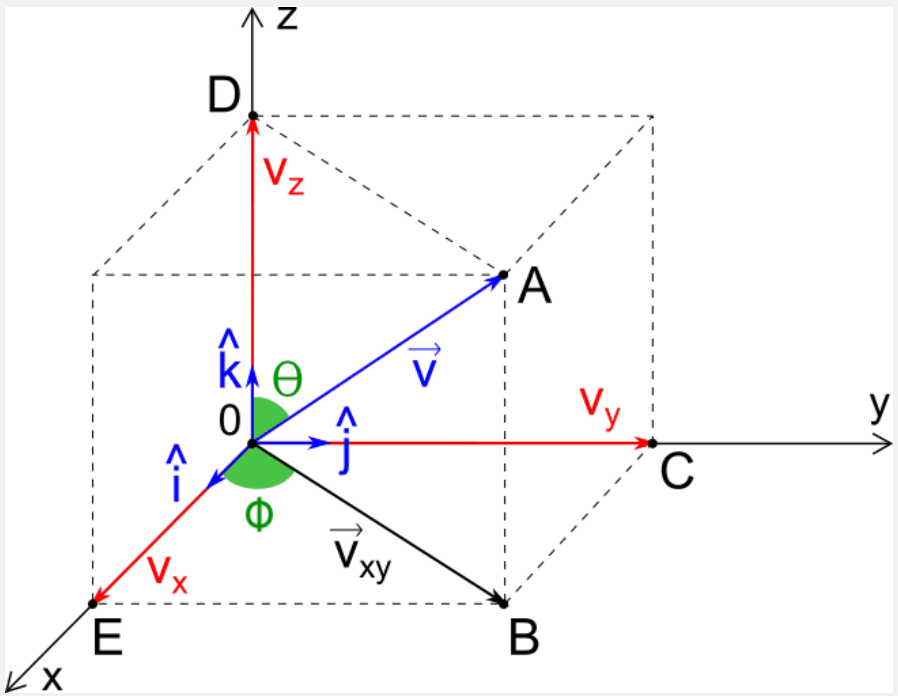
- the **same direction** as $\vec{u}$
- the **same sense** as $\vec{u}$ if $k > \mathbf{0}$, **opposite sense** if $k < \mathbf{0}$
- magnitude $k|\vec{u}| = ku$

