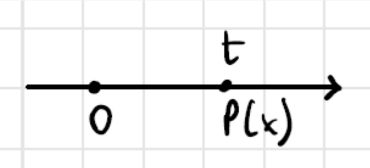
KINEMATICS pt.II

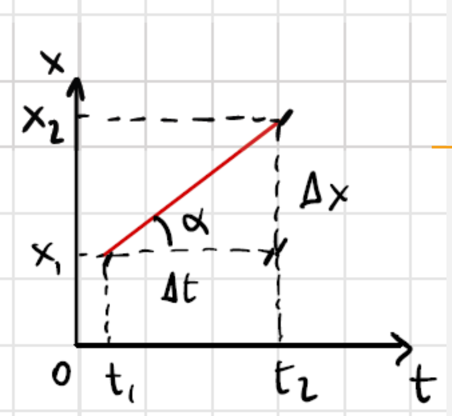
- Uniform rectilinear motion
- Uniformly accelerated rectilinear motion
- Falling objects
- Projectile motion
- Uniform circular motion

UNIFORM RECTILINEAR MOTION

Motion of a particle with constant velocity: $\vec{v} = cost$

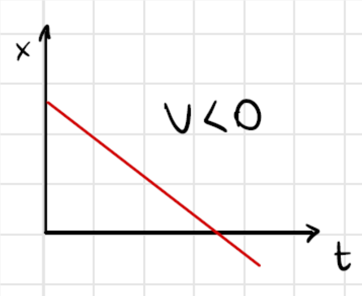
At any point, the acceleration is zero: $\vec{a} = 0$





$$v = \frac{\Delta x}{\Delta t} = \tan \alpha > 0$$

The slope of the straight line represents the velocity of the particle



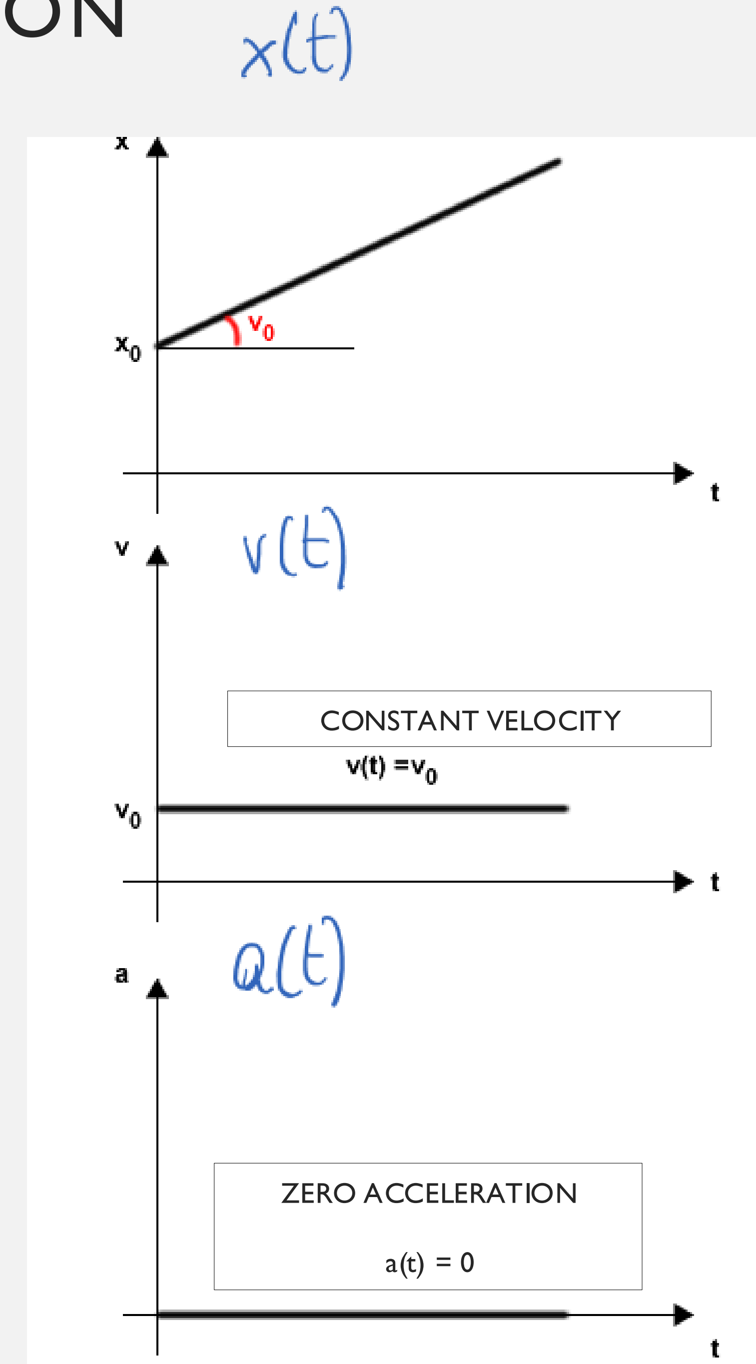
Let us consider t_1 the initial instant ($t_1 = 0$) and t_2 a generic instant ($t_2 = t$), x_0 the abscissa of the initial position and x_2 a generic position ($x_2 = x$)

$x - x_0 = vt \rightarrow \Delta x = v\Delta t$

Position equation of uniform rectilinear motion

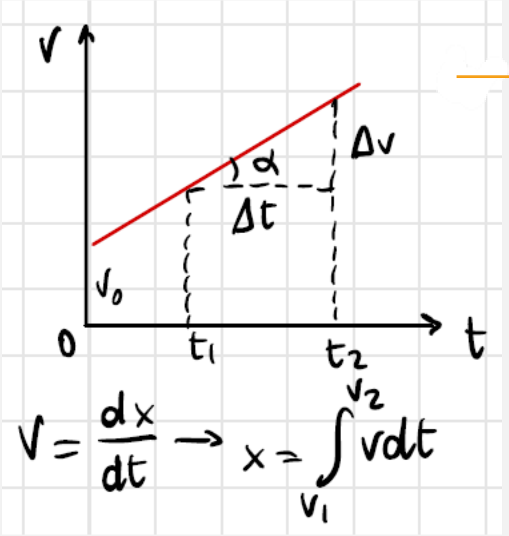
$$\boldsymbol{x = x_0 + vt}$$

$$\boldsymbol{\Delta x = v\Delta t}$$



UNIFORMLY ACCELERATED RECTILINEAR MOTION

Motion of a particle with constant acceleration: $\vec{a} = cost$



$a = \tan\alpha > 0$ →accelerated motion

The slope of the straight line represents the particle's velocity

First fundamental equation:

$$a_m = a = \frac{v - v_0}{t - 0}$$

↳ $t_0 = 0$

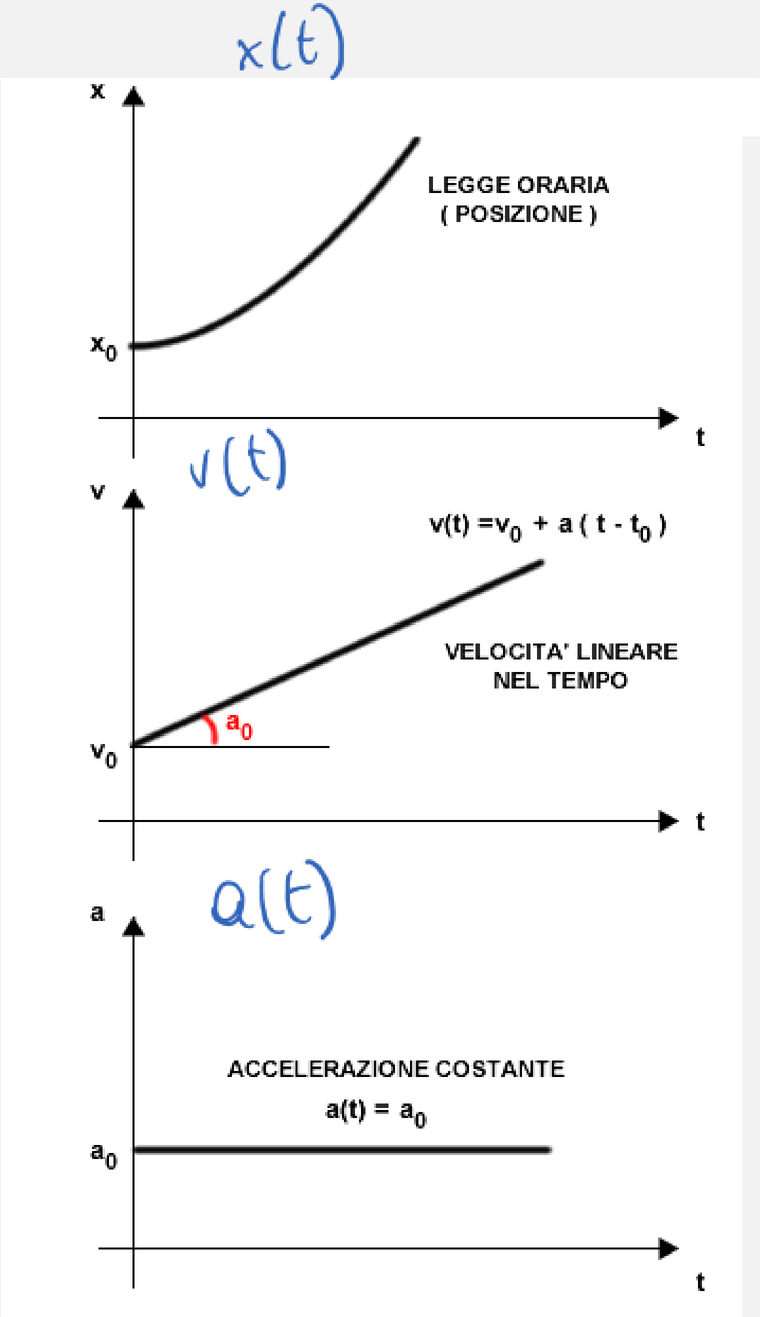
First kinematics equation

$$v = v_0 + at$$

Second fundamental equation:

