WORK and ENERGY

ENERGY

A **conservation law** is a physical principle that identifies a quantity that does not change over time.

Law of conservation of energy: The total energy in the universe is unchanged by any physical process:

$$initial\ total\ energy\ = final\ total\ energy$$

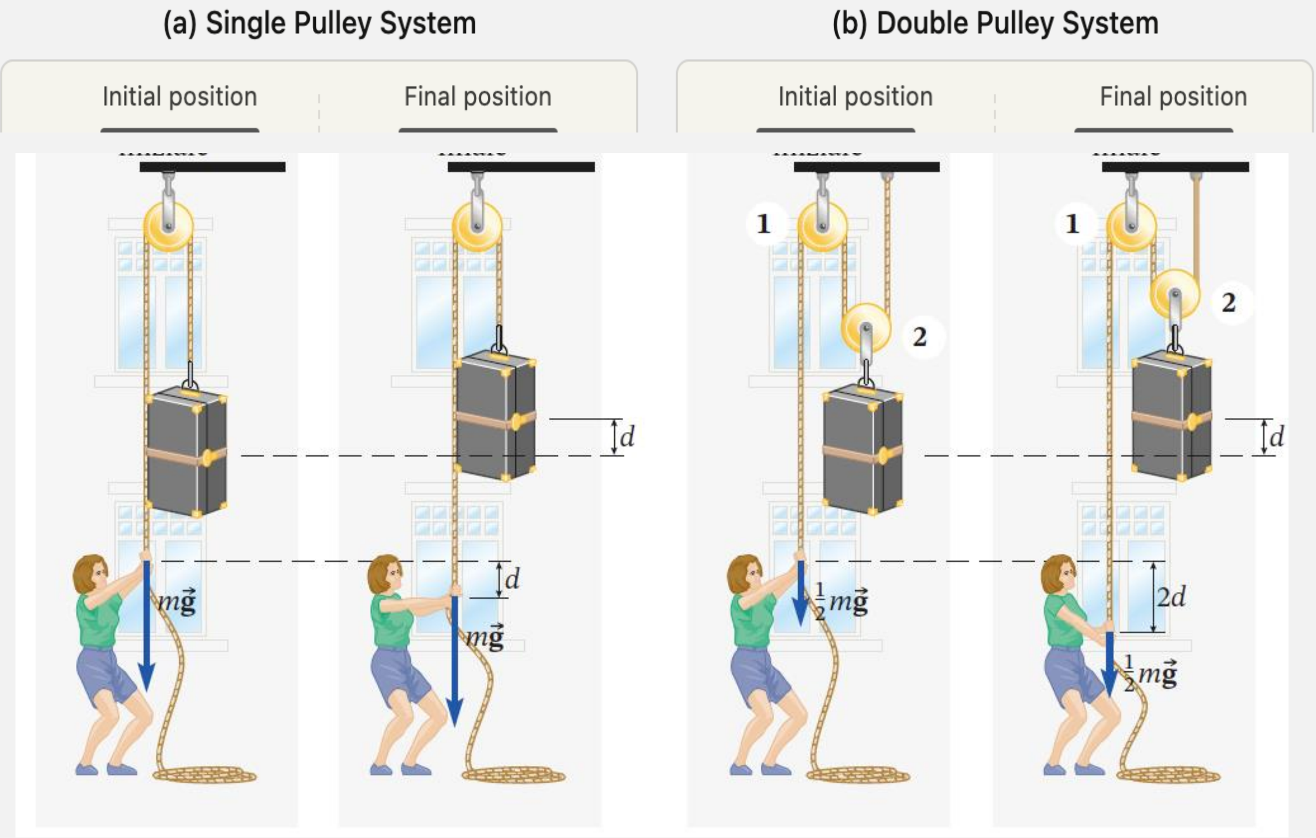
Table 6.1 Different forms of energy

Type	Description
Translational kinetic	Energy associated with the translational motion of a body (Chapter 6)
Elastic	Energy stored in an elastic body when deformed (Chapter 6)*
Gravitational	Energy associated with gravitational interaction (Chapter 6)
Rotational kinetic	Energy associated with the rotational motion of a body (Chapter 8)*
Vibrational, acoustic, seismic	Energy associated with oscillatory motions of atoms and/or molecules in a substance, driven by a mechanical wave passing through it (Chapter 11)*
Internal	Energy associated with the motion and interactions of atoms and molecules in solids, liquids, and gases. Related to the body's temperature (Chapters 12–14)*