# OPERATIONS BETWEEN VECTORS

## Vector Components / Projections on x and y

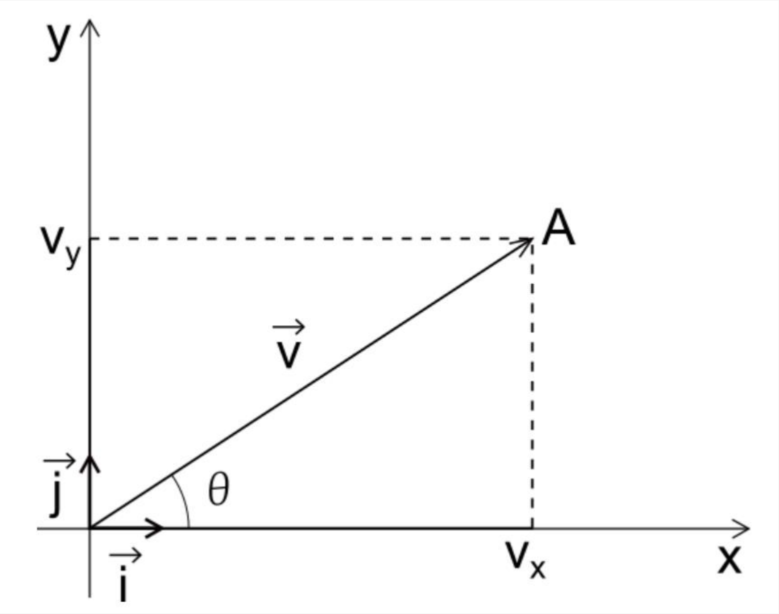


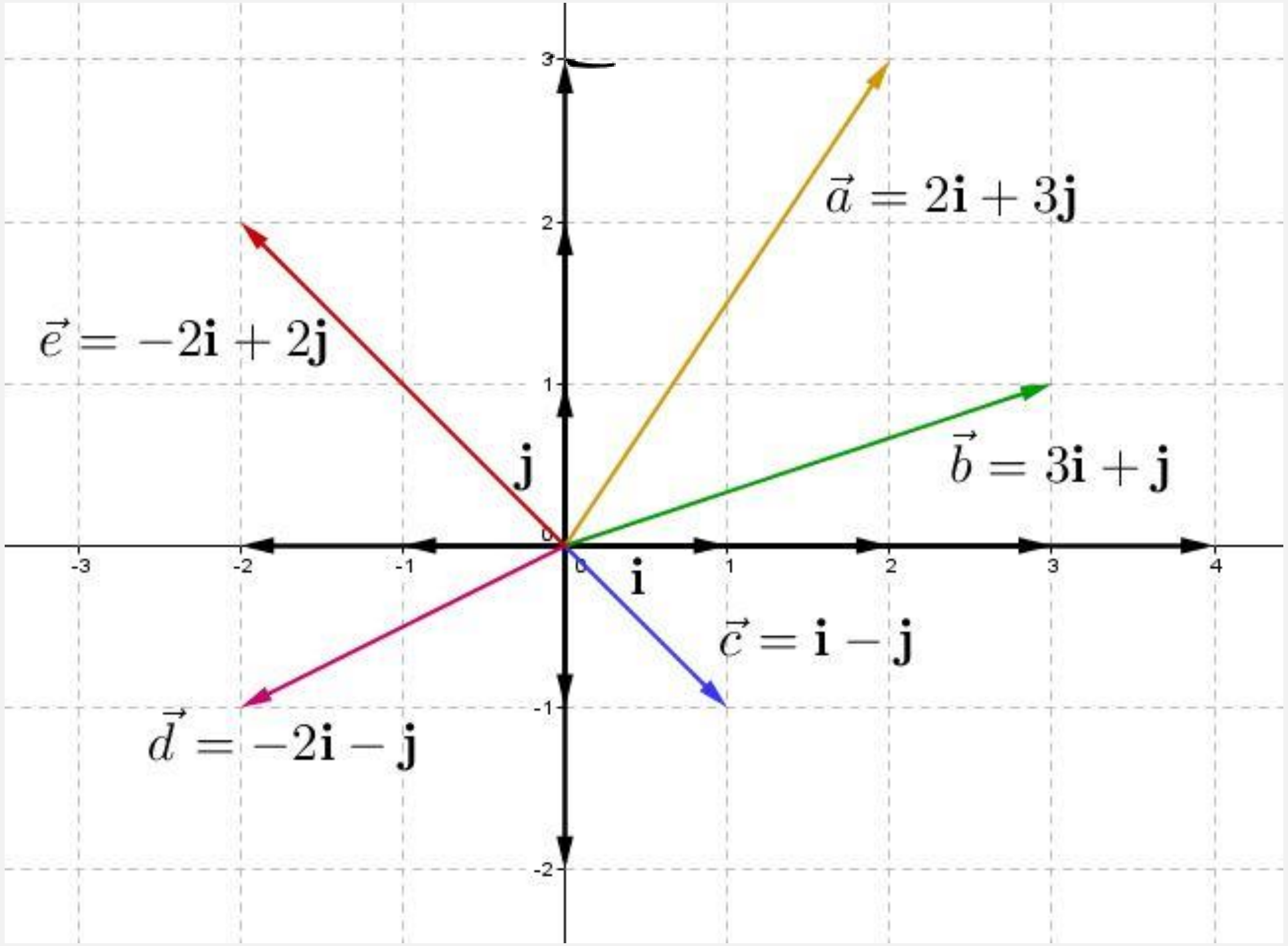



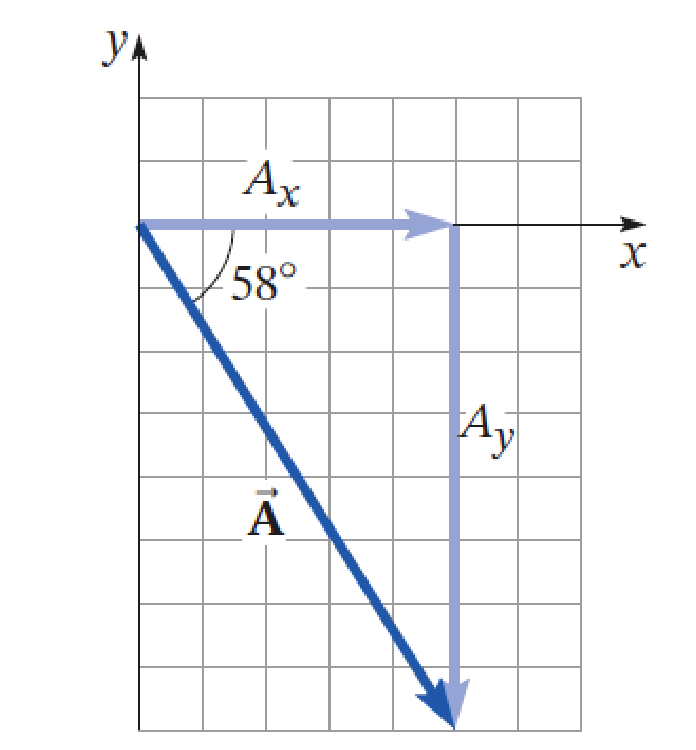
# OPERATIONS BETWEEN VECTORS

## Vector Components / Projections on x and y



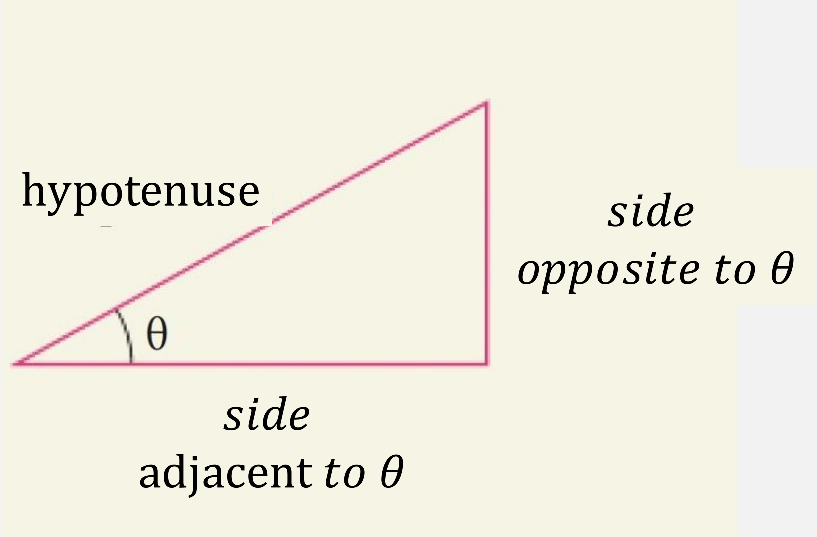
## Vector Components / Projections on x and y




$$\sin\theta = \frac{\text{side opposite to }\theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{side adjacent to }\theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{side opposite to }\theta}{\text{side adjacent to }\theta}$$



# VECTOR OPERATIONS

## Algebra with Components

$$\boldsymbol{r} = \boldsymbol{a} + \boldsymbol{b}$$

$$r_x = a_x + b_x$$

$$r_y = a_y + b_y$$

$$r_z = a_z + b_z$$

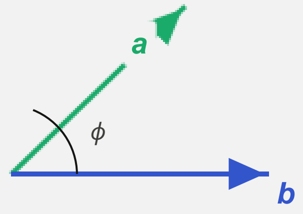
***r*** is equal to the vector (***a***+***b***): if this is true, each component of ***r*** must coincide with the corresponding component of (***a***+***b***)

*Two vectors are equal if their respective components are all equal to one another*

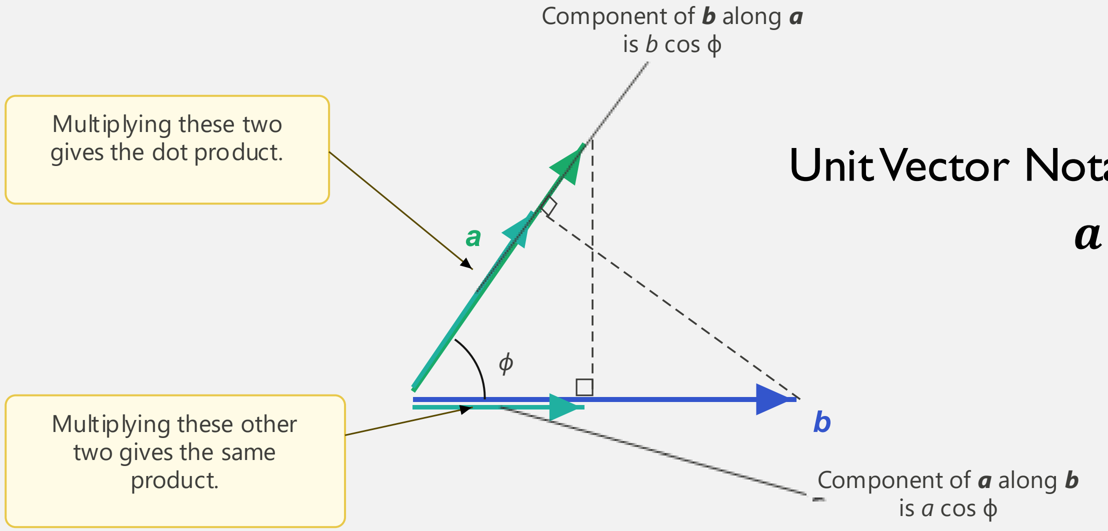
$$\boldsymbol{d} = \boldsymbol{a} + (-\boldsymbol{b}) \rightarrow \quad d_x = a_x - b_x \quad d_y = a_y - b_y \quad d_z = a_z - b_z$$

where $\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$

# Dot Product




a)




b)

Dot Product of Vectors ***a*** and ***b*** («***a*** dot ***b***»):

$$\boldsymbol{a} \cdot \boldsymbol{b} = ab\ cos\varphi$$

All the terms on the right-hand side are scalars; therefore, the product ***a·b*** is a scalar!