$$v_m = \frac{x - x_0}{t - 0}$$

$$x = x_0 + v_m t$$



UNIFORMLY ACCELERATED RECTILINEAR MOTION

$V_2 - V_1 = a(t_2 - t_1) \rightarrow V_2 = V_1 + at$

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \rightarrow \Delta v = a\Delta t$$

$$\Delta v = v_2 - v_1 \rightarrow v_2 = v_1 + \Delta v = v_1 + a\Delta t$$

$$\Delta v = v - v_0 \rightarrow v = v_0 + \Delta v = v_0 + a\Delta t$$

A body moving at velocity v_1 at time t_1 and is accelerated with constant a, it will reach velocity $v_2 = v_1 + a(t_2 - t_1)$ at time t_2

$$v_m = \frac{v_0 + v}{2}$$

$$\Delta x = v_m \Delta t = \frac{1}{2}(v_0 + v)(t - t_0) = \frac{1}{2}(v_0 + v)\Delta t$$

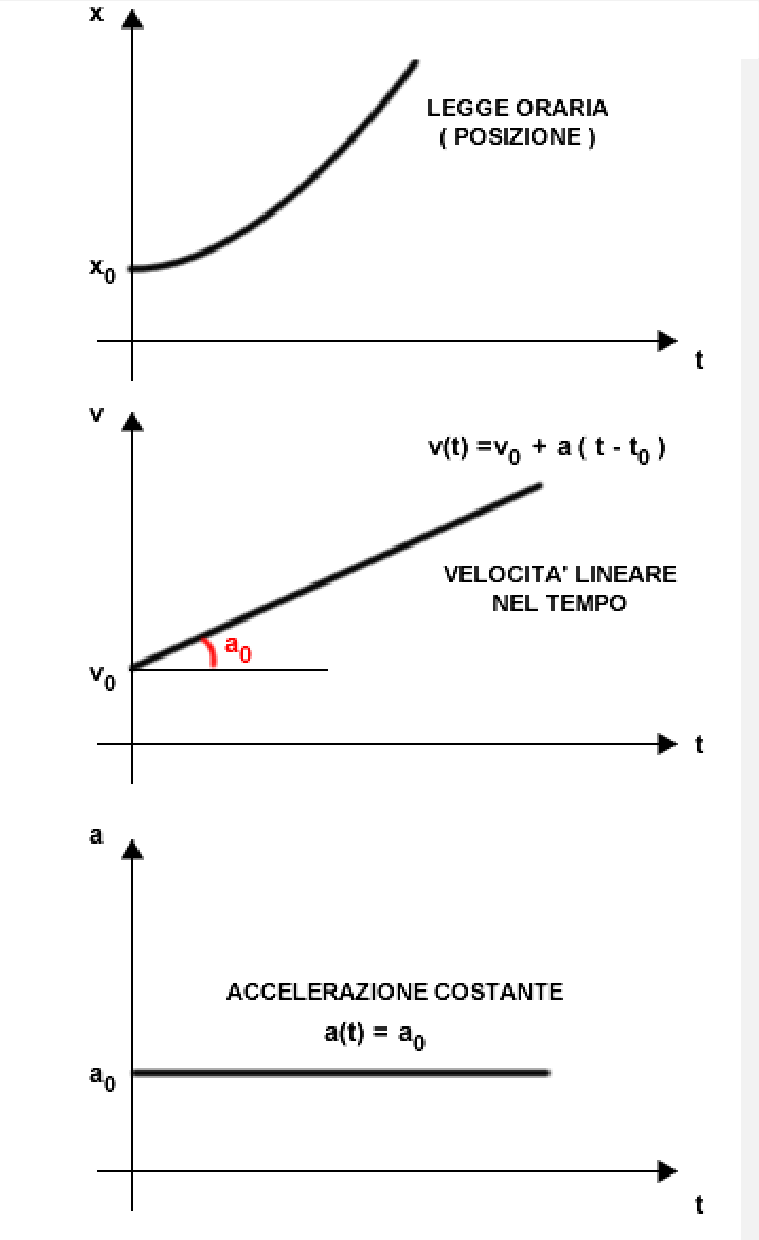
$$v = v_0 + a\Delta t \rightarrow \Delta x = \frac{1}{2}(v_0 + v_0 + a\Delta t)\Delta t = \frac{1}{2}(2v_0\Delta t) + \frac{1}{2}a\Delta t^2$$

Second equation of kinematics

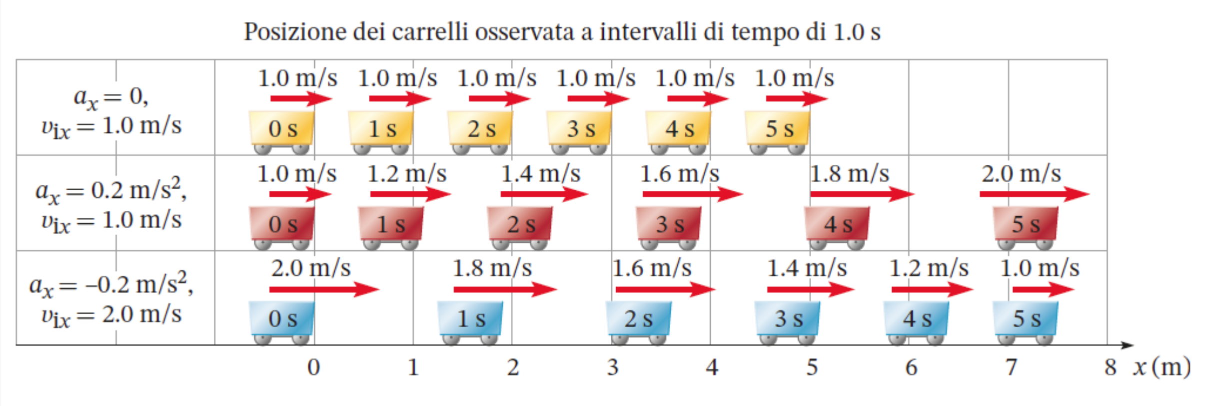
$$\Delta x = v_0\Delta t + \frac{1}{2}a\Delta t^2$$

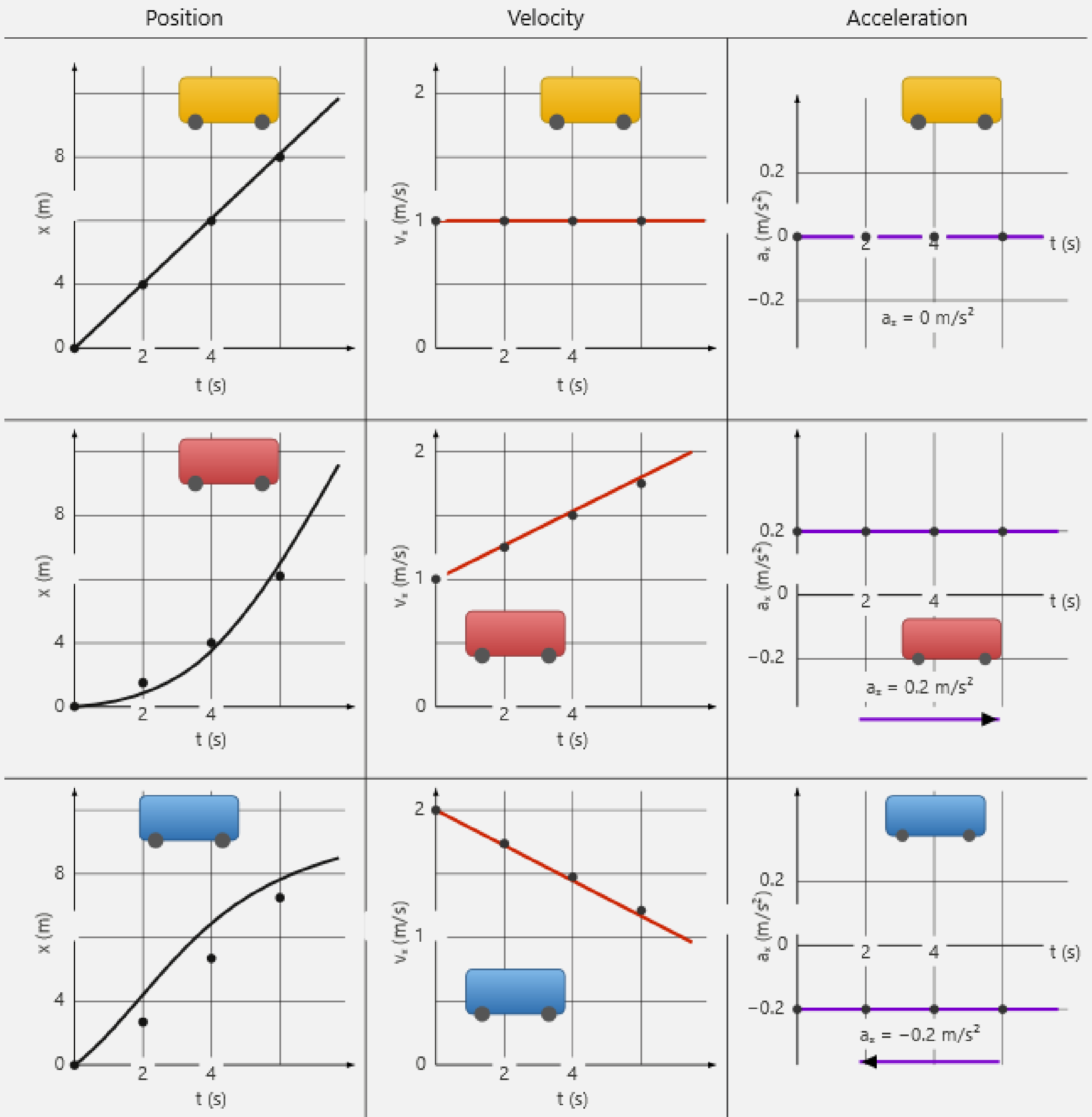
$x - x_0$

$$x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$$



UNIFORMLY ACCELERATED RECTILINEAR MOTION





UNIFORMLY ACCELERATED RECTILINEAR MOTION

Important mathematical relations

$$\Delta v = v_f - v_i = a\Delta t$$

$$\left(\frac{v_f + v_i}{2}\right)\Delta t$$

Fourth kinematics equation

$$\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$$

$$x_f - x_i$$

$$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$x = x_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