Electromagnetic	Interaction energy between electric charges and electric current; energy of the electromagnetic field, including electromagnetic waves such as light (Chapters 13, 16–20)
Rest	Total energy of a particle of rest mass m, given by Einstein's equation $E = mc^2$ (Chapter 24)
Chemical	Energy associated with the motion and interactions of electrons in atoms and molecules*
Nuclear	Energy associated with the motion and interactions of protons and neutrons in atomic nuclei (Chapter 24)

** Not a fundamental form of energy, but determined by microscopic kinetic and/or electromagnetic energy.*

At the fundamental level, there are only three types of energy: energy due to motion (kinetic energy), energy due to interactions (potential energy), and rest energy.

WORK



Trunk weight = 220 N
Height to reach = 4 m

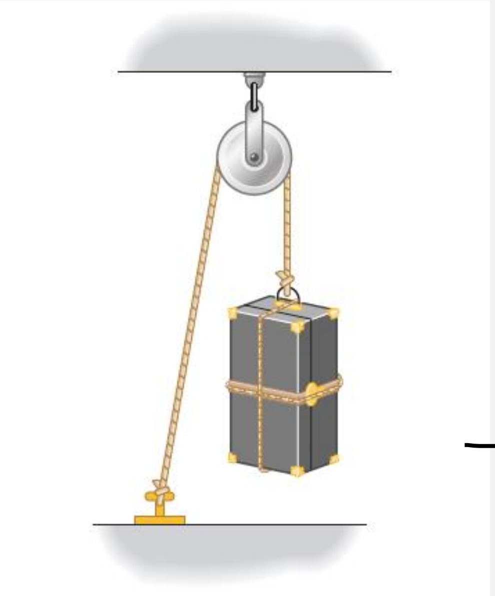
Amount of energy transferred when a force acts on a moving object

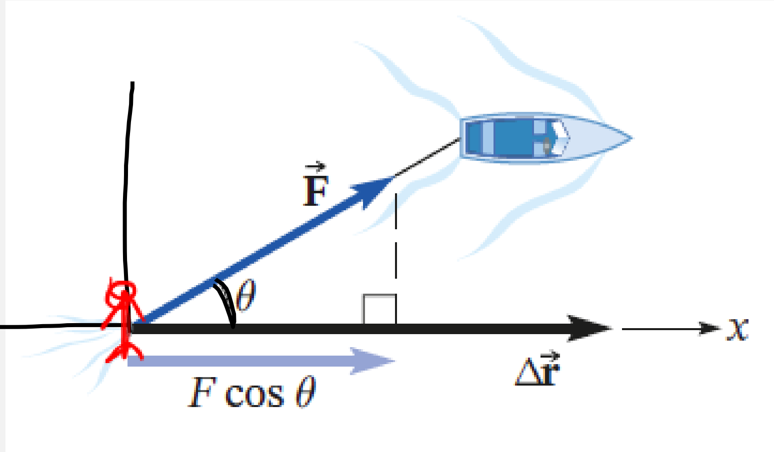
$220N \cdot 4m = 880N \cdot m$

$110N \cdot 8m = 880N \cdot m$ → **WORK (W)**

$N \cdot m = \text{Joule (J)}$

WORK





Only the component of force in the direction of displacement does work:

The work done by a constant force is the product of the magnitude of the displacement and the component of the force in the direction of that displacement

If there is no displacement, no work is done and there is no energy transfer

Work done by a constant force $\vec{F}$ acting on a body whose displacement is Δr

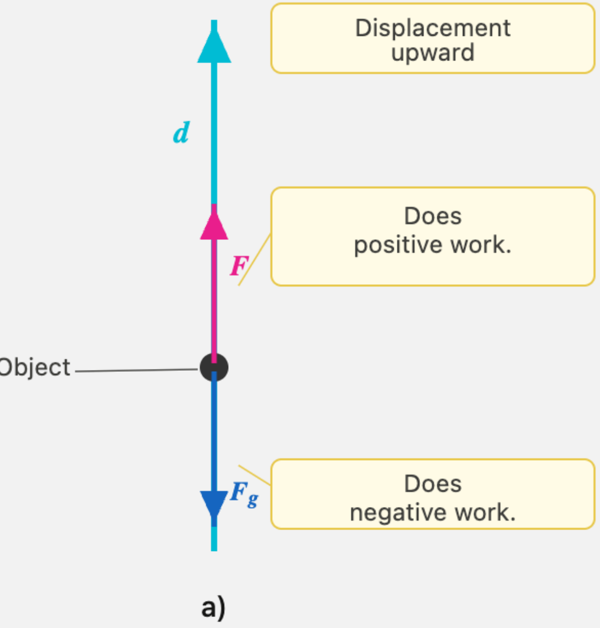
$$W = F\Delta r\cos\theta \leftrightarrow W = \vec{F}\cdot\Delta\vec{r}$$

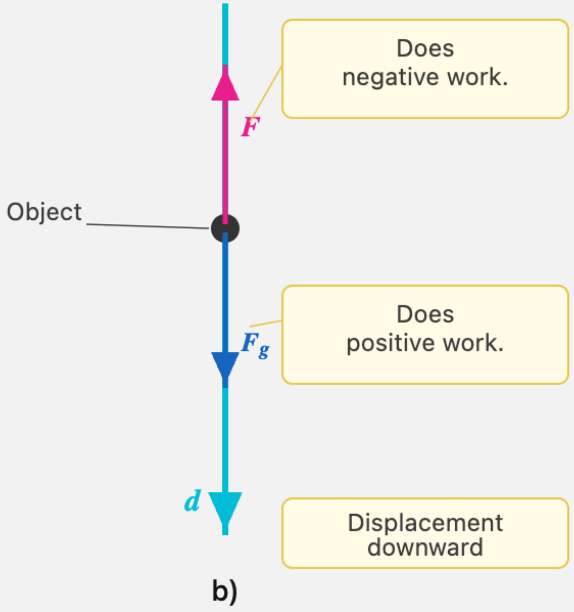
$F\Delta r \cos 90° = 0$
↓ 0

- $\theta < 90° \rightarrow W > 0$ $(\cos\theta > 0)$ POSITIVO
- $\theta > 90° \rightarrow W < 0$ $(\cos\theta < 0)$ NEGATIVO
- $\theta = 90° \rightarrow W = 0$ $(\cos\theta = 0)$ NULLO