Unit Vector Notation:

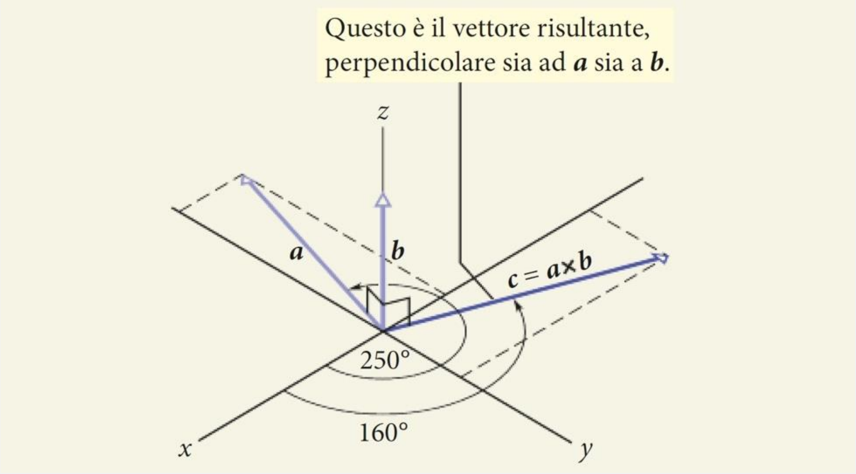
$$\boldsymbol{a} \cdot \boldsymbol{b} = (a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}) \cdot (b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k})$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_xb_x + a_yb_y + a_zb_z$$

[Distributive Property]

# VECTOR OPERATIONS

## Vector Product

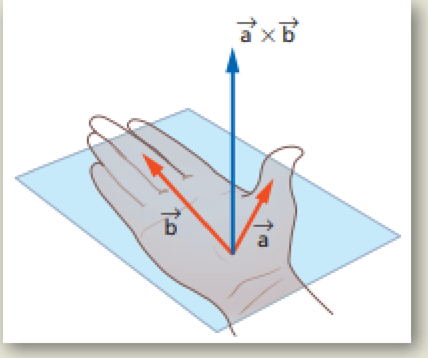



**Prodotto vettoriale**

Il prodotto vettoriale di due vettori $\vec{a}$ e $\vec{b}$ è il vettore $\vec{c}$ che ha:

- **modulo** uguale ad $ab \operatorname{sen} \alpha$;
- **direzione** perpendicolare al piano individuato dai due vettori;
- **verso** dato dalla regola della mano destra, illustrata nella figura.

Si indica con $\vec{a} \times \vec{b}$.




Vector product of vectors ***a*** and ***b*** ("***a*** vector ***b***") → vector ***c***, whose magnitude is:

$$\boldsymbol{c} = ab\, sin\varphi$$

No Commutative Property: $\boldsymbol{a} \times \boldsymbol{b} \neq \boldsymbol{b} \times \boldsymbol{a}$

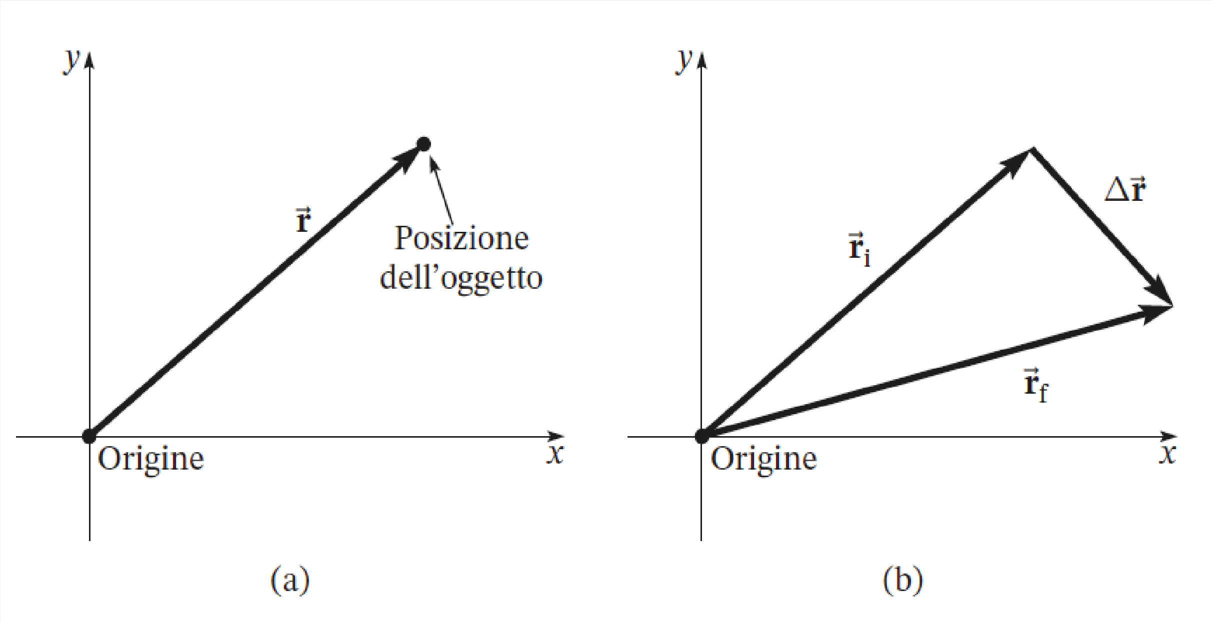
Yes Distributive Property: $\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c}$

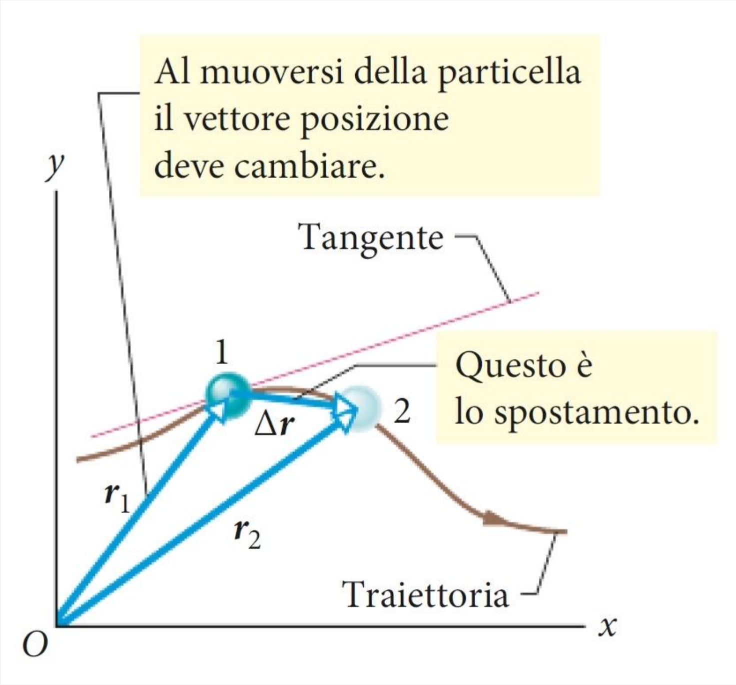
With ***a*** // ***b*** → ***a*** × ***b*** = 0