Third kinematics equation

$$v_f^2 - v_i^2 = 2a\Delta x$$

SPECIAL MOTIONS

Example

We are designing an airport. A certain airplane must reach a speed of at least 27.8 m/s before takeoff and can accelerate up to 2m/s^2. If the runway is 150 m long, can this airplane reach the speed required for takeoff? If not, what is the minimum length the runway should have?

$x_i = 0\,m$

$v_i = 0\,m/s$

$a = 2\,m/s^2$

$x_f = 150\,m$

v_f?

$$v_f^2 - v_i^2 = 2a\Delta x = 2a(x_f - x_i)$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$= (0\,m/s)^2 + 2 \cdot 2\,m/s^2 (150\,m - 0\,m)$$

$$= 4\,m/s^2 (150\,m) = 600\,m^2/s^2$$

$\hookrightarrow v_f = 24.5\,m/s$ → no x il decollo!

$x_i = 0\,m$

$v_i = 0\,m/s$

$a = 2\,m/s^2$

$v_f = 27.8\,m/s$

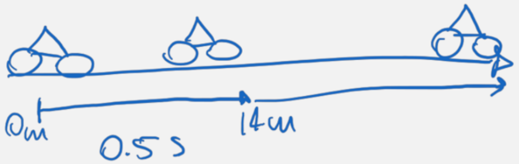
x_f?

$$v_f^2 - v_i^2 = 2a(x_f - x_i)$$

$x_f = 193\,m$ → nuova pista

Example

Braking distance. Calculate the total stopping distance for an initial speed of 28 m/s and assuming that the acceleration of an automobile is a=-6.0 m/s^2. Assume that the reaction time is 0.50 s.



$v = cost = 28\,m/s$ $\quad a = cost = -6\,m/s^2$

$a = 0\,m/s^2$ $\quad v_i = 28\,m/s$

$t = 0.5\,s$ $\quad v_f = 0\,m/s$

(1) $t = 0.5\,s$

$v_i = 28\,m/s$

$v_f = 28\,m/s$

$a = 0\,m/s^2$

$x_i = 0\,m$

$\rightarrow \Delta x = v\Delta t$

$x_f - x_i = v(t_f - t_i)$

$x_f = v \cdot t_f$

$x_f = 28\,m/s \cdot 0.5\,s = 14\,m$

(2) x_f?

$x_i = 14\,m$

$v_i = 28\,m/s$

$v_f = 0\,m/s$

$a = -6\,m/s^2$

$\rightarrow v_f^2 - v_i^2 = 2a\Delta x$

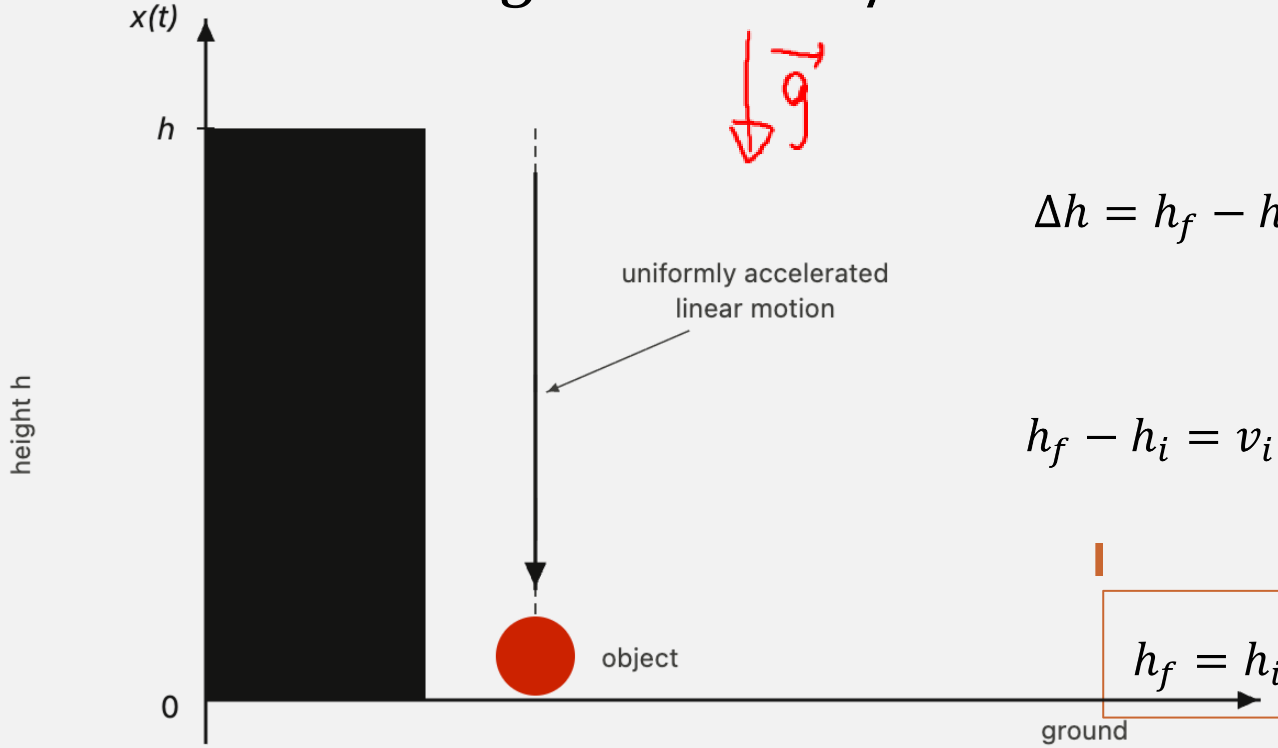
$v_f^2 - v_i^2 = 2a(x_f - x_i) \rightarrow x_f - x_i = \frac{v_f^2 - v_i^2}{2a} \rightarrow x_f = x_i + \frac{v_f^2 - v_i^2}{2a}$

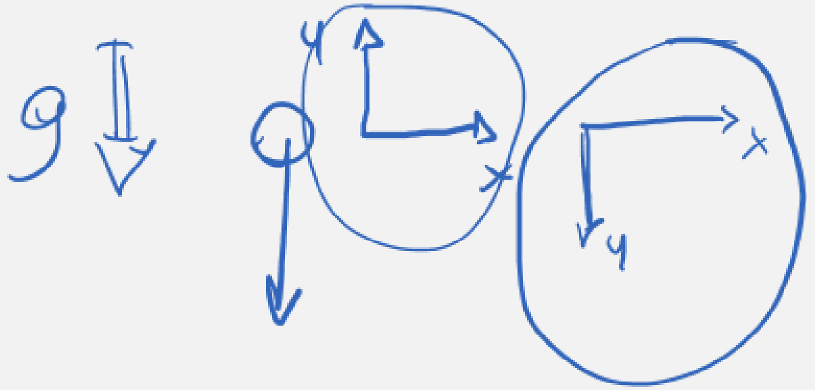
$= 14\,m + \frac{(0\,m/s)^2 - (28\,m/s)^2}{2(-6\,m/s^2)}$

$\leftarrow$ **79 m**

FALLING OBJECTS

$$\vec{g} = 9.80\, m/s^2$$





$$\Delta h = h_f - h_i = v_i \Delta t + \frac{1}{2} g \Delta t^2$$

$$h_f - h_i = v_i(t_f - t_i) + \frac{1}{2} g (t_f - t_i)^2$$

1

$$h_f = h_i + v_i t_f + \frac{1}{2} g t_f^2$$

$$v_f^2 - v_i^2 = 2g\Delta h \rightarrow v_f^2 = v_i^2 + 2g(h_f - h_i) \rightarrow$$

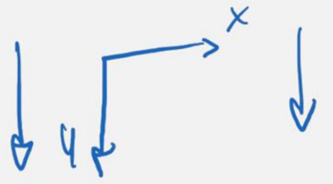
2

$$v_f = \sqrt{v_i^2 + 2g(h_f - h_i)}$$

FALLING OBJECTS

Example

Suppose that a ball is dropped from a tower 70 m high. How far will it have fallen after 1 s, 2 s, and 3 s? Assume h is positive downward and neglect air resistance.