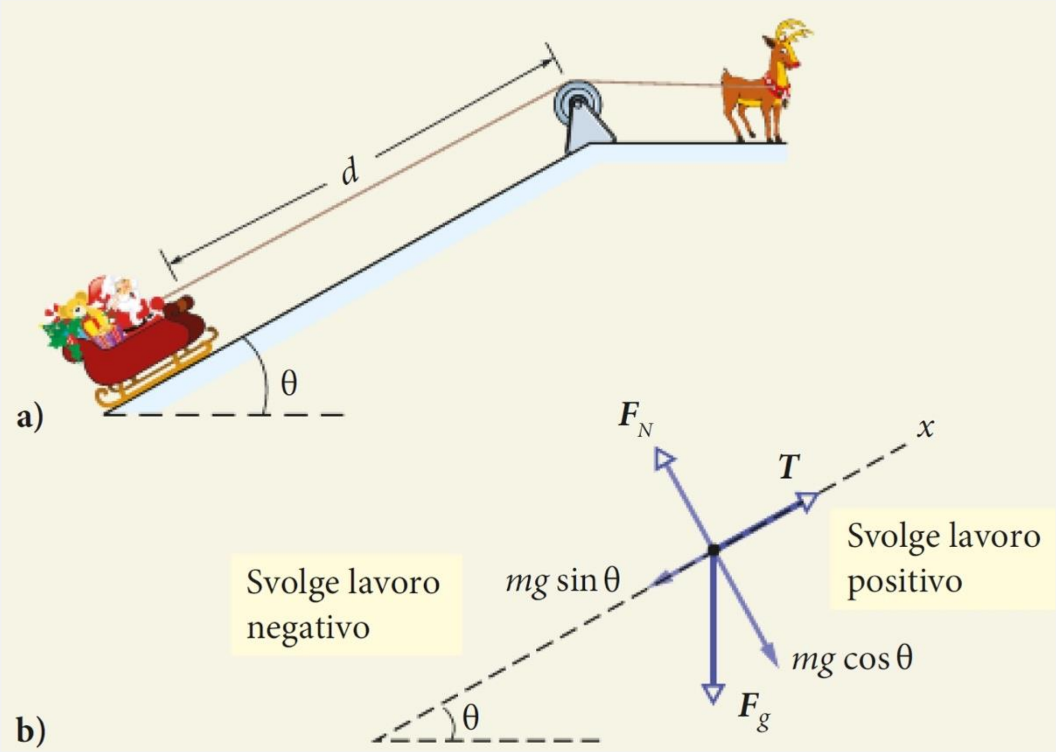
WORK

$$W = F\Delta r\cos\theta \leftrightarrow W = \vec{F}\cdot\Delta\vec{r}$$





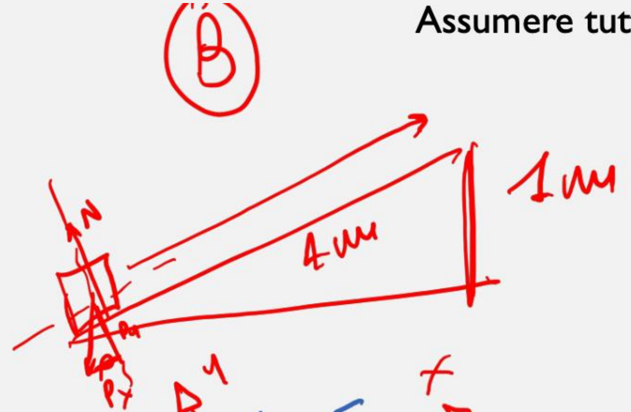
- $\theta < 90° \rightarrow W > 0 \ (\cos\theta > 0)$
- $\theta > 90° \rightarrow W < 0 \ (\cos\theta < 0)$
- $\theta = 90° \rightarrow W = 0 \ (\cos\theta = 0)$



Example

A body weighing $1400N$ must be moved onto a platform 1 m above the ground. It can be lifted directly, or pushed along an inclined plane of $4m$. Assuming negligible friction between the body and the inclined plane: A) determine the work done in lifting the body 1 m vertically (upward at constant speed); B) determine the work done in pushing the body $4m$ along the inclined plane (force applied parallel to the plane); C) determine the work done by the normal force exerted on the body by the ramp surface. Assume all forces are constant.

Assumere tutte le forze costanti.



