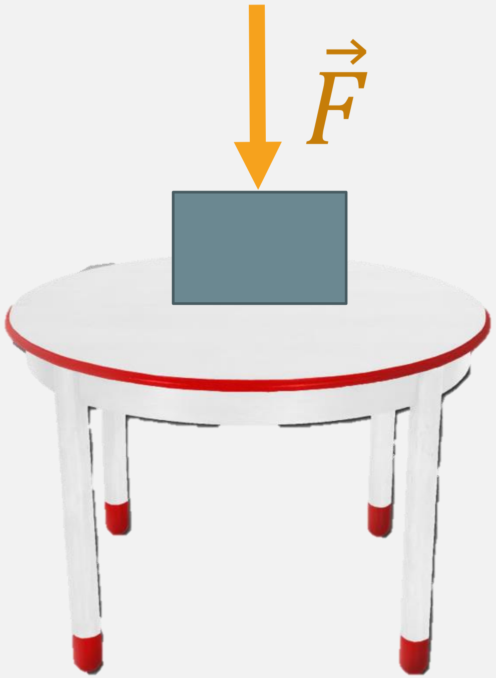
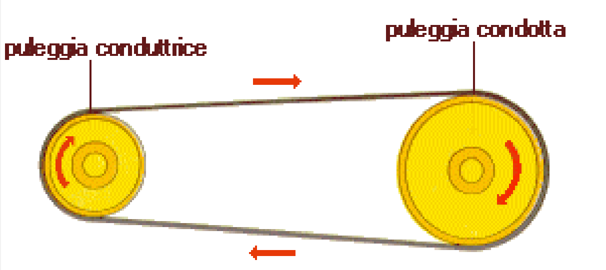
DYNAMICS pt.II

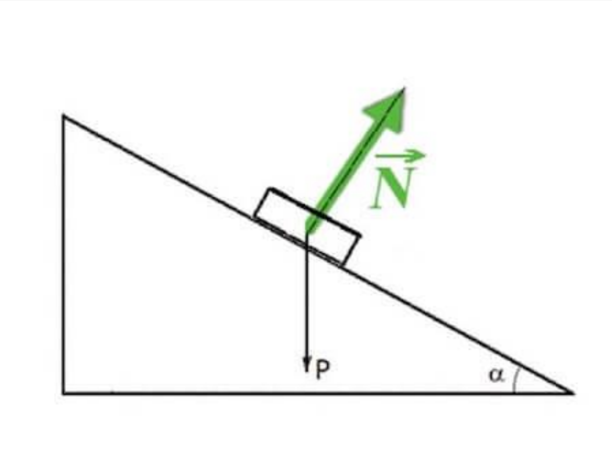
1. Constraint forces
   a) Normal force
   b) Tension
   c) Friction

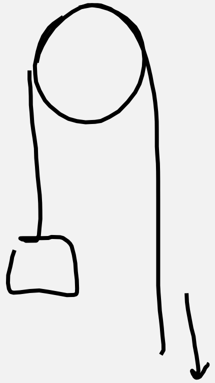
CONTACT FORCES

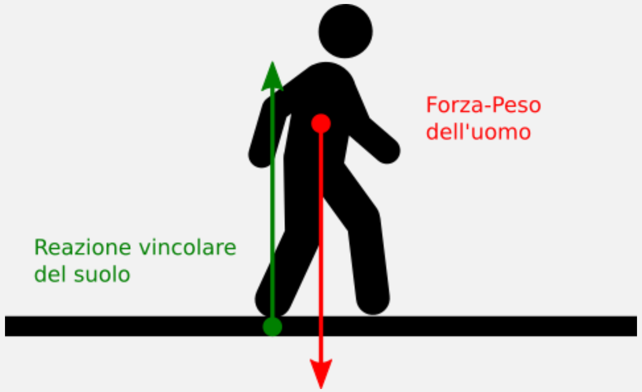
CONSTRAINT FORCE or CONSTRAINT REACTION

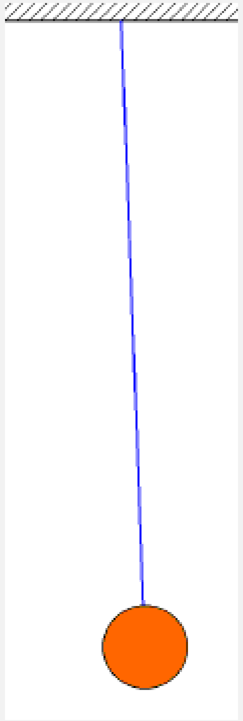










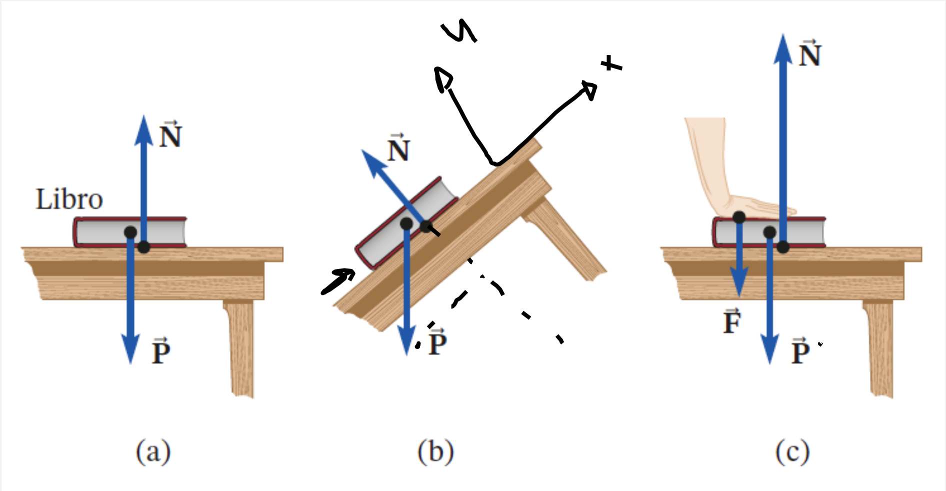


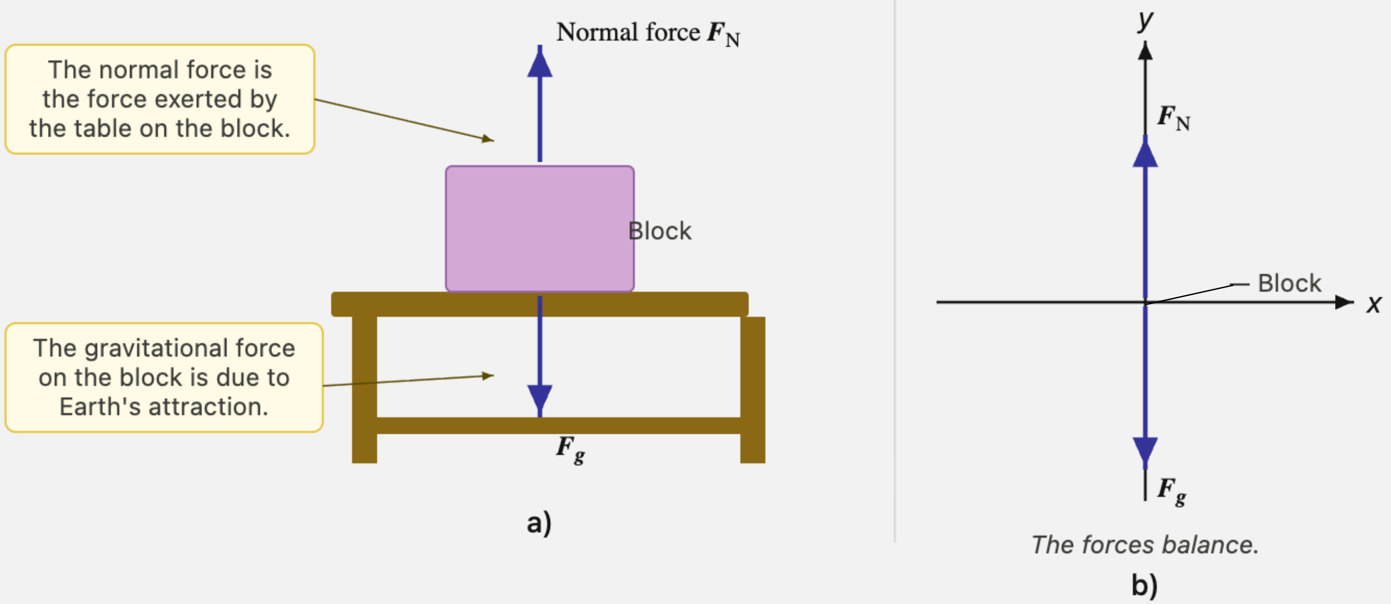
The constraints to which a body is subjected can themselves exert forces.
The action of the constraint is represented by a force called constraint reaction.