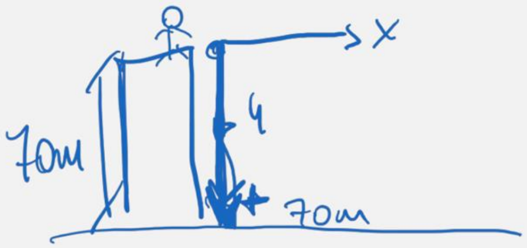


$h_i = 0\,m$

$h_f = 70\,m$

$v_i = 0\,m/s$

$g = 9.8\,m/s^2$



$t_1 = 1\,s$

$t_2 = 2\,s$

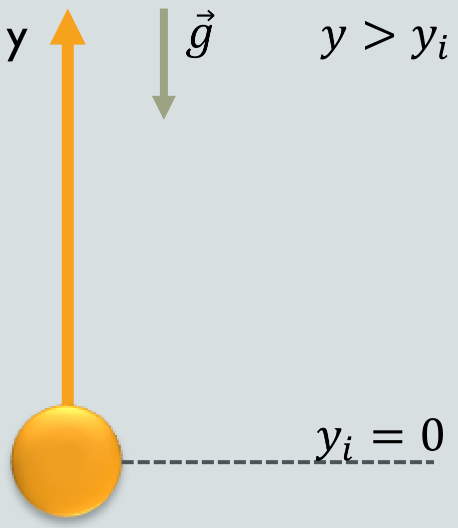
$t_3 = 3\,s$

(1) $h_f = \cancel{h_i} + \cancel{v_i t} + \frac{1}{2} g t^2 = \frac{1}{2} \cdot 9.8\,m/s^2 \cdot 1\,s^2 = 4.9\,m$

(2) $h_f = \frac{1}{2} \cdot 9.8\,m/s^2 \cdot (2s)^2 = 19.6\,m$

(3) $h_f = \frac{1}{2} \cdot 9.8\,m/s^2 \cdot (3s)^2 = 44.1\,m$

OBJECTS THROWN UPWARD



$$\begin{cases} y = y_i + v_i t + \frac{1}{2} a t^2 \\ v = v_i + at \end{cases}$$

ASCENT

$y_i = 0$

$t_i = 0 \rightarrow t_u - t_i = t_u$

$v_f = v_{y\,max} = 0$

$$\begin{cases} y = v_i t - \frac{1}{2} g t^2 \\ v = v_i - gt \end{cases}$$

$$\begin{cases} y_{max} = v_i t_u - \frac{1}{2} g t_u^2 \\ 0 = v_i - g t_u \end{cases}$$

→ Having reached a height y_{max} at time t_s we have $v=0$.

$$\begin{cases} y_{max} = g t_u^2 - \frac{1}{2} g t_u^2 \\ v_i = g t_u \end{cases}$$

$$\begin{cases} y_{max} = \frac{1}{2} g t_u^2 \\ v_i = g t_u \end{cases}$$

$$t_u = \sqrt{\frac{2 y_{max}}{g}}$$

DESCENT

$y_i = y_{max}$

$v_i = v_{y\,max} = 0$

$y_f = 0$

$t_i = 0 \rightarrow t_d - t_i = t_d$

$$y_f = y_i + v_i t_d - \frac{1}{2} g t_d^2$$

$$0 = y_{max} - \frac{1}{2} g t_d^2$$

$$t_d = \sqrt{\frac{2 y_{max}}{g}}$$

FALLING OBJECTS

Example

A boy throws a ball from a terrace 4 m high with a speed of 10 m/s upward. Determine:

a. The maximum height reached by the ball
b. The time the ball takes to reach the maximum height
c. The time the ball takes to reach the ground
d. The speed with which the ball reaches the ground

$v_i = 10\,m/s$

$v_{y max} = 10\,m/s$

$a = -g = -9.8\,m/s^2$

$\rightarrow v_f^2 - v_i^2 = 2a\Delta x$

$v_{y max}^2 - v_i^2 = -2g(y_{max} - y_i)$

$(0\,m/s)^2 - (10\,m/s)^2 = -2 \cdot 9.8\,m/s^2 (y_{max} - 4\,m)$

$y_{max} = 9.1\,m$ (dal suolo)

$t_s?$ $\rightarrow$ $v_i = 10\,m/s$

$y_i = 4\,m$

$y_{max} = 9.1\,m$

$a = -9.8\,m/s^2$

$\rightarrow$ opzione 1: $v_{y max} = v_i + at$

$0\,m/s = 10\,m/s - 9.8\,m/s^2 \cdot t$

$t = 1.02\,s$

opzione 2

$t_s = \sqrt{\frac{2 y_{max}}{g}} = \sqrt{\frac{2 \cdot 5.1\,m}{9.8}} = 1.02\,s$

t_{TOT}? $\rightarrow t_D + t_S$

$t_S \rightarrow 1.02 s$

$t_D \rightarrow \sqrt{\frac{2 y_{max}}{g}} = \sqrt{\frac{2 \cdot 9.1 m}{9.8 m/s^2}} = 1.36 s \Rightarrow t_{TOT} = 1.02 s + 1.36 s = 2.38 s$

Velocità con cui cade? $\rightarrow$ posso considerare solo la discesa =

$v_f = v_i + at = v_i - gt = 0 m/s - 9.8 m/s^2 \cdot 1.36 = -13.33 m/s$ (verso il basso

$v_i \rightarrow 0 m/s$ (v_{ymax})

$t \rightarrow 1.36 s$ (t_D)

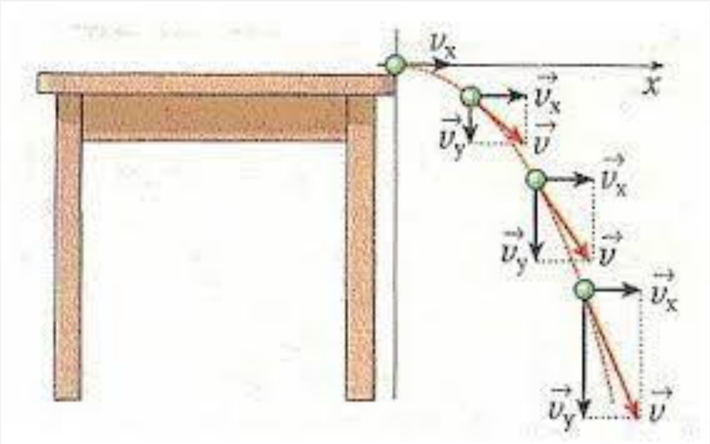
Oppure posso considerare tutto il tragitto, ma cambia v_i e cambia il t =

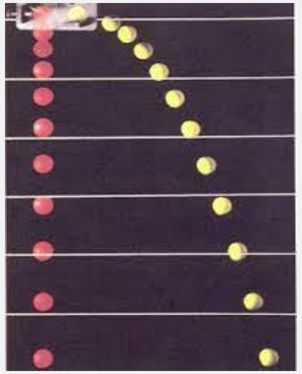
$v_f = v_i + at = v_i - gt = 10 m/s - 9.8 m/s^2 \cdot 2.38 s = -13.32 m/s$

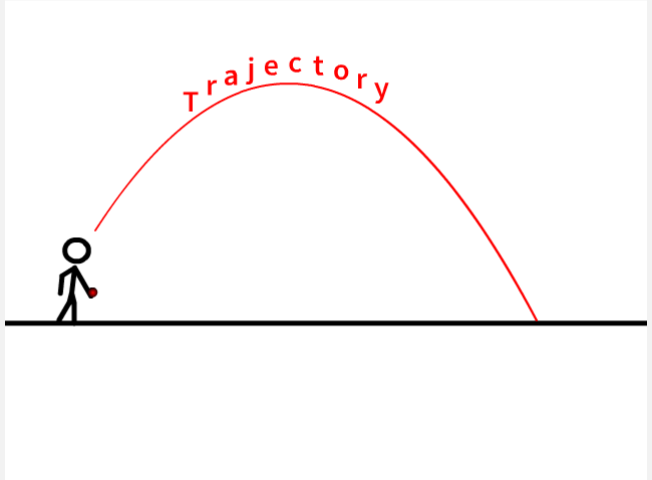
$v_i \rightarrow 10 m/s$

$t \rightarrow t_{TOT}$

PARABOLIC (OR PROJECTILE) MOTION







$$\Delta v = v_f - v_i = a\Delta t$$

$$\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$$

$$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$v_f^2 - v_i^2 = 2a\Delta x$$