$P = 1400N$

$\Delta y = 1m$

$\Delta x = 4m$

$W = \vec{F} \cdot \vec{\Delta y} = F \Delta y \cdot \cos\theta$

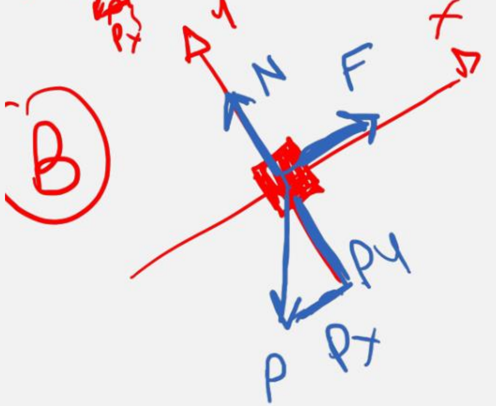
$\Sigma F_y = F - P$

$F = P$

$= mg \cdot \Delta y \cdot \cos\theta$

$1400N \cdot 1m \cdot \cos 0°$

$1400\ N \cdot m = J$

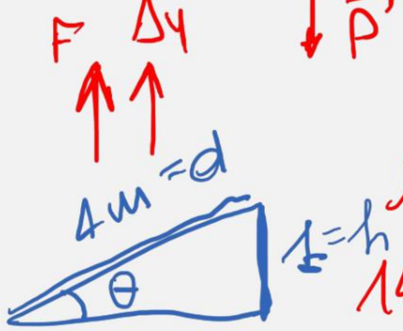


$\Delta x = 4m$

$\Sigma F_x = 0 = F - P_x$

$F - P_x = 0$

$F - mg \sin\theta = 0$



$\theta \Rightarrow \sin\theta = \frac{h}{d} = \frac{1}{4} = 0.25$

$F = mg \sin\theta = 350N$

$F = 350N$

W_F ? $\rightarrow \vec{F} \cdot \vec{\Delta x} = F\Delta x \cos\theta = F\Delta x \cdot \cos 0^\circ = 350N \cdot 4m = \underline{1400} \, N \cdot m = J$

$+1$

(C) W_N ? $N \rightarrow$ suy

$\cos 90^\circ = 0$

$W_N = \vec{N} \cdot \vec{\Delta x} = N \cdot \Delta x \cdot \cos\theta$

$= N \cdot \Delta x \cdot 0$

$= 0$

TOTAL WORK

NET WORK

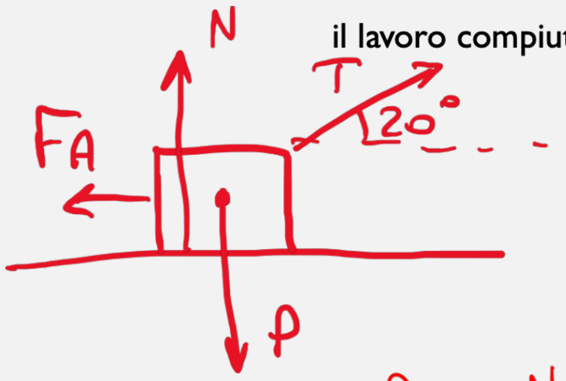
$$W_{TOT} = W_1 + W_2 + \cdots + W_n$$

$$W_{TOT} = F_{TOT} \Delta r \cos \theta$$

Example

We pull a body along a horizontal plane by means of a rope. The mass of the body is $26\ kg$. The rope makes an angle of $20°$ with the horizontal plane. Assume the coefficient of friction between the body and the plane is $\mu_D = 0.16$. The body moves at constant speed $v = 3\ km/h$ and travels $120\ m$: A) what is the work done in pulling the body? B) what is the work done by the forces exerted by the plane? C) what is the total work done on the body?

il lavoro compiuto dalle forze esercitate dal piano? C) qual è il lavoro totale compiuto sul corpo?