With ***a*** ⊥ ***b*** → |***a*** × ***b***| = max

# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

## Position Vector – Displacement Vector



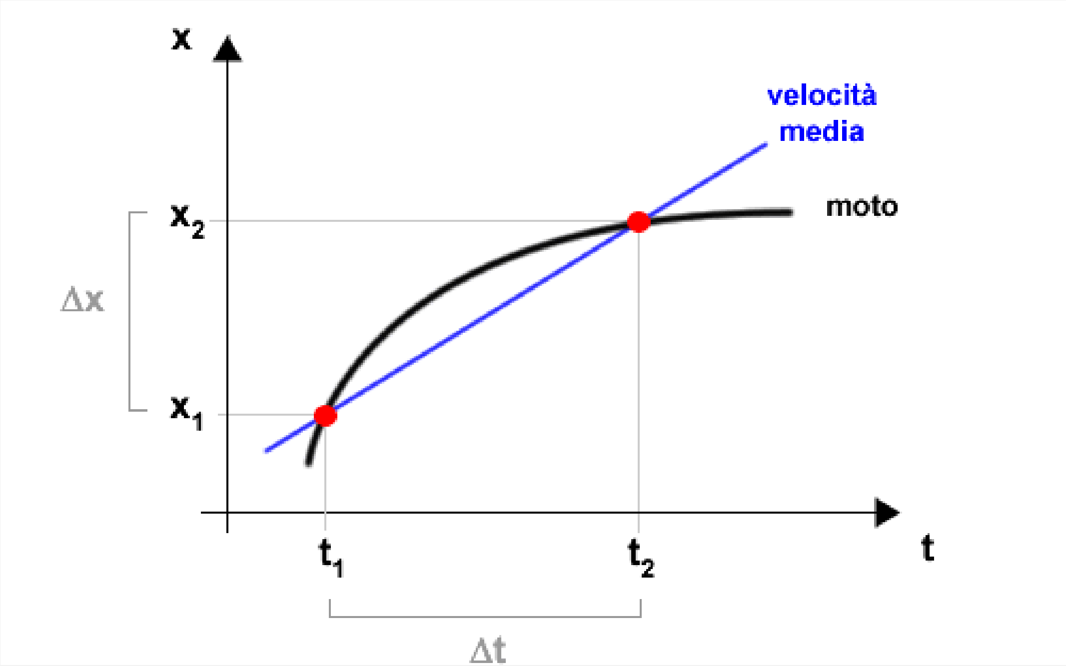

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$ Position Vector

$$\Delta\vec{r} = (x_f - x_i)\cdot\hat{\imath} + (y_f - y_i)\cdot\hat{\jmath} = \Delta x\cdot\hat{\imath} + \Delta y\cdot\hat{\jmath}$$

Displacement Vector

# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

## Average Velocity Vector



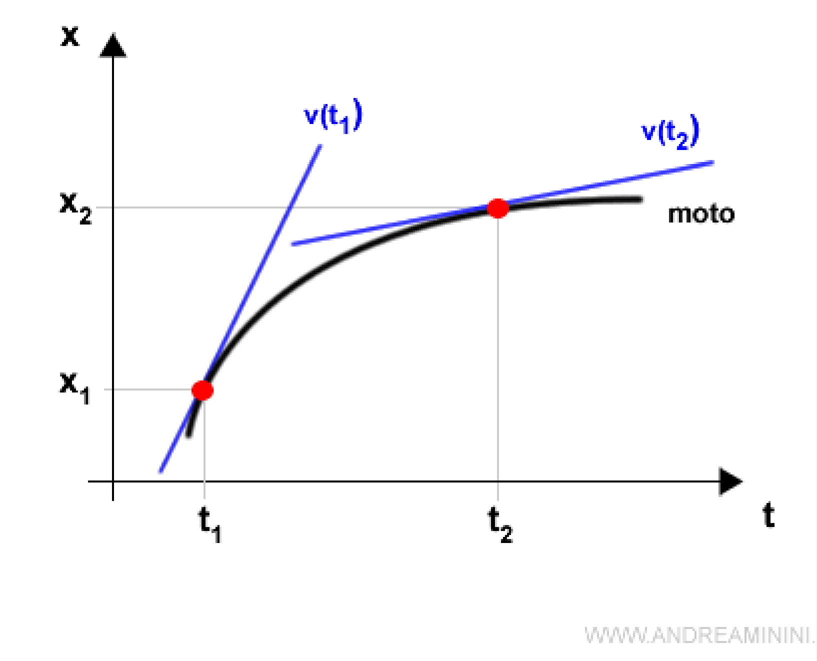

$$\vec{v}_m = \frac{\Delta\vec{r}}{\Delta \mathrm{t}} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \qquad [L]/[t] = [Lt^{-1}]$$

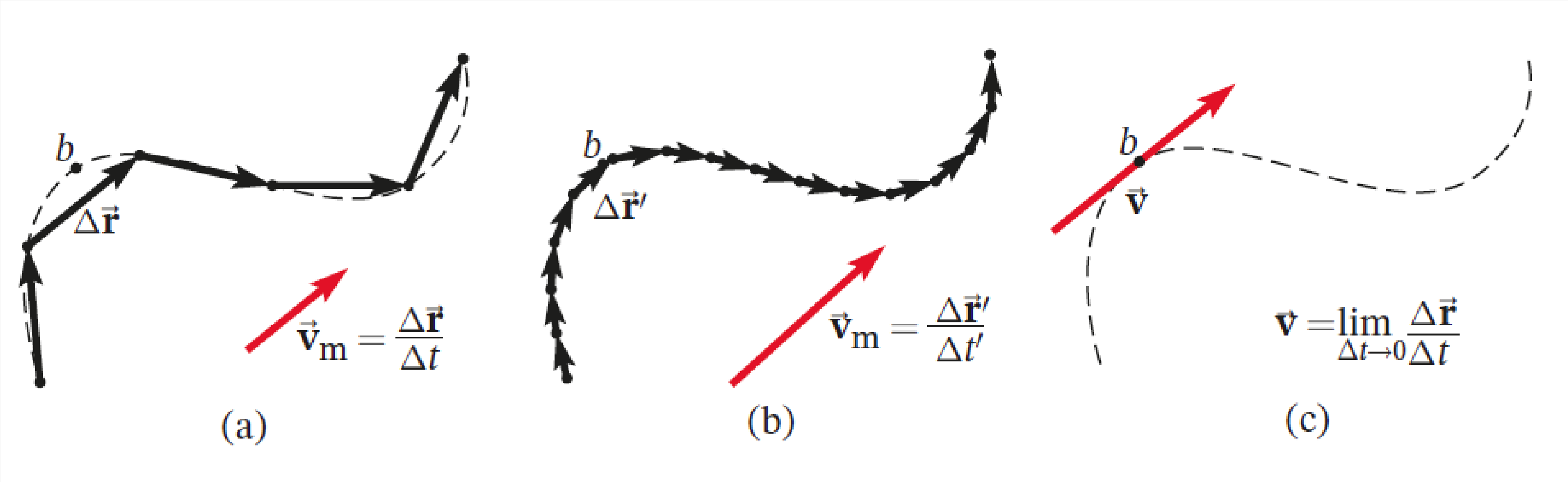
$$\vec{v}_m = \frac{\Delta x\boldsymbol{i} + \Delta y\boldsymbol{j} + \Delta z\boldsymbol{k}}{\Delta \mathrm{t}} = \frac{\Delta x}{\Delta t}\boldsymbol{i} + \frac{\Delta y}{\Delta t}\boldsymbol{j} + \frac{\Delta z}{\Delta t}\boldsymbol{k}$$

$$v_{m,x} = \frac{\Delta x}{\Delta t}\ , \qquad v_{m,y} = \frac{\Delta y}{\Delta t}$$

# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

## Instantaneous Velocity Vector



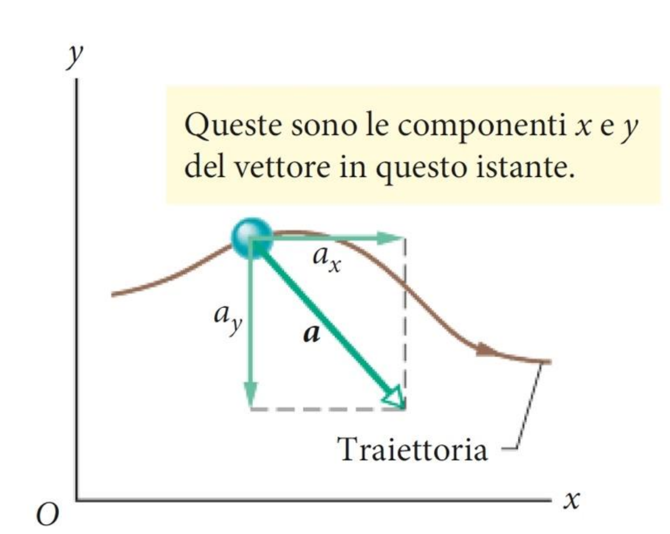

In Scalar Form:

$$\vec{v} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$v_x = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}\ , \qquad v_y = \lim_{\Delta t\to 0}\frac{\Delta y}{\Delta t}$$