AXIS x → $a_x = 0$

$$\Delta v_x = 0\ (v_x = cost)$$

$$\Delta x = v_x\Delta t$$

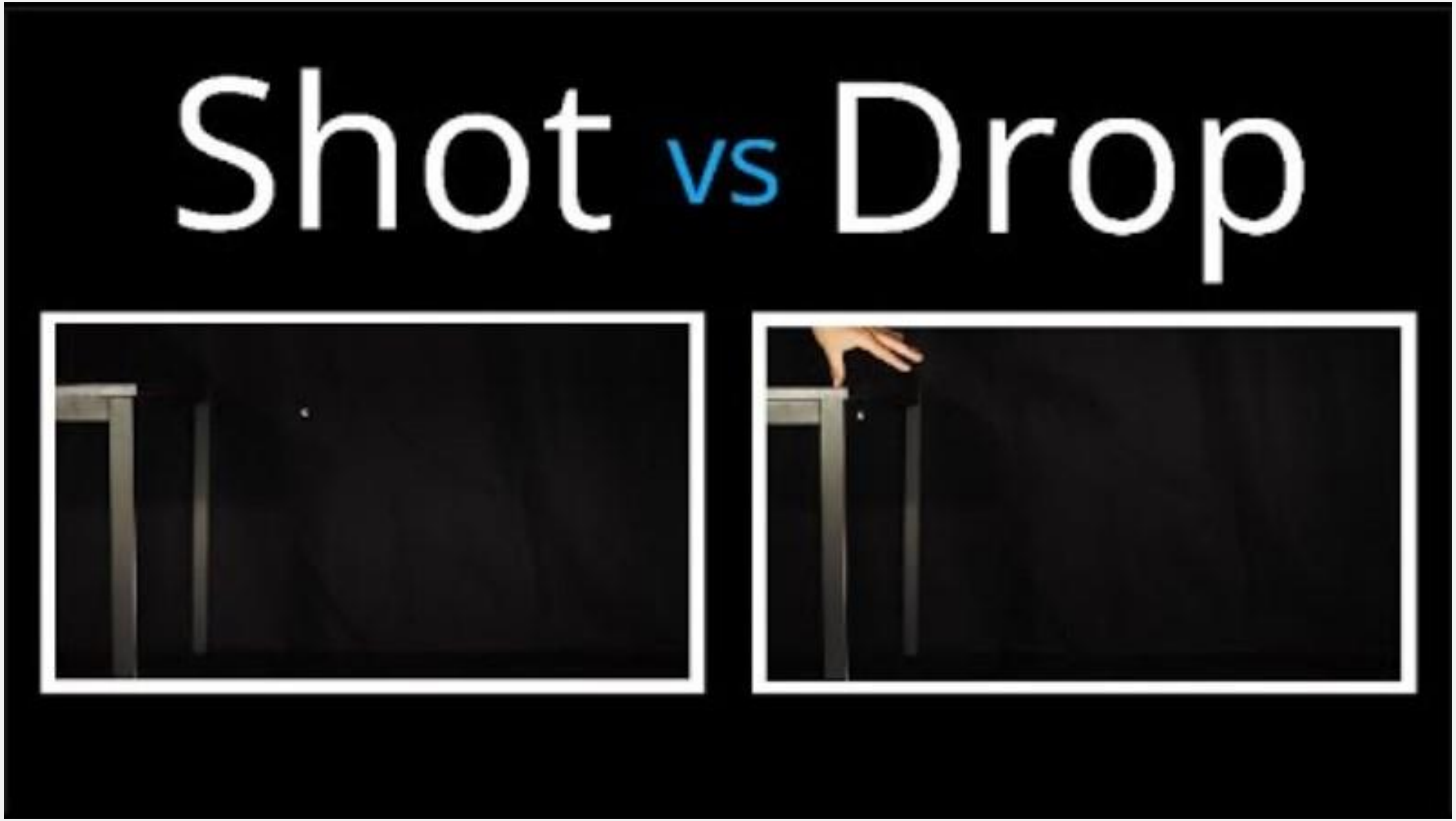
AXIS y → $a_y = cost$

$$\Delta v_y = a_y\Delta t = -g\Delta t$$

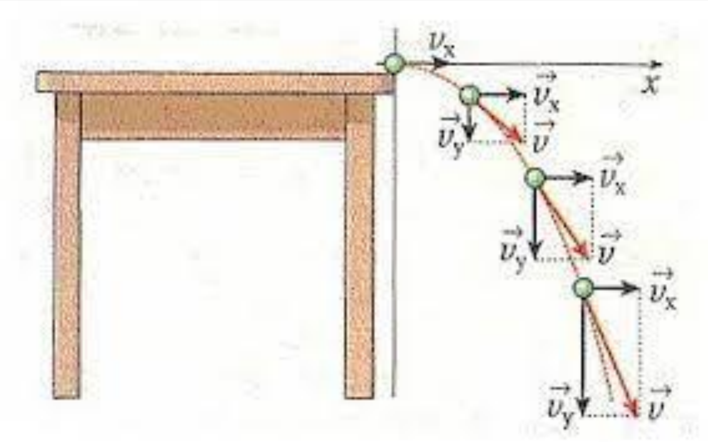
$$\Delta y = \frac{1}{2}(v_{fy} + v_{iy})\Delta t$$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

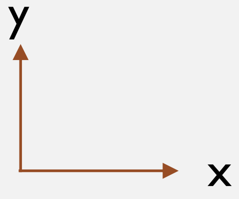
$$v_{fy}^2 - v_{iy}^2 = 2a_y\Delta y$$


Shot vs Drop

PARABOLIC MOTION



$\vec{a} = \vec{g}$

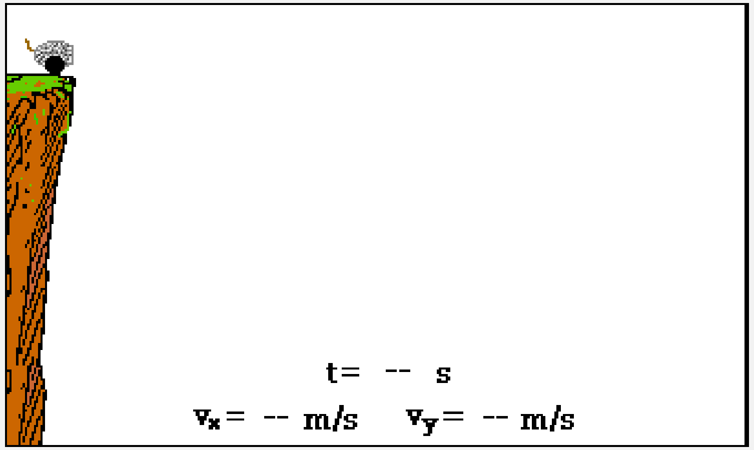


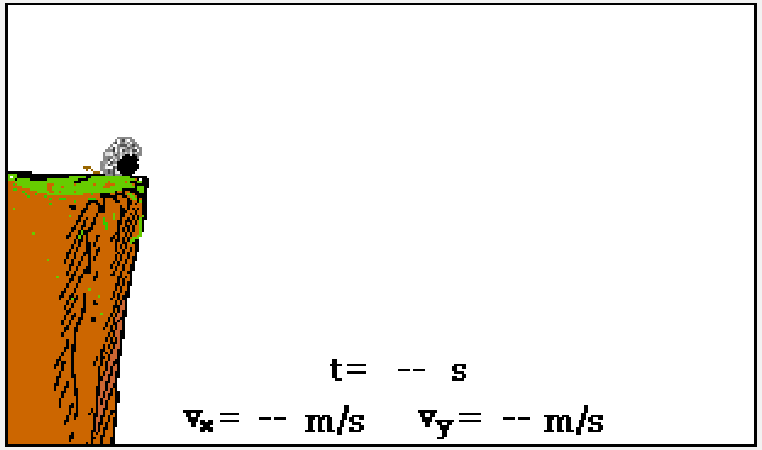
The ball leaves the table ($t = 0$): the ball is subject to gravitational acceleration:

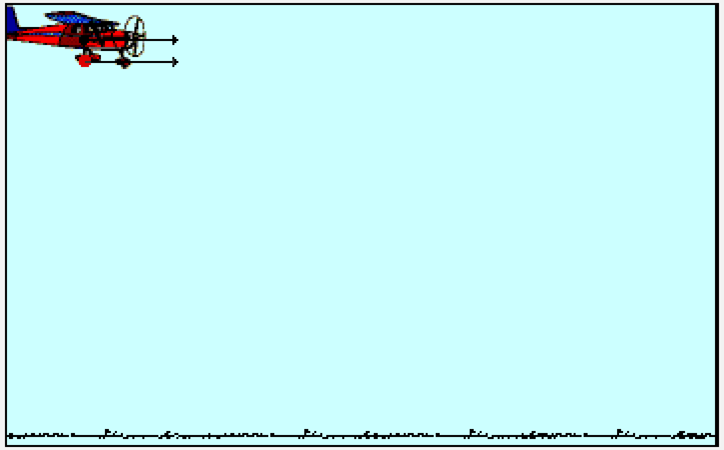
- v_y is initially 0, then increases
- $a_y = g$

$$\Delta v_y = a_y \Delta t \rightarrow v_y = -gt$$

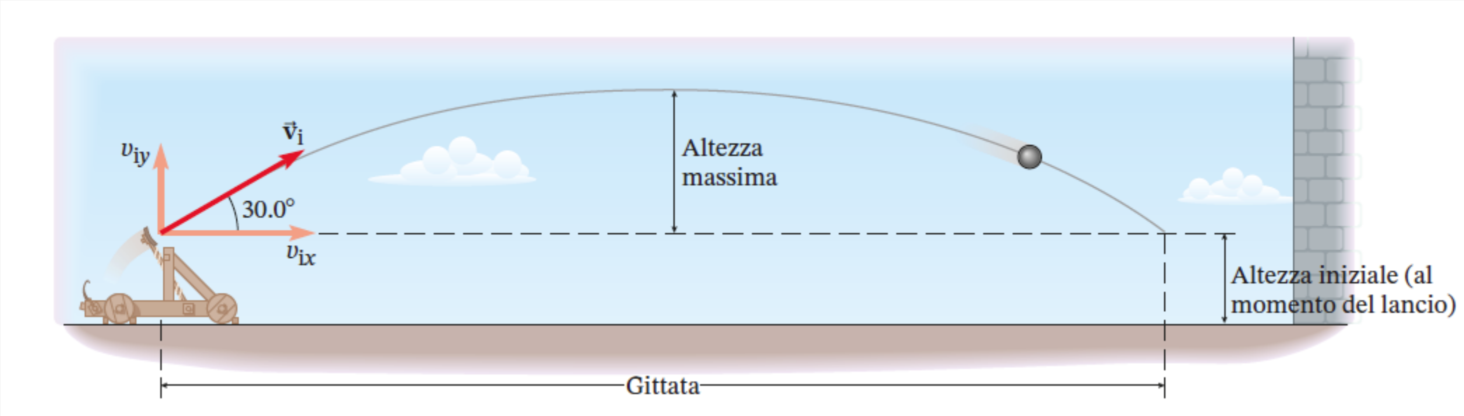
Vertical displacement (y) → $\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \rightarrow \Delta y = -\frac{1}{2}g(\Delta t)^2$ (setting y_i=0)