NORMAL FORCE

SUPPORT CONSTRAINT FORCE

Force acting perpendicularly to the contact surface between two bodies





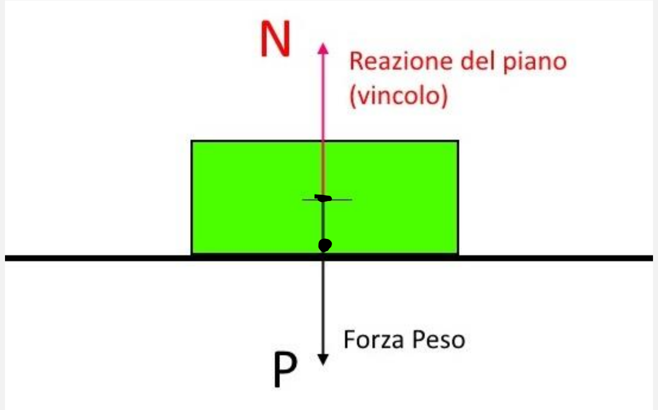
The normal force is always directed perpendicular to the contact surface and its magnitude can only be determined after analyzing all the other forces involved

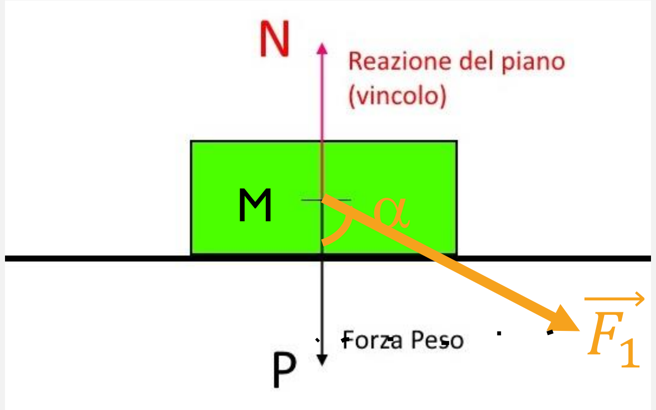
→ the constraint reaction is always orthogonal to the surface constituting the constraint and directed outward from it

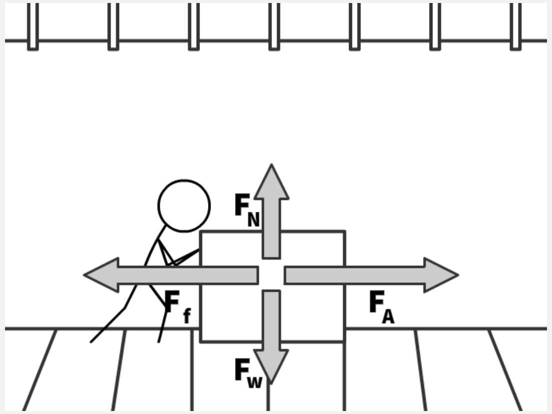
NORMAL FORCE

SUPPORT CONSTRAINT FORCE

Force acting perpendicularly to the contact surface between two bodies







The body is in equilibrium, therefore the sum of the forces acting on it equals zero:

$$\vec{N} + \overrightarrow{W} = 0$$
$$\vec{N} = -\overrightarrow{W}$$
$$\vec{N} = -M\vec{g}$$

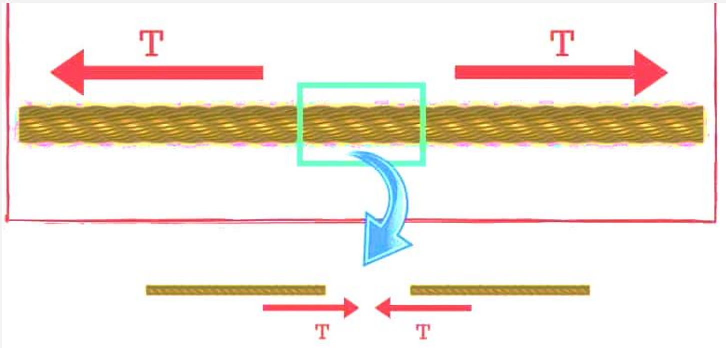
In addition to the gravitational force, there is an external force $\overrightarrow{F_1}$ which is oblique and forms an angle α con l'asse verticale:

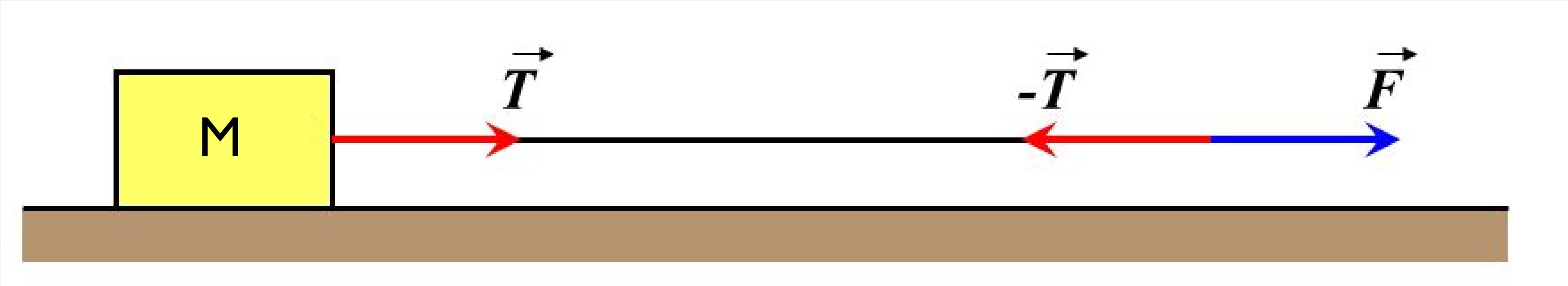
If the body is in equilibrium, the constraint reaction is given by the sum of the gravitational force and the vertical component of $\overrightarrow{F_1}$:

$$\left|\vec{N}\right| = F_1 \cos\alpha + \mathrm{W}$$

TENSION

IDEAL ROPES:
- Negligible mass
- Inextensible
- Flexible





$$m_{corda}\vec{a} = \vec{F} - \vec{T}$$

(Newton's second law: ma=sum of applied forces)

$$m_{corda} = 0$$

$$0\vec{a} = \vec{F} - \vec{T} \rightarrow \vec{F} = \vec{T}$$

If the rope is ideal, applying a force $\vec{F}$ all’estremità della fune è esattamente equivalente all’applicazione della forza $\vec{F}$ directly to the mass m