Same direction and same sense as the displacement at that instant

# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

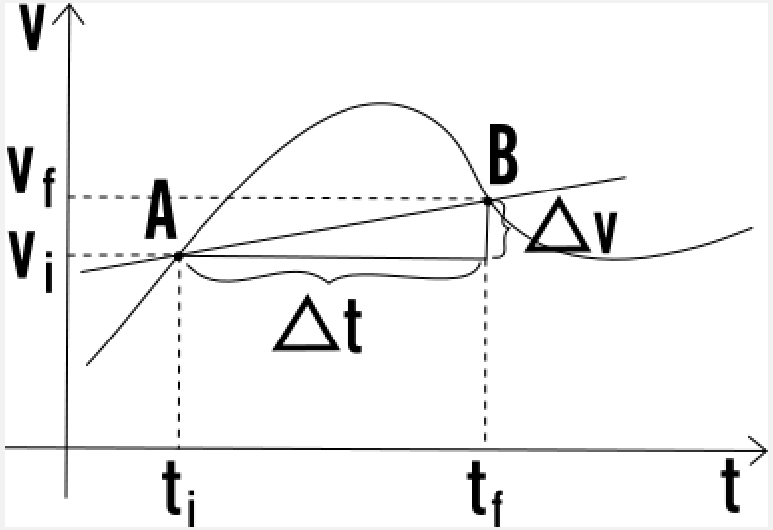



## Average Acceleration Vector

$$\vec{a}_m = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t}$$

$$a_{m,x} = \frac{\Delta v_x}{\Delta t}, \quad a_{m,y} = \frac{\Delta v_y}{\Delta t}$$

Same direction and same sense as $\Delta\vec{\boldsymbol{v}}$ $[Lt^{-1}]/[t] = [Lt^{-2}]$

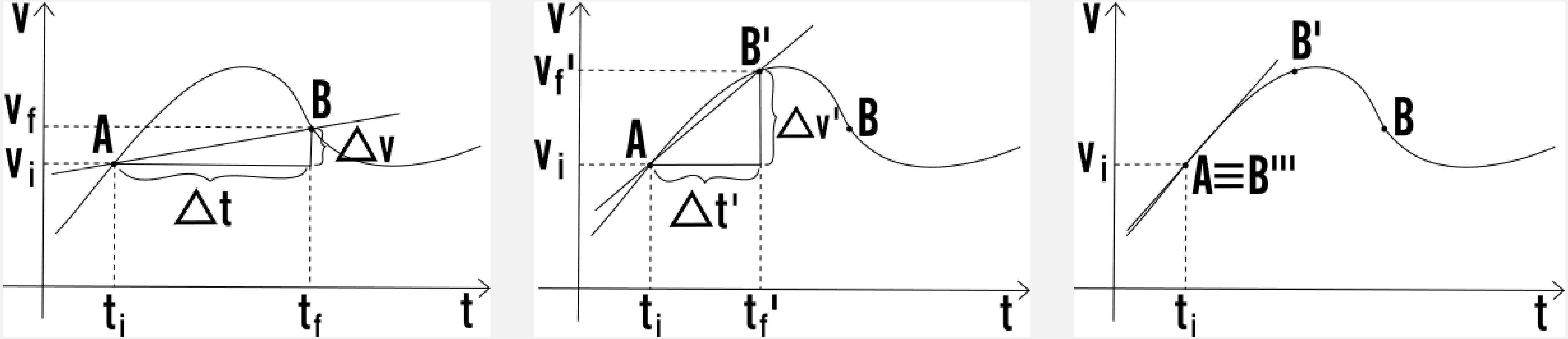


# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

## Instantaneous Acceleration Vector

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

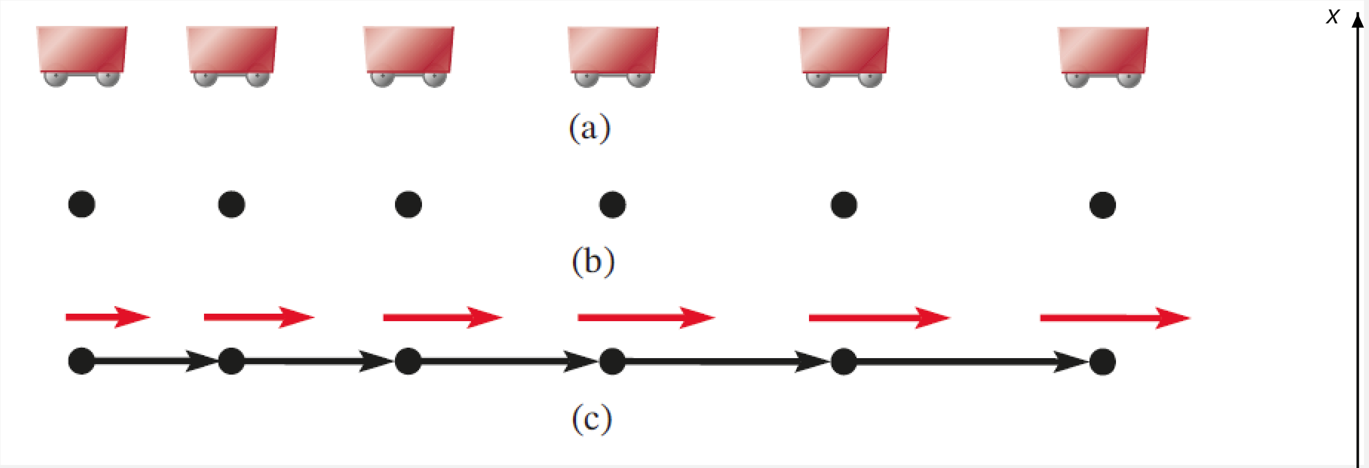
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad , \qquad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