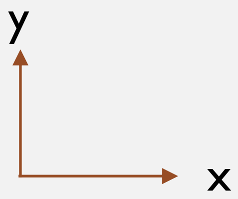






PARABOLIC MOTION

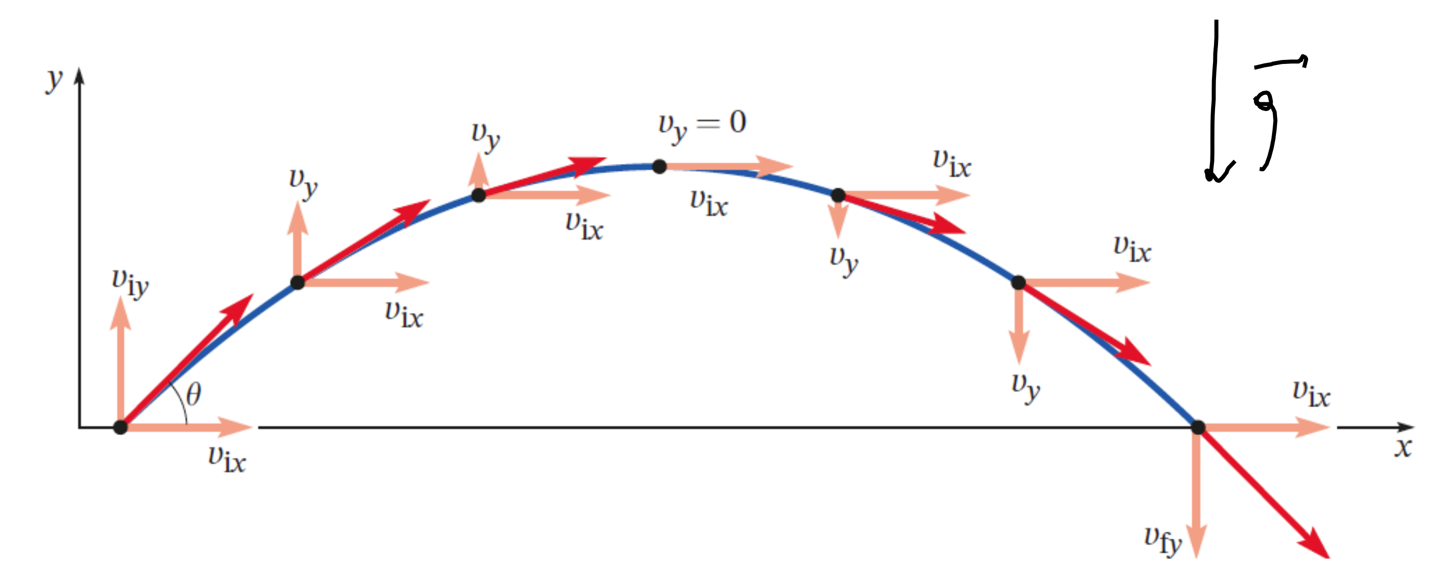




Here the velocity has a vertical component v_{iy}

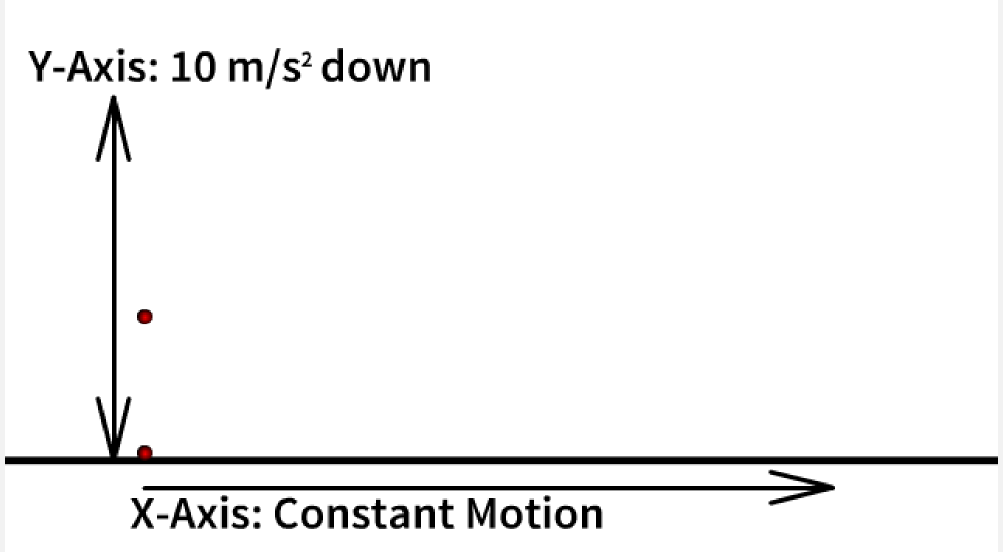
The body is launched with an initial velocity $\vec{v_i}$ → this forms an angle θ with the horizontal axis:

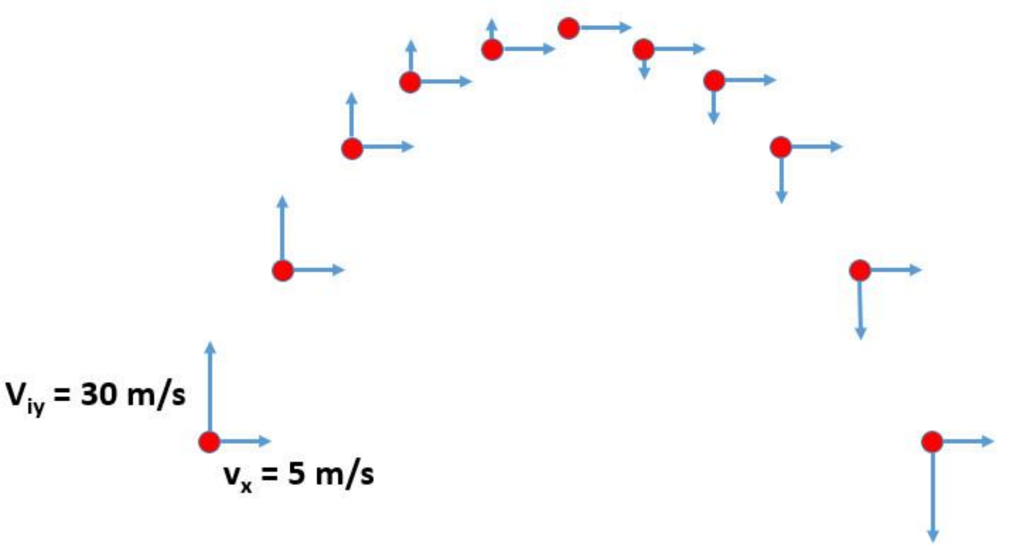
$$v_{ix} = v_i cos\theta \text{ and } v_{iy} = v_i sin\theta$$

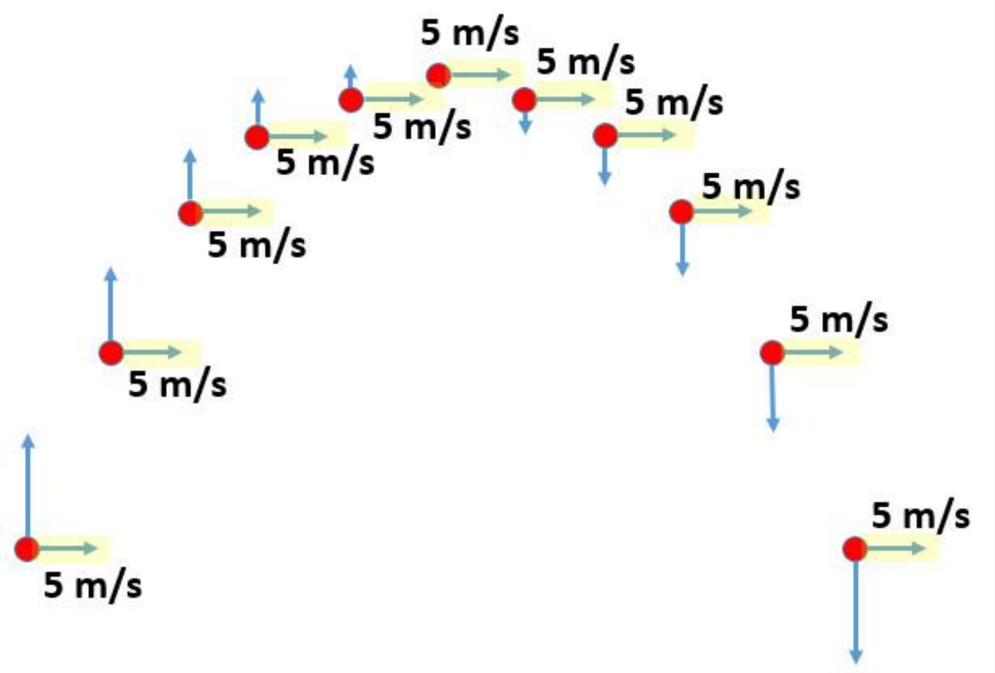


The angle θ is called the angle of elevation. During the flight, the only force acting is the weight force: a_x=0 and v_x=const

Vy is initially positive, decreases during the ascent, becomes zero at the maximum height, and finally increases in magnitude during the descent, but with a negative direction.