TENSION

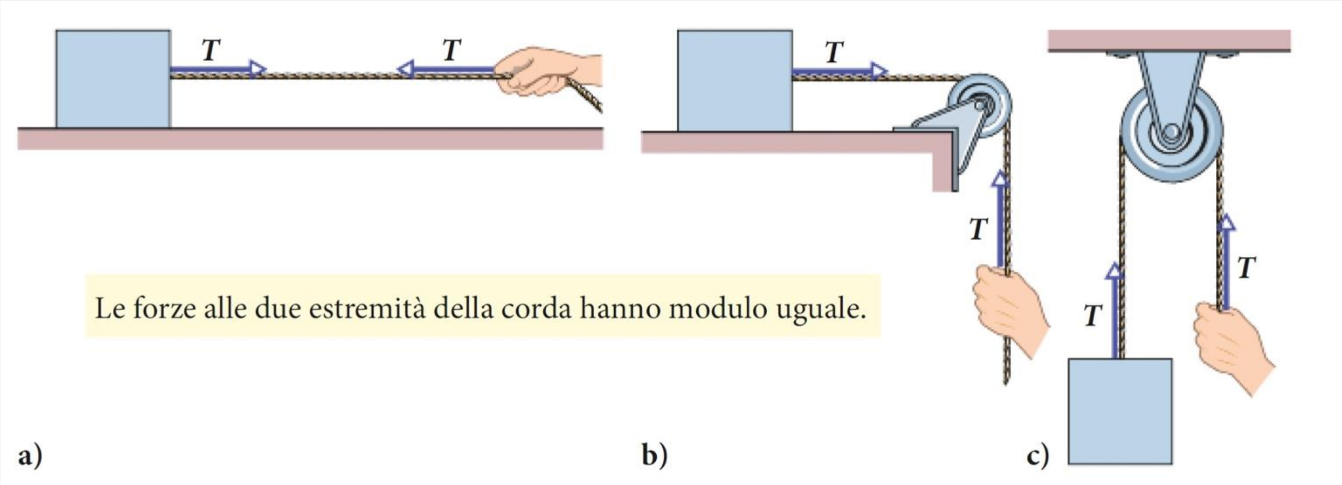
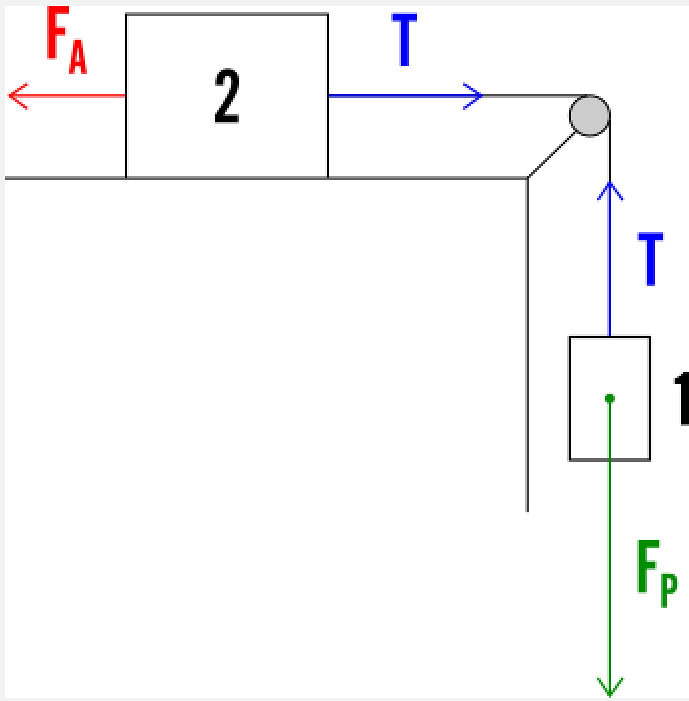


Figura 5.9 a) La corda, ben tesa, è in tensione. Se la sua massa è trascurabile, la corda tira il corpo e la mano con una forza ***T***, anche se la corda scorre attorno a una carrucola priva di massa e di attrito come in **b)** e in **c)**.

PULLEY

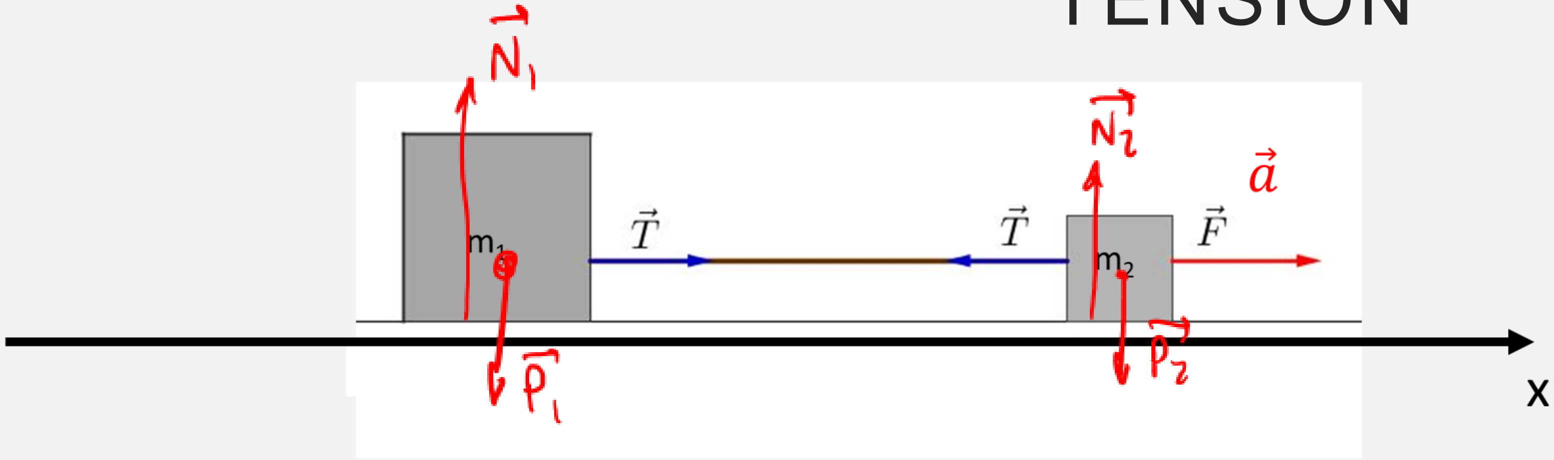
(b) A vertical force is applied to move a body horizontally

(c) A body is lifted by applying a vertical force directed downward



If the system is in equilibrium, it means that body 1 is not heavy enough to move body 2 (friction)

TENSION



- m_1 and m_2 , connected by an ideal rope.
- On m_2 a force $\vec{F}$ directed to the right is applied.
- m_1 will have an acceleration $\overrightarrow{a_1}$ and m_2 will have an acceleration $\overrightarrow{a_2}$
- The rope is ideal: it does not stretch/shorten: the system moves as a rigid body → unique acceleration for the whole system $\vec{a}_1 = \vec{a}_2 = \vec{a}$

On m_1 the gravitational force and constraint reaction act vertically, cancelling each other out → the vertical net force is equal to zero
On m_1 the only effective force is tension:

$\Sigma F = m_1 a = T$

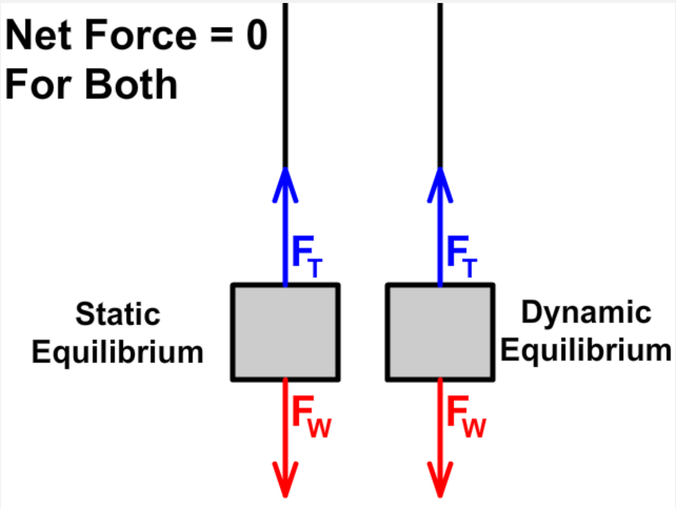
$$m_1 a = T$$

On m_2 the gravitational force and constraint reaction act vertically, cancelling each other out → the vertical net force is equal to zero
On m_2 act $\vec{F}$ and $\vec{T}$, which have opposite directions:

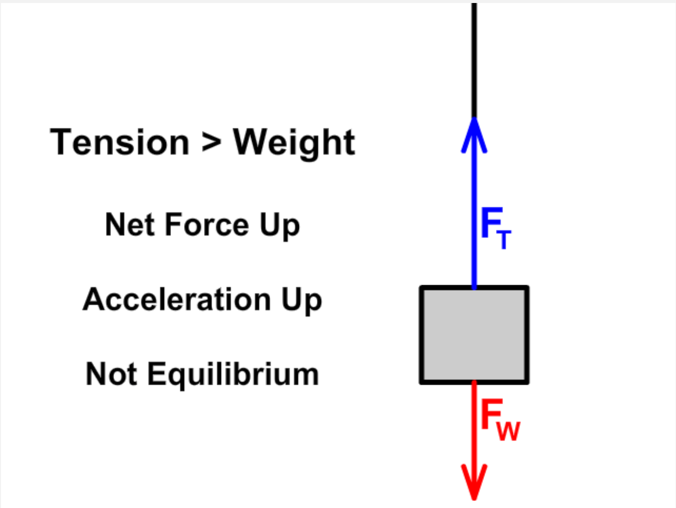
$\Sigma F = m_2 a$

$$m_2 a = F - T$$

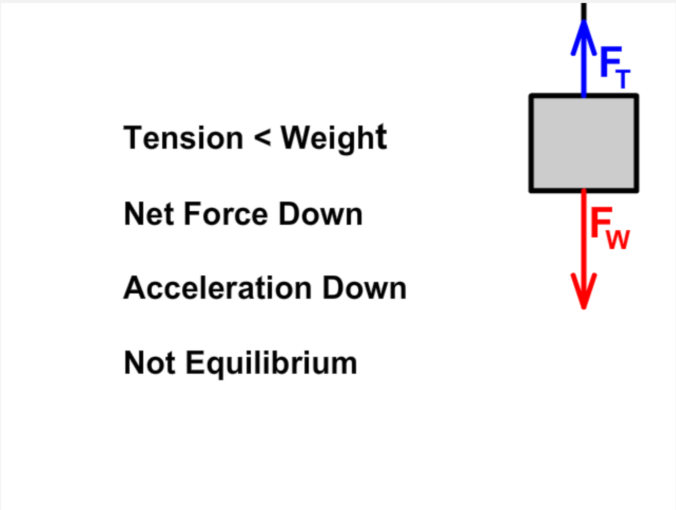
$$\begin{cases} m_1 a = T \\ m_2 a = F - T \end{cases} \qquad a = \frac{F}{m_1 + m_2} \qquad T = \frac{m_1 F}{m_1 + m_2}$$



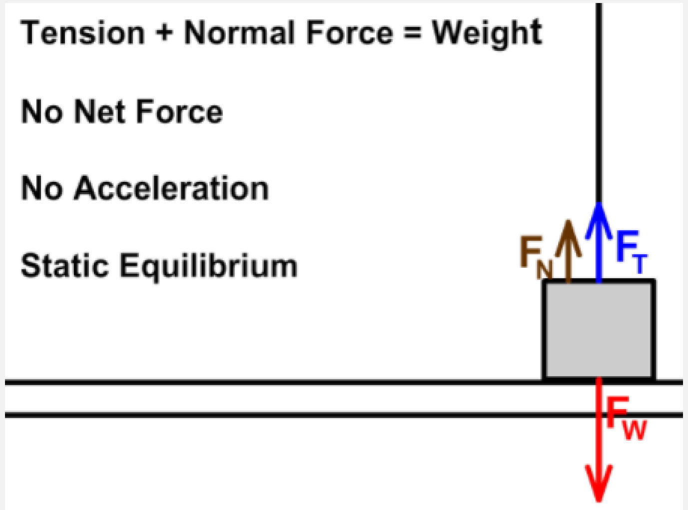
Net Force = 0
For Both
F
T
Static
Equilibrium
Dynamic
Equilibrium
F
W



Tension > Weight
Net Force Up
Acceleration Up
Not Equilibrium
F
T
F
W

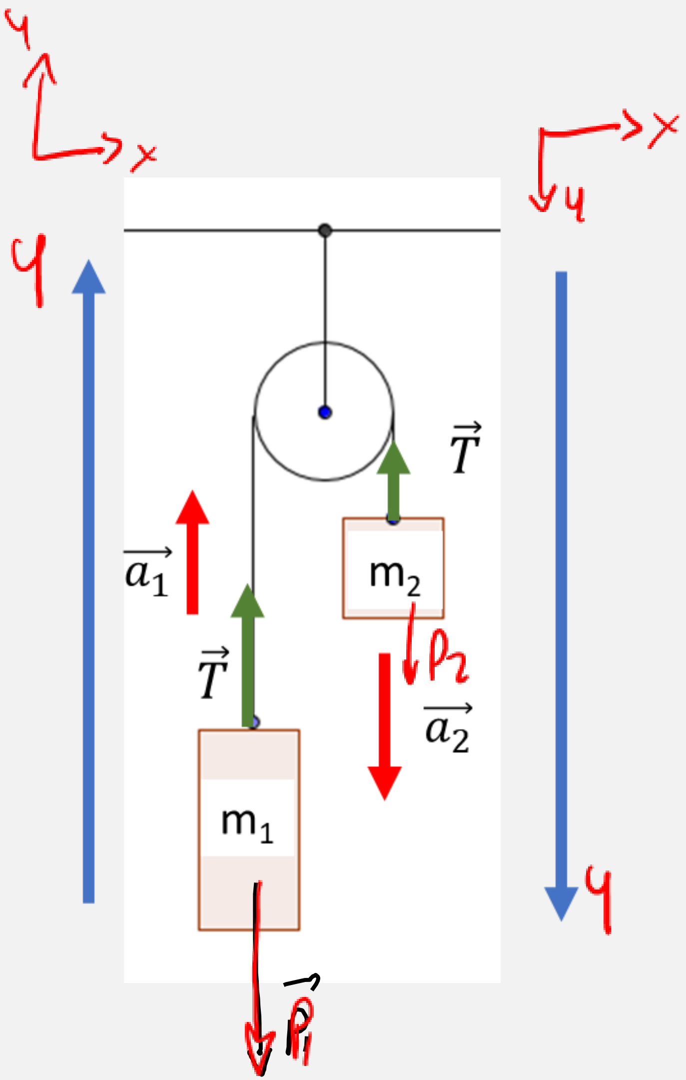


Tension < Weight
Net Force Down
Acceleration Down
Not Equilibrium
F
T
F
W



Tension + Normal Force = Weight
No Net Force
No Acceleration
Static Equilibrium
F
N
F
T
F
W

TENSION – PULLEYS



Two masses are suspended from an ideal rope (m_1 and m_2)

$\Sigma F_1 = m_1 a$

$$\begin{cases} m_1 a_1 = T - m_1 g \\ m_2 a_2 = m_2 g - T \end{cases}$$

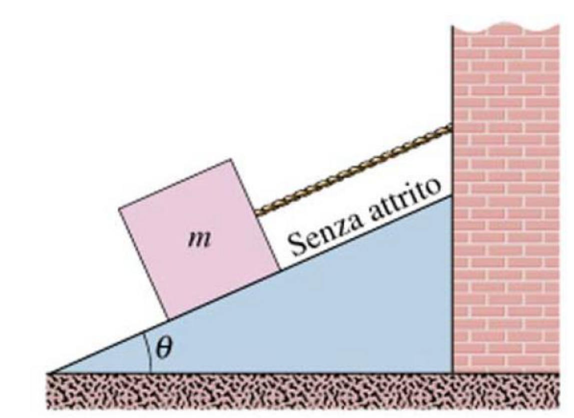
Since the acceleration is in fact unique,