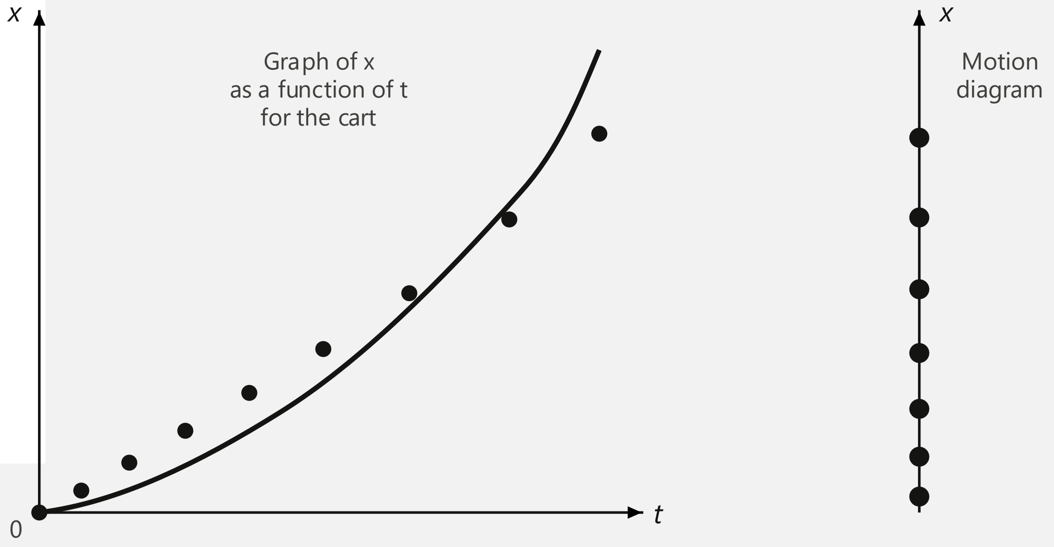


# POSITION, DISPLACEMENT, VELOCITY AND ACCELERATION VECTORS

## Motion Diagram

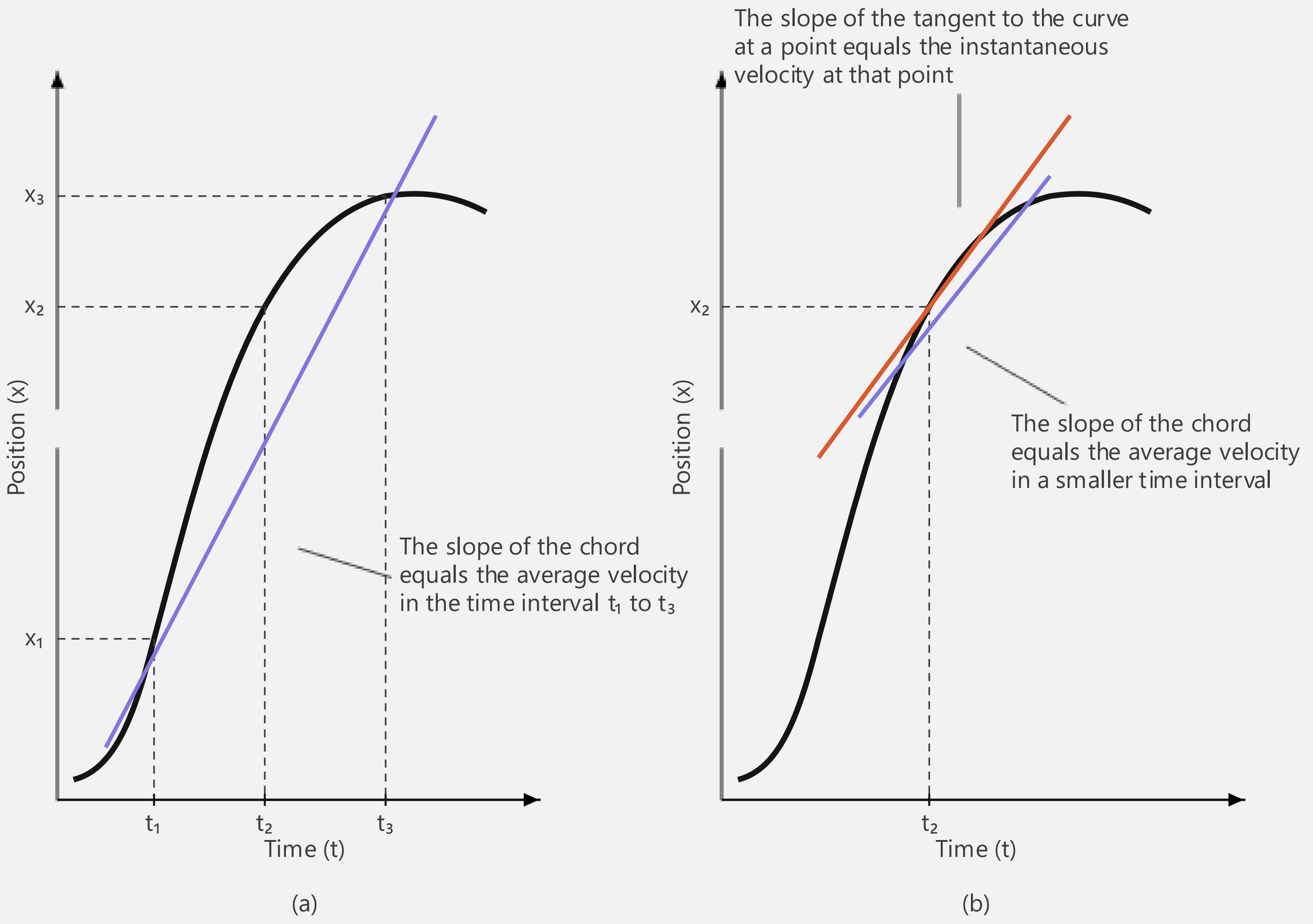
A motion diagram shows an object's position as a function of time



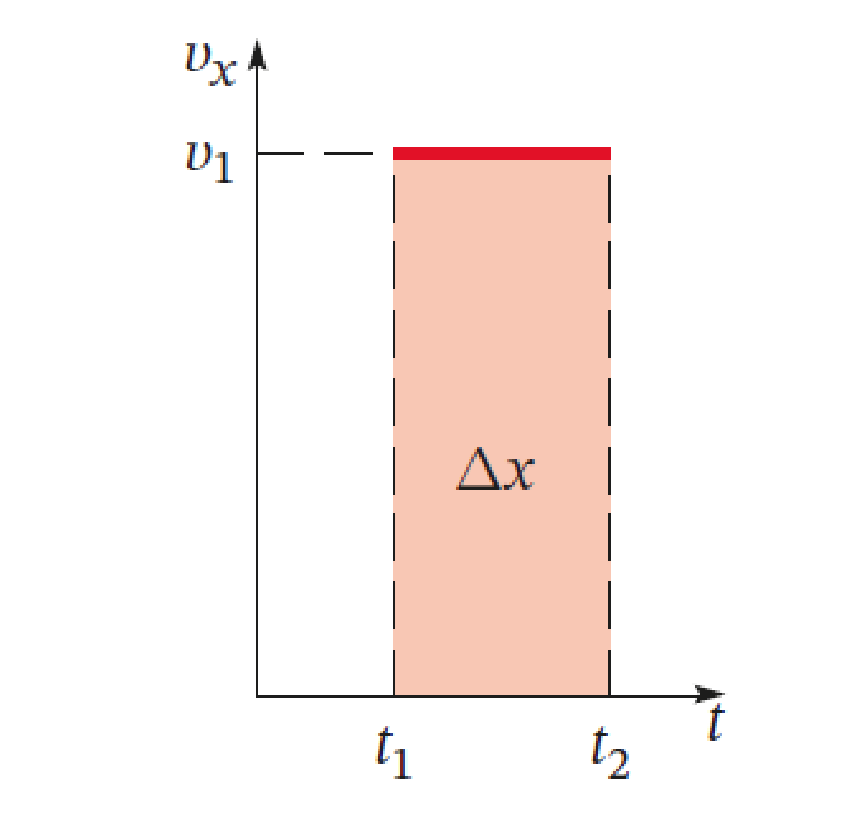
# POSITION, DISPLACEMENT, VELOCITY AND ACCELERATION VECTORS