Y-Axis: 10 m/s² down
X-Axis: Constant Motion


V_{iy} = 30 m/s
v_x = 5 m/s


5 m/s
5 m/s
5 m/s
5 m/s
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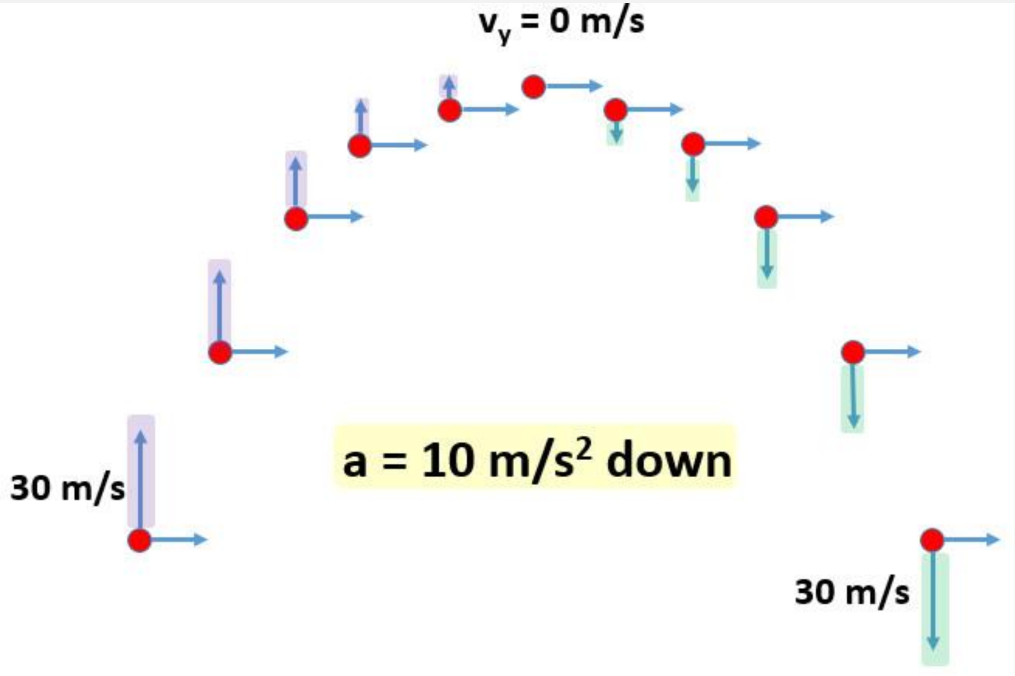
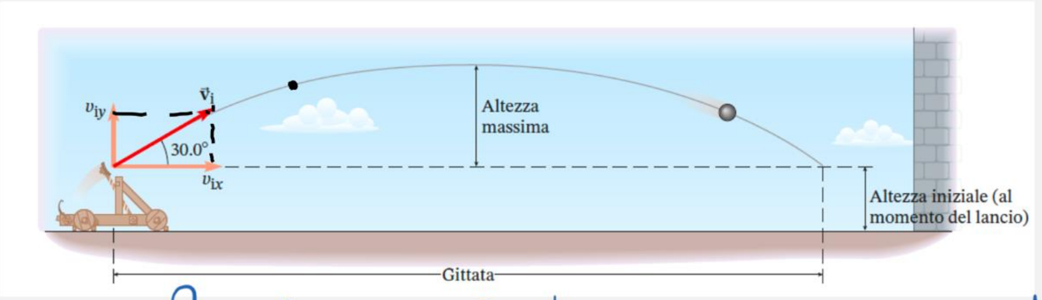
5 m/s


v_y = 0 m/s
30 m/s
a = 10 m/s² down
30 m/s

PARABOLIC MOTION

Example

A catapult launches a 32 kg stone with a speed of 50.0 m/s at an elevation angle of 30.0°. A) What is the maximum height reached by the stone? B) What is the flight time (the time interval during which the projectile remains in the air before falling back to the ground)? C) What is its range (the horizontal distance traveled by the projectile before falling back to the ground)?



$v_i = 50\,m/s$ $(\theta = 30°)$

$v_{ix} = v_i \cdot \cos\theta = 43.3\,m/s$

$v_{iy} = v_i \cdot \sin\theta = 25\,m/s$

$y_{max}? \rightarrow v_{y\,max} = 0\,m/s$

$\Delta y = \frac{1}{2}(v_{y\,max} + v_{iy}) \cdot \Delta t = \frac{1}{2} \cdot v_{iy}\,\Delta t$

(v_m; $v_{y\,max} = 0\,m/s$)

$v_{y\,max} - v_{iy} = a_y \Delta t = -g\Delta t \rightarrow \Delta t = \frac{-v_{iy}}{-g}$ ($v_{y\,max} = 0\,m/s$)

$\Delta y = \frac{1}{2} \cdot v_{iy}\left(\frac{-v_{iy}}{-g}\right)$

$= \frac{1}{2}\,\frac{v_{iy}^2}{g} = \frac{1}{2} \cdot \frac{(25\,m/s)^2}{9.8\,m/s^2} = 31.9\,m$

Tempo di volo → dall'equazione di prima = $\Delta t = \frac{-V_{iy}}{-g} = \frac{V_{iy}}{g} = \frac{25 m/s}{9.8 m/s^2} = 2.55 s$
(t_{TOT})

questo è t_s

$t_{TOT} = 2t_s = 5.10 s$

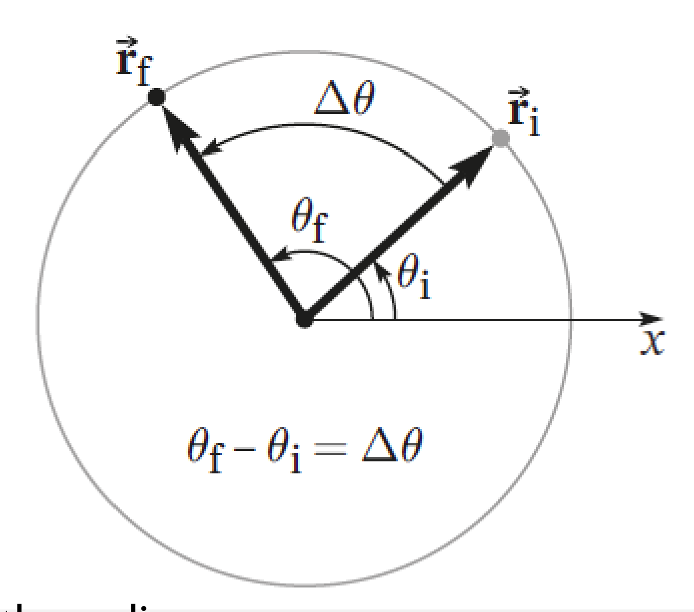
Gittata? (solo x) → $\Delta x = v\Delta t = V_{ix} \cdot t_{TOT} = 43.3 m/s \cdot 5.10 s = 220.8 m$

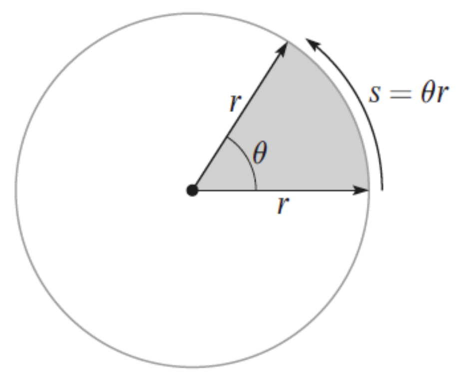
UNIFORM CIRCULAR MOTION

Definition of **angular displacement**: $\Delta\theta = \theta_f - \theta_i$

Definition of **average angular velocity**: $\omega_m = \frac{\Delta\theta}{\Delta t}$

Definition of **instantaneous angular velocity**: $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$





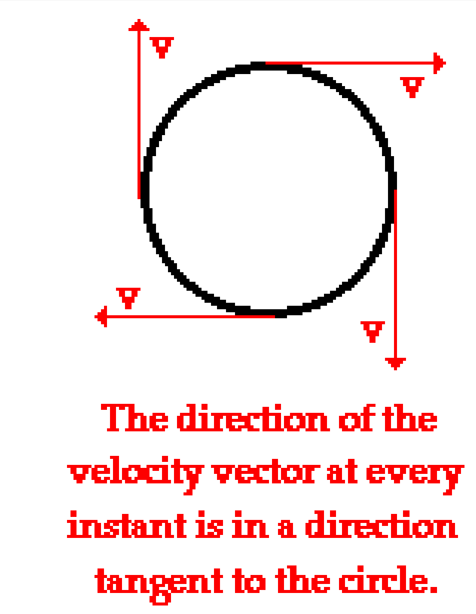
In many situations the most convenient measure of angle is the radian.