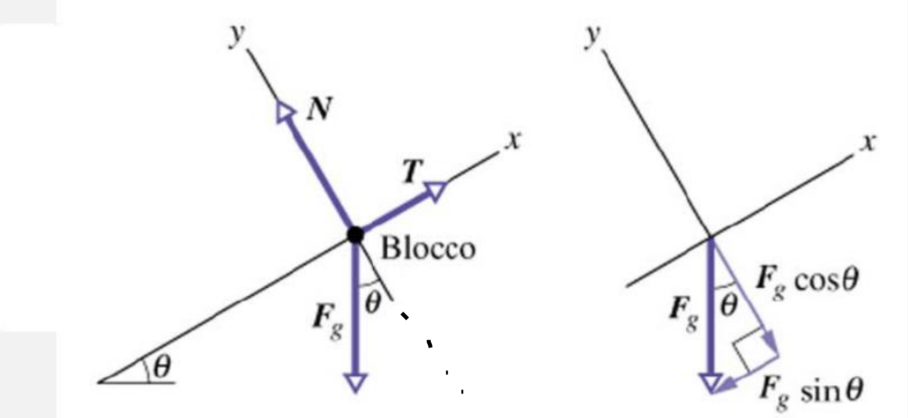
$$a = \frac{m_2 - m_1}{m_1 + m_2} g$$

Three possible scenarios:

1. $m_1 = m_2$: system in equilibrium, because $m_2 - m_1 = 0$, hence $a = 0$
2. $m_1 < m_2$: the pulley moves in the direction of m_2
3. $m_1 > m_2$: it will be m_1 that descends → the acceleration will be < 0

TENSION





$\theta = 30°, m = 15kg$
Determine the tension in the rope and the constraint reaction, knowing that the system is in equilibrium.

Equilibrium → the resultant of the forces is equal to 0: $\vec{T} + \vec{N} + \vec{F_g} = m\vec{a} = 0$

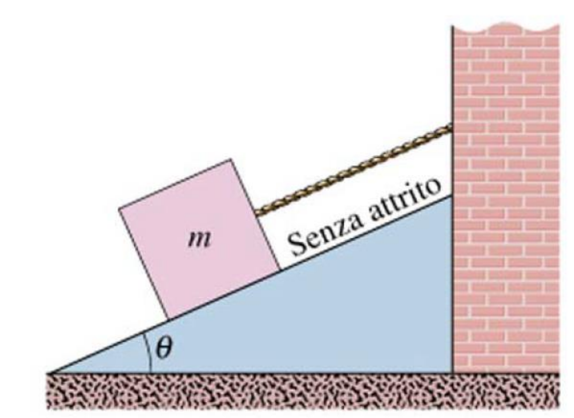
Along the x-axis: tension and the component of the gravitational force parallel to the inclined plane ($F_g \sin\theta$):

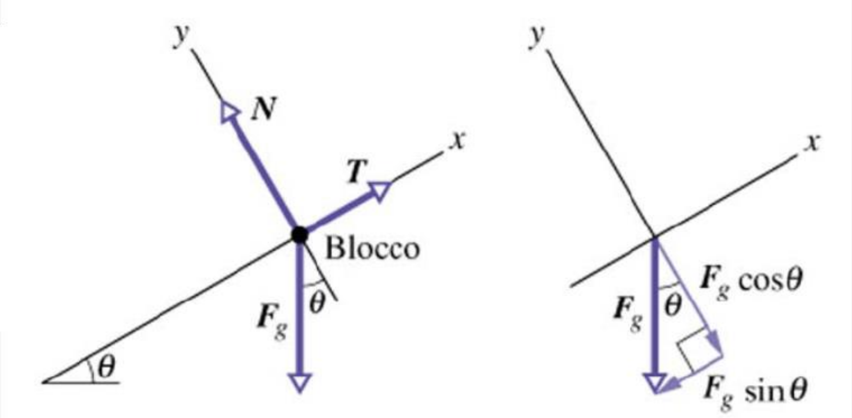
$$T - mg\sin\theta = 0$$

Along the y-axis: constraint reaction and the component of the gravitational force perpendicular to the inclined plane ($F_g\, cos\,\theta$):

$$N - mg\cos\theta = 0$$

TENSION





$\theta = 30°, m = 15kg$

Determine the tension in the rope and the constraint reaction, knowing that the system is in equilibrium.

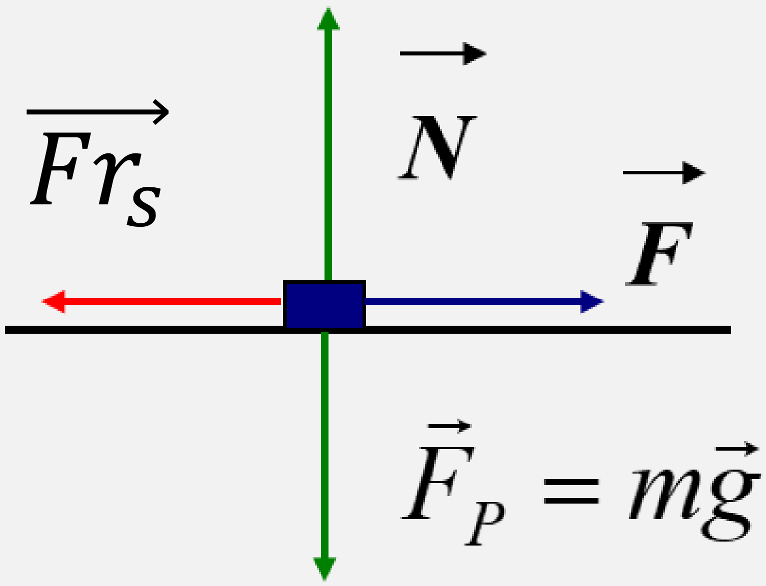
The rope breaks: what is the acceleration of the block?

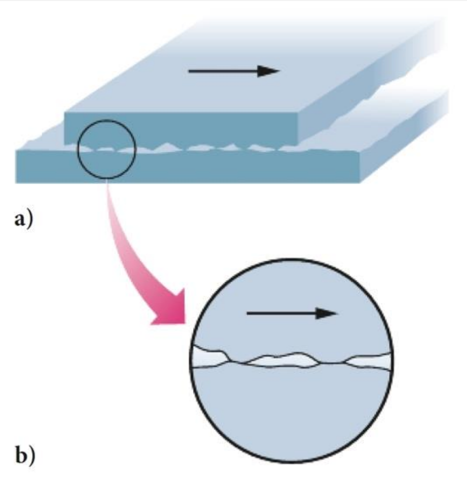
Along the y-axis the resultant is always equal to zero: $N - mg\cos\theta = 0$

Along the x-axis, the tension disappears: the only component effective in making the body move is $mg\sin\theta$

Value of the acceleration

STATIC FRICTION





Body on a horizontal plane subjected a force $\vec{F}$ but does not move: $\vec{F} = \overrightarrow{Fr_S}$

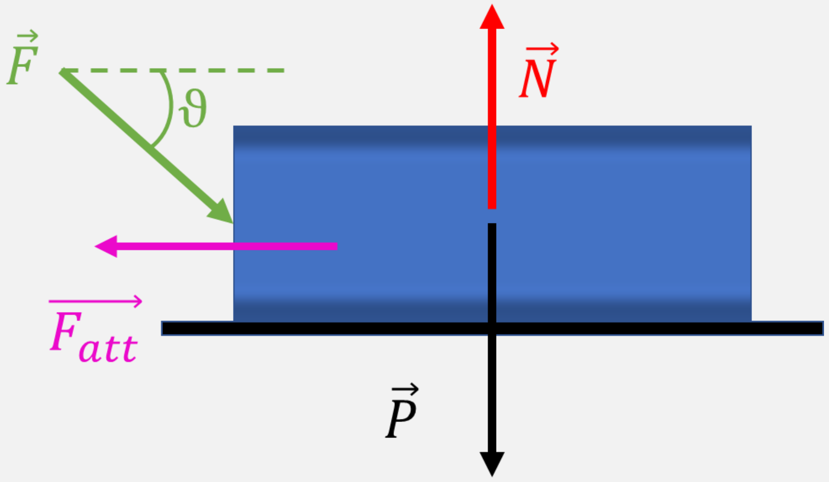
$$\vec{F} \leq \overrightarrow{\mathrm{Fr}}_{s\,max} = \mu_S \overrightarrow{F_P} = \mu_S \vec{N}$$