## Graphical Relationship Between Position and Velocity



# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

## Graphical Relationship Between Position, Time, and Velocity

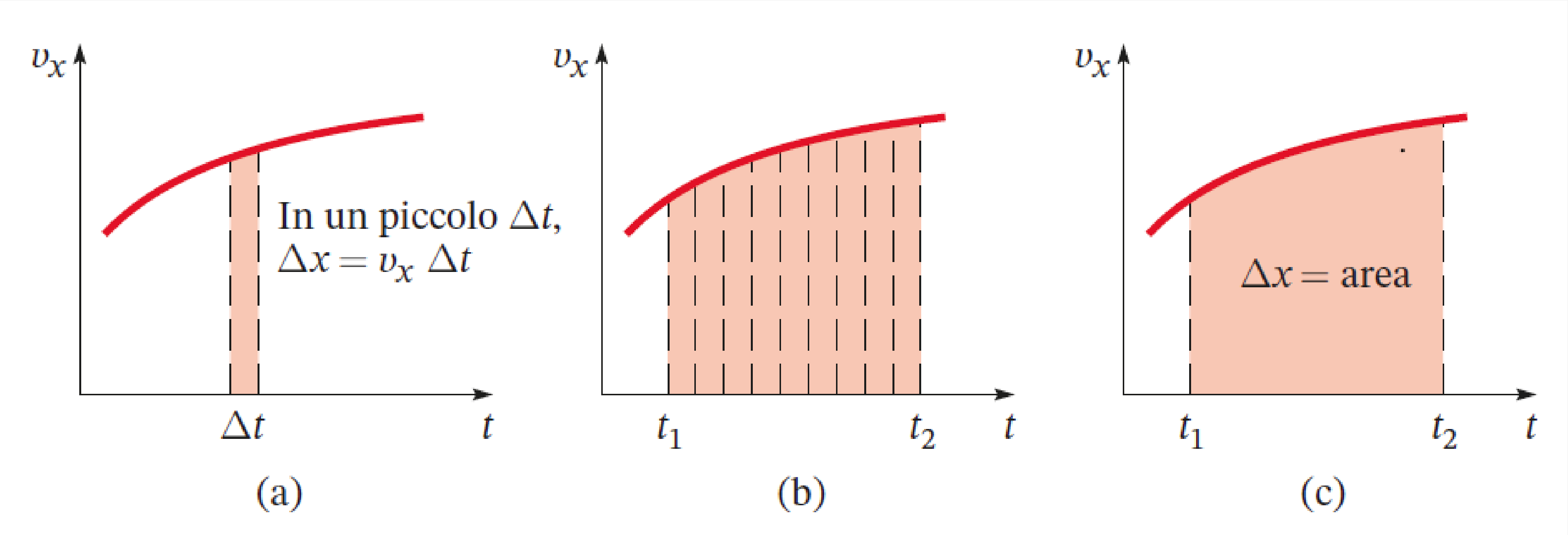



$$v_x = v_{m,x} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x \Delta t \text{ (for constant } v_x\text{)}$$

# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

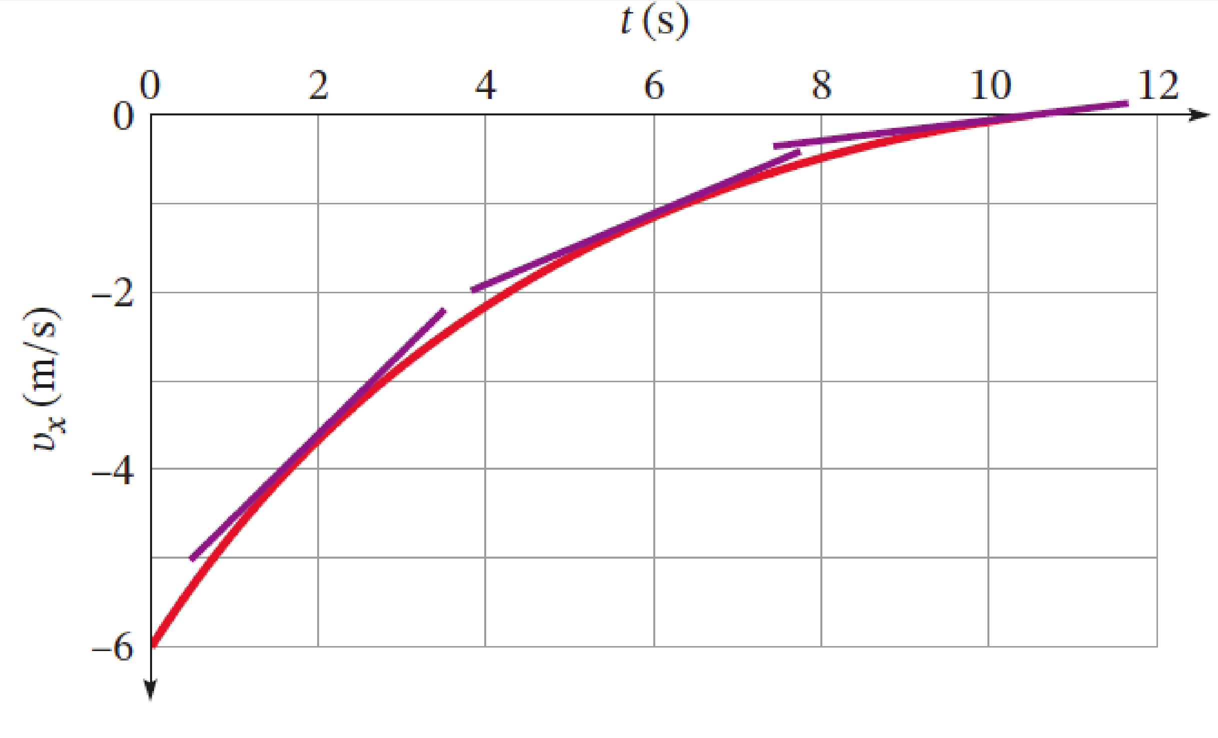
## Graphical Relationship Between Position, Time and Velocity




for non − constant $v_x$

# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

## Graphical Relationship Between Velocity and Acceleration




Same graphical relationship as for displacement and velocity:

- $a_x$ is the slope of the tangent at a given point on the curve $v_x(t)$
- $\Delta v_x$ is the area under the curve $a_x(t)$ over a given time interval

# POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

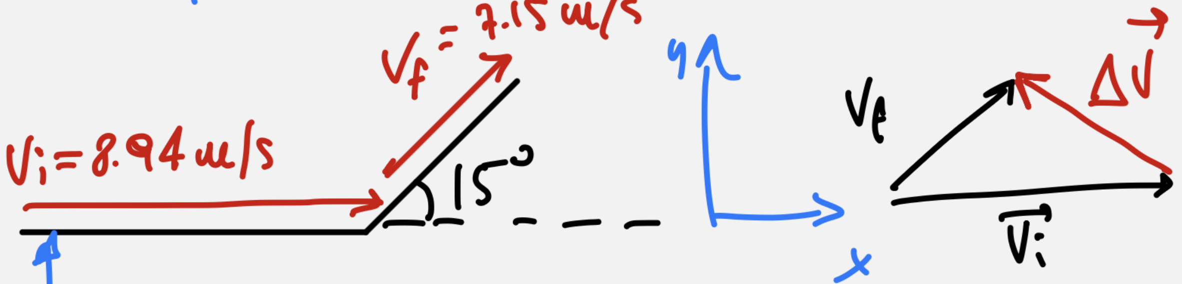
## Example

A skater is moving along a level road with a speed of 8.94 m/s; after 120.0 s the road begins to incline (at an angle of 15°) and the skater maintains a speed of 7.15 m/s.

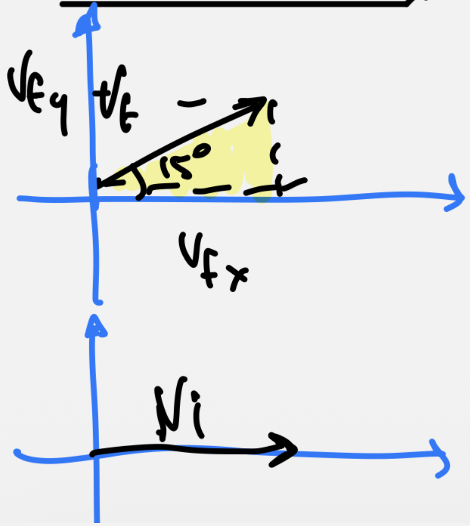
(a) What is the change in the skater's velocity?