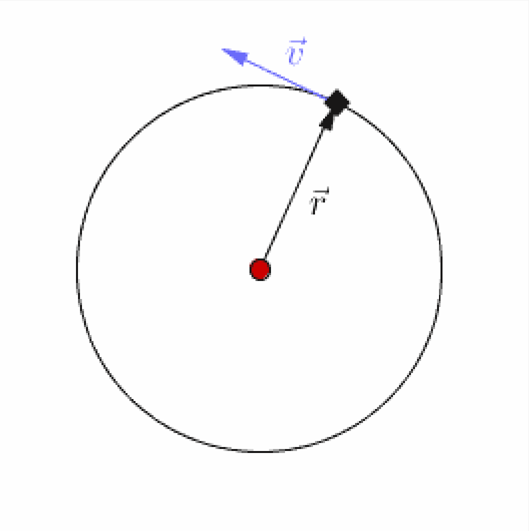
$\theta(in\ radian) = \frac{s}{r}$ $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi rad$ $360° = 2\pi rad$

A particle moving with uniform circular motion has a velocity with constant magnitude, while its direction changes over time: at every instant of time, the direction of the instantaneous velocity is in fact tangent to the circular trajectory

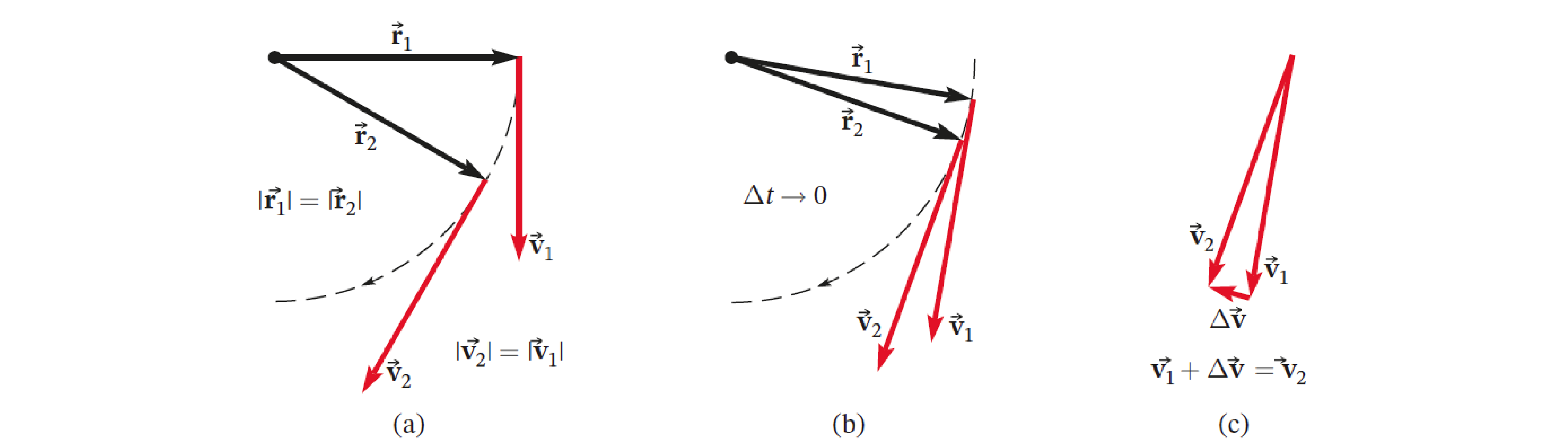
↓

Since the direction of the particle's velocity changes continuously,
the particle must possess a non-zero acceleration





UNIFORM CIRCULAR MOTION



Portion of a circular trajectory of radius r, along which a point-like body moves with uniform circular motion. At the points considered, the two velocity vectors are tangent to the trajectory and have the same magnitude

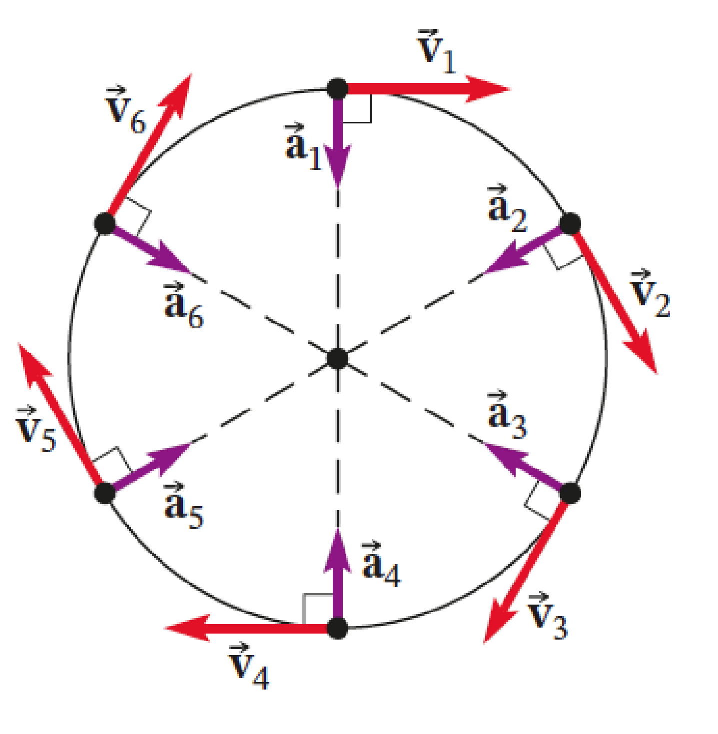
As the time interval between the two positions considered becomes progressively smaller, the two position vectors become increasingly close

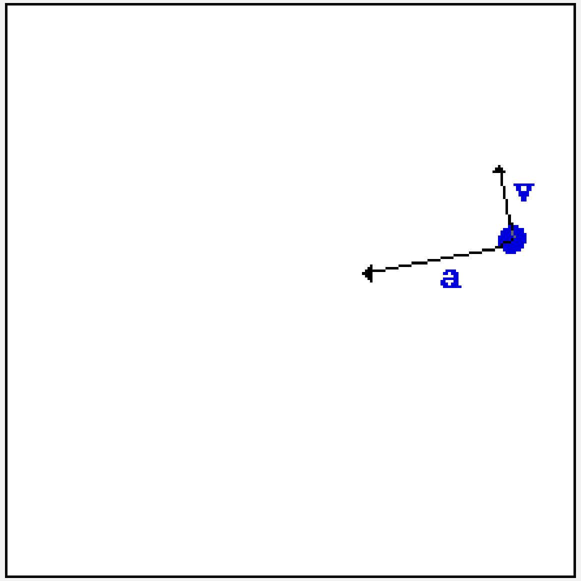
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

We determine the change in velocity Δv over a small time interval. When $\Delta t \rightarrow 0$, the angle between the two vectors tends to 0 and Δv becomes perpendicular to the velocity itself. Δv is directed along the radius of the circumference and oriented toward the center

UNIFORM CIRCULAR MOTION

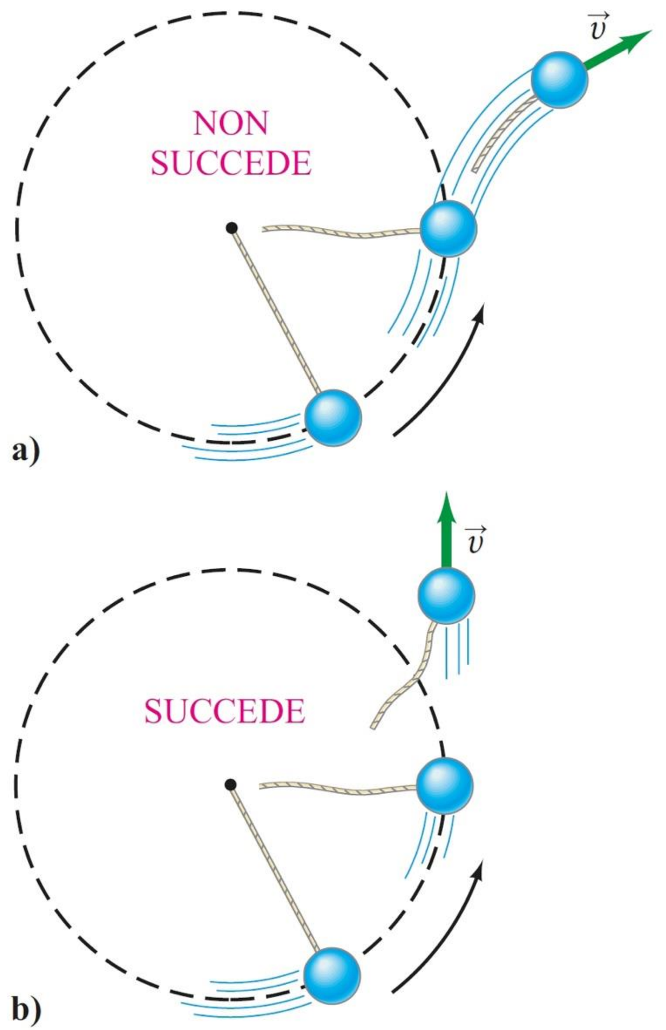
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$





a has the same direction and orientation as Δv → therefore, the acceleration will also be radially directed toward the center of the circumference:

The acceleration of a body moving with uniform circular motion is called radial acceleration $\overrightarrow{a_r}$ or centripetal acceleration



a)

b)

Figura 5.16 Se esistesse una forza centrifuga, la pallina dovrebbe muoversi verso l'esterno come in **a)**, una volta lasciata libera. In realtà, si allontana dalla traiettoria originaria, proseguendo in direzione tangenziale alla circonferenza come in **b)**.
Per esempio, in **c)** le scintille prodotte da una mola in rapida rotazione vengono proiettate lungo linee rette tangenti al bordo della mola.

c)