Horizontal support plane:

$$\vec{F}_P = -\vec{N} \qquad \left|\vec{F}_P\right| = \left|\vec{N}\right| = mg$$

$$Fr_{s\,max} = \mu_s mg$$

STATIC FRICTION



Equilibrium → the net force is zero

Along the x-axis: horizontal component of the applied force ($F \cos\theta$) and friction force ($\overrightarrow{\mathrm{Fr}}$)

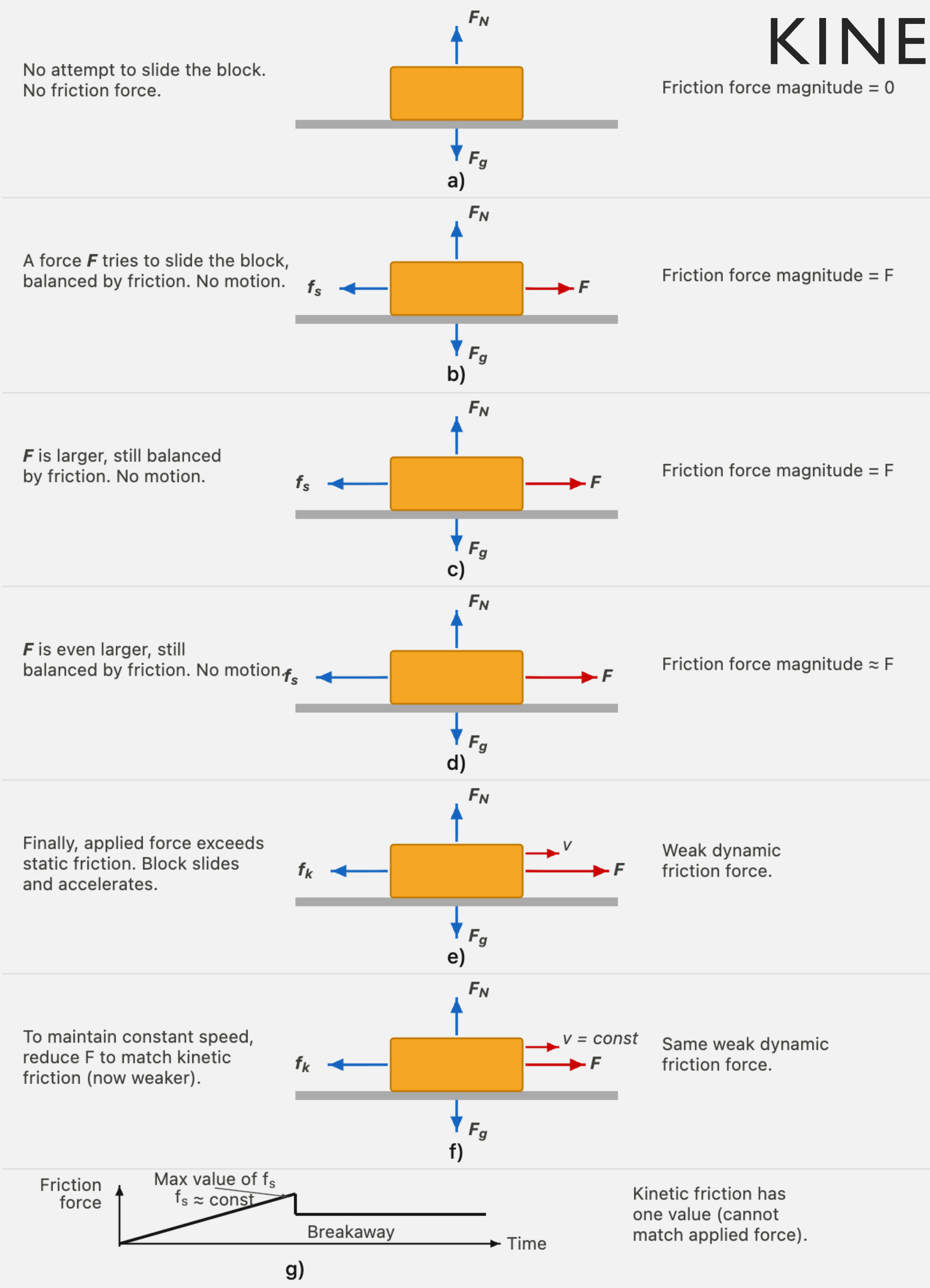
$$F\cos\theta - Fr = 0$$

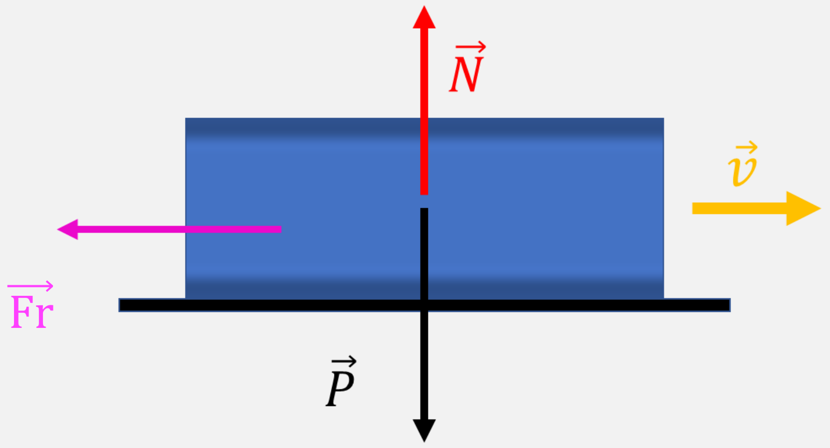
Along the y-axis: constraint reaction and vertical component of the force ($F_g \sin\theta$):