(b) What is the skater's average acceleration over the 120 s time interval?

$8.94\ m/s - 7.15\ m/s = 1.79\ m/s$ → variaz. modulo




$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$




$$v_{fx} = v_f \cos\theta = 7.15\ m/s \cdot 0.97 = 6.91\ m/s$$

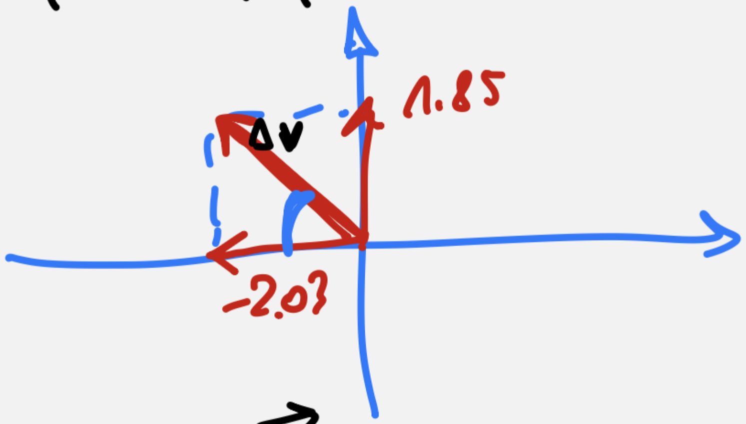
$$v_{fy} = v_f \cdot \sin\theta = 7.15\ m/s \cdot 0.26 = 1.85\ m/s$$

$$v_{ix} = v_x = 8.94\ m/s$$

$$v_{iy} = 0$$

$v_x = v_{fx} - v_{ix} = (6.91 - 8.94)\,m/s = -2.03\,m/s$

$v_y = v_{fy} - v_{iy} = (1.85 - 0)\,m/s = +1.85\,m/s$




$$\Delta v = \sqrt{(v_x)^2 + (v_y)^2} \longrightarrow 2.74\,m/s$$

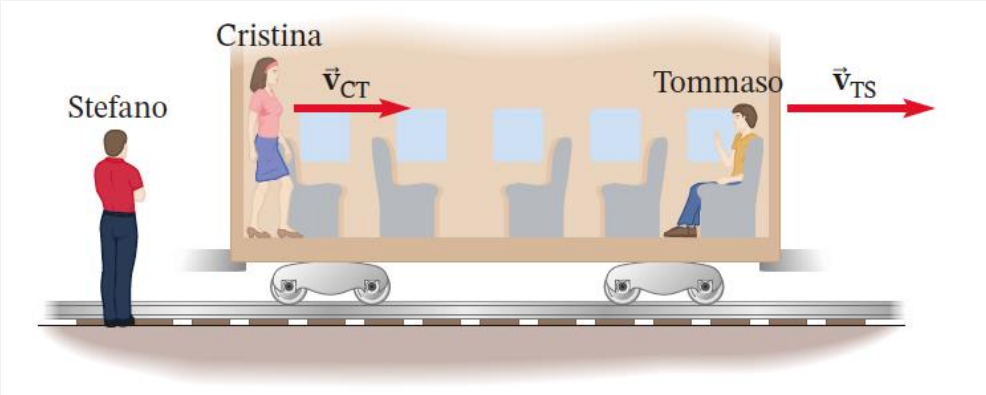
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{1.85\,m/s}{2.03\,m/s} = 42.3^\circ$$

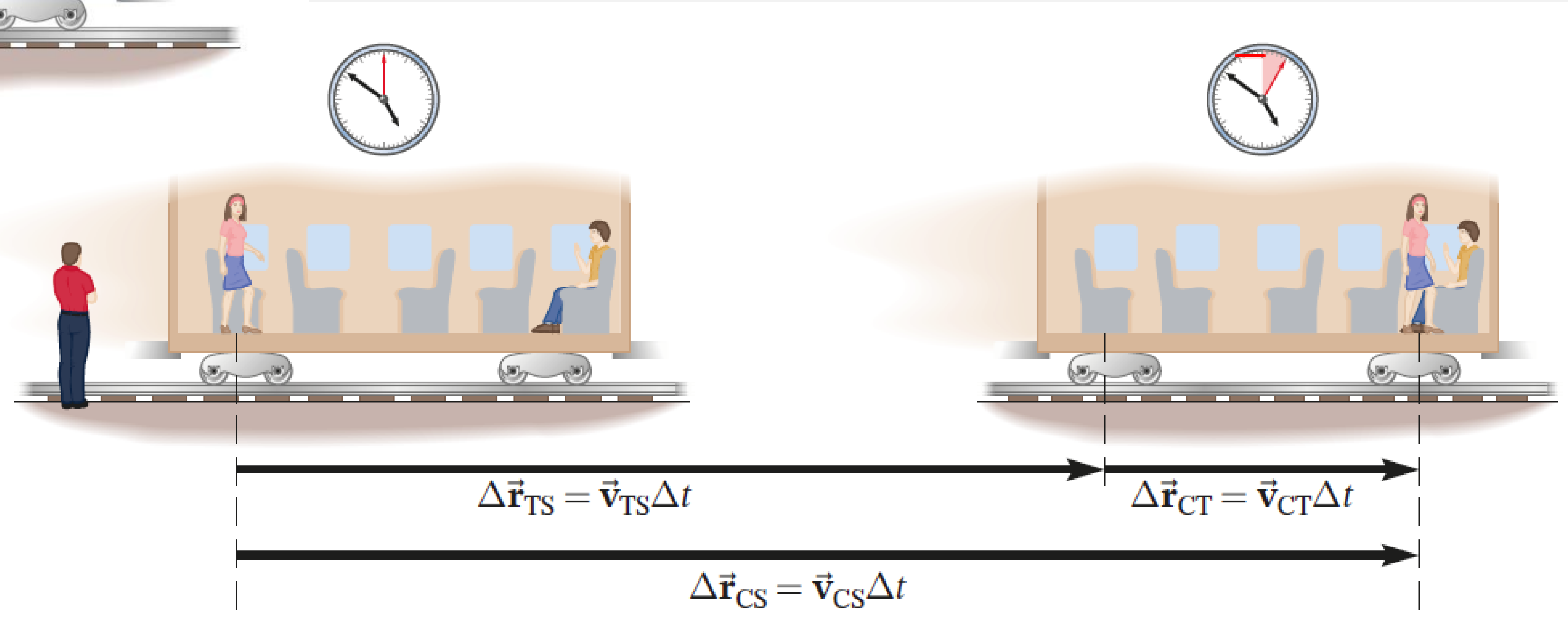
$$\vec{a}_m = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{2.74\,m/s}{120\,s} = 0.023\,m/s^2$$

direzione di $\vec{a}_m$ = direzione di $\Delta\vec{v}$ → $42.3^\circ$ rispetto all'asse orizzont.

# POSITION AND DISPLACEMENT VECTORS, VELOCITY AND ACCELERATION

## Relative Velocity and Reference Frames




$$\Delta\vec{r}_{CS} = \Delta\vec{r}_{CT} + \Delta\vec{r}_{TS}$$

$$\vec{v}_{CS} = \vec{v}_{CT} + \vec{v}_{TS}$$