$$mg + F\,sin\,\theta - N = 0 \rightarrow F\,sin\,\theta + mg = N$$

$$\mu_s = \frac{Fr}{N} \qquad \mu_s = \frac{F\cos\theta}{F\sin\theta + mg}$$

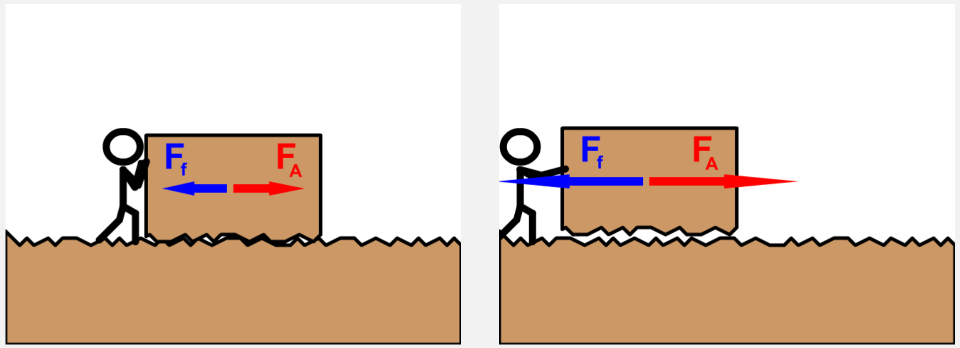
KINETIC FRICTION



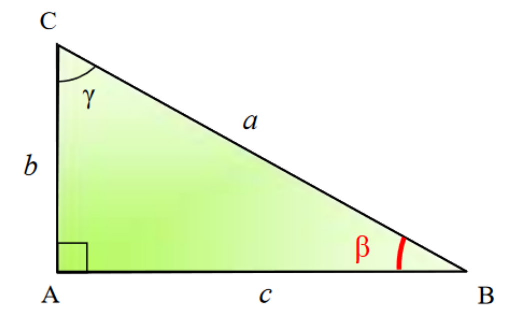


$$Fr = \mu_D N$$

$$\mu_S > \mu_D$$



INCLINED PLANE

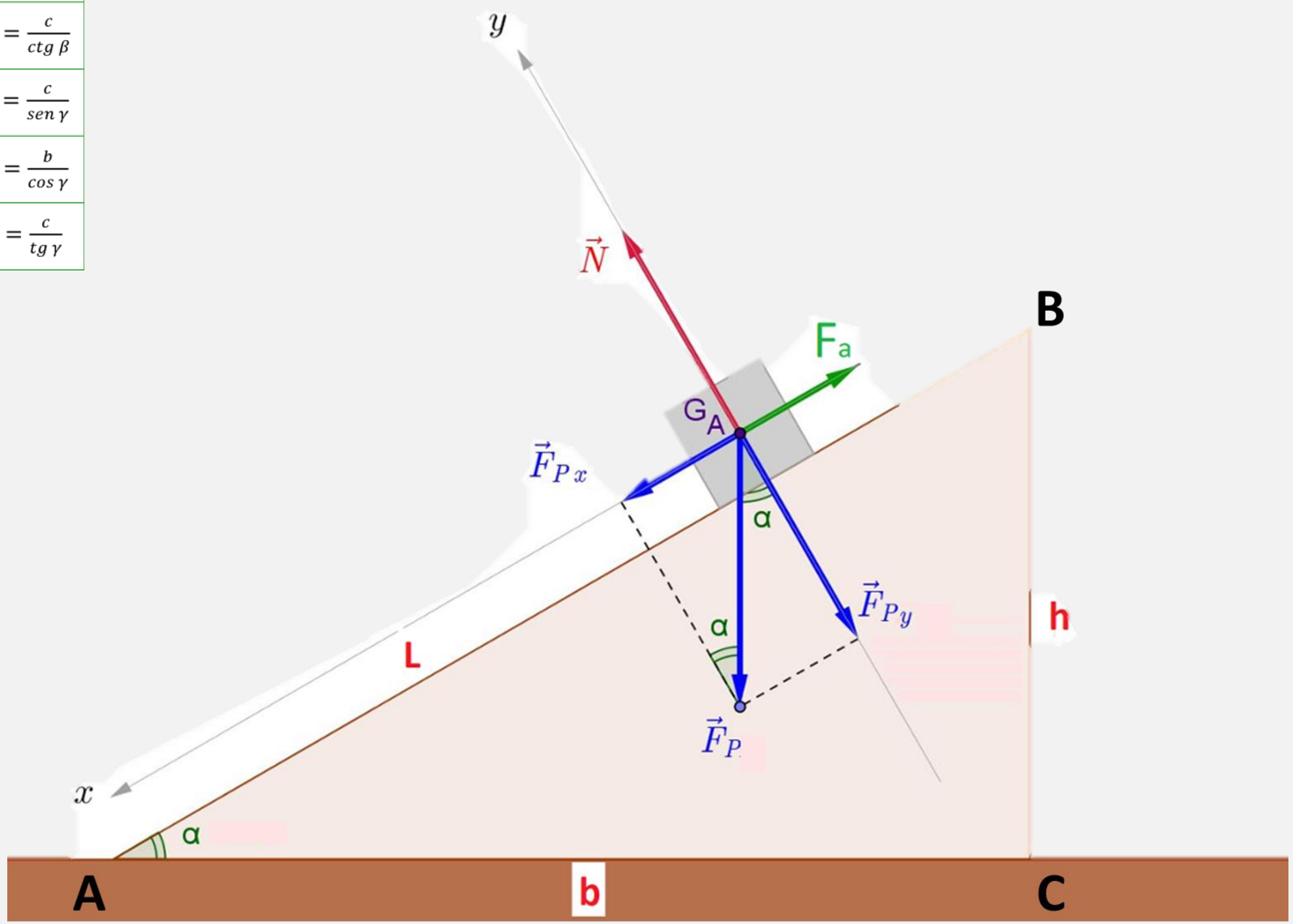


$sen\,\beta = \frac{b}{a}$	$\longrightarrow$	$b = a \cdot sen\,\beta$	e	$a = \frac{b}{sen\beta}$
$cos\,\beta = \frac{c}{a}$	$\longrightarrow$	$c = a \cdot cos\,\beta$	e	$a = \frac{c}{cos\,\beta}$
$tg\,\beta = \frac{b}{c}$	$\longrightarrow$	$b = c \cdot tg\,\beta$	e	$c = \frac{b}{tg\,\beta}$
$ctg\,\beta = \frac{c}{b}$	$\longrightarrow$	$c = b \cdot ctg\,\beta$	e	$b = \frac{c}{ctg\,\beta}$
$sen\,\gamma = \frac{c}{a}$	$\longrightarrow$	$c = a \cdot sen\,\gamma$	e	$a = \frac{c}{sen\,\gamma}$
$cos\,\gamma = \frac{b}{a}$	$\longrightarrow$	$b = a \cdot cos\,\gamma$	e	$a = \frac{b}{cos\,\gamma}$
$tg\,\gamma = \frac{c}{b}$	$\longrightarrow$	$c = b \cdot tg\,\gamma$	e	$b = \frac{c}{tg\,\gamma}$

$$F_{Px} = F_P \sin\alpha$$

$$F_{Py} = F_P \cos\alpha = N$$

$$F_a = \mu_S N = \mu_S F_P \cos\alpha = \mu_S mg \cos\alpha$$

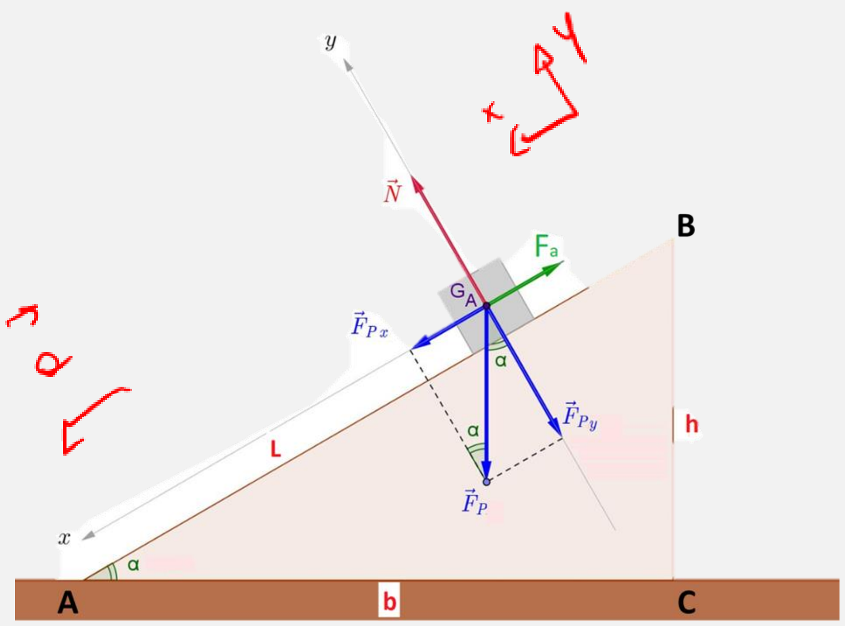


INCLINED PLANE

CASE WITHOUT FRICTION

$$F_{Px} = F_P \sin\alpha$$

$$ma = F_{Px} = mg\sin\alpha \rightarrow a = g\sin\alpha$$



CASE WITH FRICTION

$$F_a = \mu_D N = \mu_D F_P \cos\alpha = \mu_D mg\cos\alpha$$

$$ma = F_{Px} - F_a = mg\sin\alpha - \mu_D mg\cos\alpha$$

$$a = g(\sin\alpha - \mu_D\cos\alpha)$$

EQUILIBRIUM CASE

$F_{As} = F_{Px}$

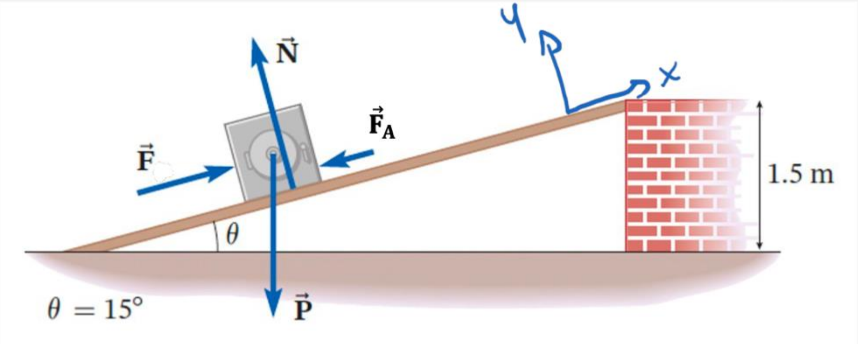
$$\mu_S mg\cos\alpha = mg\sin\alpha$$

$$\sin\alpha = \mu_S\cos\alpha \rightarrow \mu_S = \tan\alpha$$

$\frac{\sin\alpha}{\cos\alpha}$

Example

A safe must be moved along an inclined plane whose upper end is 1.5m above the ground. The mass of the safe is 510kg. The coefficient of static friction along the surface of the inclined plane is 0.42, while the coefficient of kinetic friction is 0.33. The plane makes an angle of 15° with the horizontal plane. A) Assuming the pushing force is parallel to the inclined plane, what magnitude will be needed to set the safe in motion on the ramp? B) Once the safe has been set in motion, what force must be applied to move it at constant velocity?



(1) STAT.

$\Sigma F_x = m a_x = 0$

↳ $F - F_{A_s} - P = 0$

$F - \mu_s mg\cos\theta - mg\sin\theta = 0$

$F = \mu_s mg\cos\theta - mg\sin\theta = 3300N$

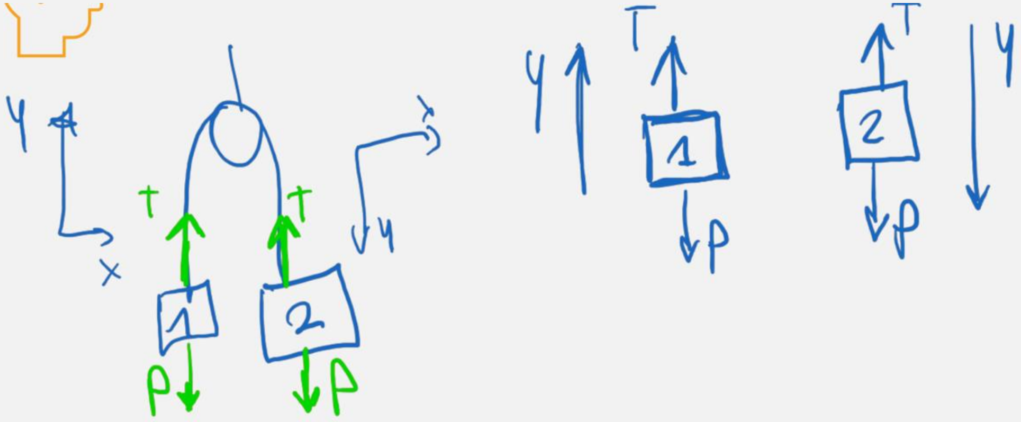
(2) DINAM.

$F - F_{A_d} - P = 0$ → $V = cost \rightarrow a = 0$

$F - \mu_d mg\cos\theta - mg\sin\theta = 0$ $F = 2900N$

Example

Given an ideal pulley with an ideal rope and two masses, m1=26kg and m2=42kg. Calculate the motion of the system.



(1)

$$\Sigma F = m_1 a$$

$$T - P = m_1 a$$

$$T - m_1 g = m_1 a$$

(2)

$$\Sigma F = m_2 a$$

$$P - T = m_2 a$$

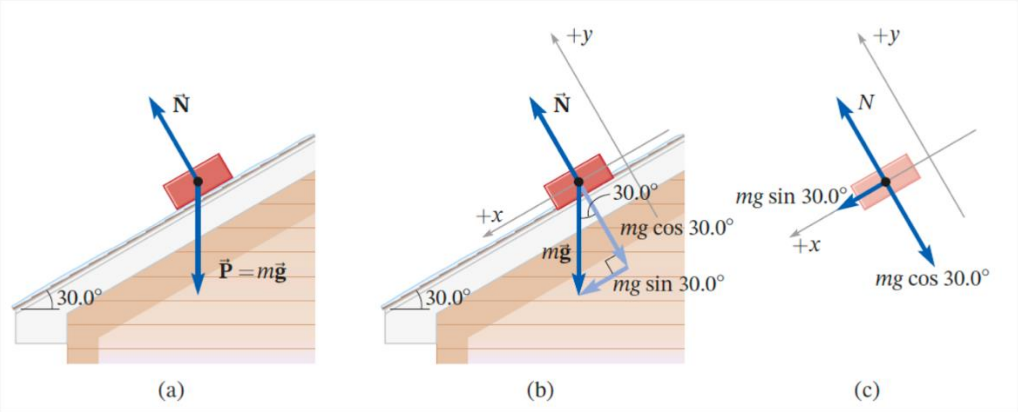
$$m_2 g - T = m_2 a$$

$$\begin{cases} T = m_1 a + m_1 g \\ T = m_2 g - m_2 a \end{cases}$$

$$a = \frac{m_2 - m_1}{m_1 + m_2} g = 2.31\ m/s^2$$

Example

A brick with a mass of 1kg slides on the icy surface of a roof, inclined at 30°. If the brick starts from rest, what velocity will it have after 0.9s, i.e. when it reaches the edge of the roof? Neglect friction.



$$\Sigma F_x = m \cdot a_x = mg \sin\theta = mg \sin 30°$$

$$a_x = g \cdot \sin 30°$$

accelerazione costante:

$$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t$$

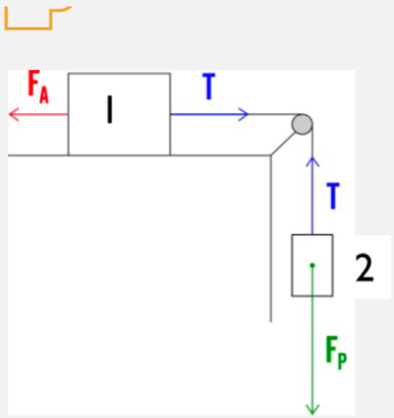
Sappiamo che $\Delta t = 0.9s$ e $v_{ix} = 0 m/s$ →

$$v_{fx} = v_{ix} + a_x \Delta t$$

$$= 0\ m/s + 9.8\ m/s^2 \cdot 0.9s = 4.4\ m/s$$

Example

Two boxes are connected by an ideal rope running over a pulley. The coefficient of kinetic friction between the surface and box 1 is 0.2. Find the acceleration of the system.



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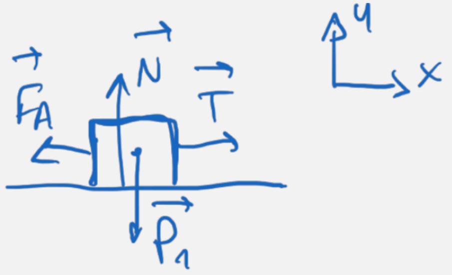

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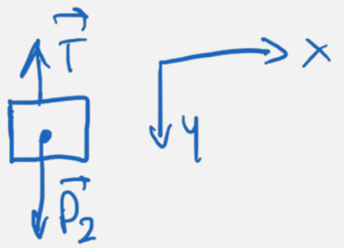

$$\vec{N} = m_1 g = 5\,kg \cdot 9.8\,m/s^2 = 49\,N$$

$$\Sigma F_{x_1} = T - F_A = m_1 a$$

non
lo conosco

①

$$f_A = \mu_D N = 0.2 \cdot 49\,N = 9.8\,N$$

$$T = m_1 a + F_A$$

② $P_2 = m_2 g = 19.6\,N$

$$\Sigma F_{2y} = m_2 a = P_2 - T$$

$$m_2 a = m_2 g - T$$

$$T = m_2 g - m_2 a$$

$$m_1 a + F_A = m_2 g - m_2 a$$

$$a = \frac{m_2 g - F_A}{m_1 + m_2} = 1.4\,m/s^2$$