$m = 26\ kg$ $\quad d = 120\ m$ $\quad W_T?$ $\quad W_{TOT}?$

$\mu_D = 0.16$ $\quad v = 3\ km/h$ $\quad W_{FA}?$ $\quad W_N?$

$$\Sigma F_x = T_x - F_A = T\cos\theta - \mu_D N = 0$$
$$\Sigma F_y = N - P + T_y = N - P + T\sin\theta = 0$$

$$\Rightarrow \begin{cases} T\cos\theta = \mu_D N \\ N = P - T\sin\theta \end{cases} \Rightarrow \begin{cases} N = \dfrac{T\cos\theta}{\mu_D} \\ N = P - T\sin\theta \end{cases} \Rightarrow \frac{T\cos\theta}{\mu_D} = P - T\sin\theta$$

$$\Rightarrow T\cos\theta = \mu_D(P - T\sin\theta) \Rightarrow T\cos\theta = \mu_D P - \mu_D T\sin\theta \Rightarrow T\cos\theta + \mu_D T\sin\theta = \mu_D P$$

$$\Rightarrow T(\cos\theta + \mu_D \sin\theta) = \mu_D P \Rightarrow T = \frac{\mu_D P}{\cos\theta + \mu_D \sin\theta} = 41.18\ N$$

$$T\cos\theta = 38.71\ N$$

$$W_T = \vec{T} \cdot \vec{d} = Td\cos\theta = 38.71\,N \cdot 120\,m \cong 4645\,J$$

$$F_A = \mu_D N = T\cos\theta \text{ (eq. precedenti)} = 38.71\,N$$

$$W_{FA} = \vec{F_A} \cdot \vec{d} = F_A \cdot \cos\theta \cdot d \cong -4645\,J$$

$$W_{TOT} = 0\,J$$

KINETIC ENERGY

$$W_{TOT} = F_{TOT}\Delta x$$

$$F_{TOT} = m\vec{a}$$

$$W_{TOT} = ma_x\Delta x$$

Constant force constant acceleration

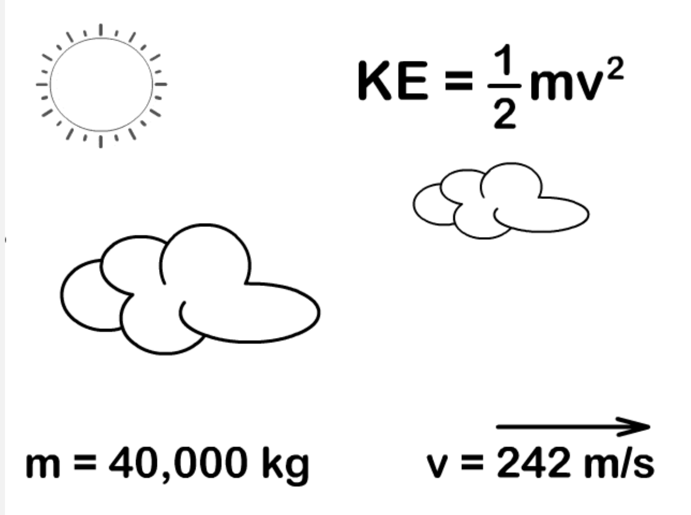
$$v_{fx}^2 - v_{ix}^2 = 2a_x\Delta x$$

$$a_x\Delta x = \frac{1}{2}(v_{fx}^2 - v_{ix}^2)$$

$$W_{TOT} = \frac{1}{2}m(v_{fx}^2 - v_{ix}^2)$$

$$v_f^2 - v_i^2 = (v_{fx}^2 + v_{fy}^2) - (v_{ix}^2 + v_{iy}^2) = v_{fx}^2 - v_{ix}^2$$

$$W_{TOT} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$



KINETIC ENERGY

$$W_{TOT} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

→ $\frac{1}{2}mv^2$

The total work done on the body equals the change in the quantity $\frac{1}{2}mv^2$

TRANSLATIONAL KINETIC ENERGY OF THE BODY K

$$K = \frac{1}{2}mv^2$$

WORK-ENERGY THEOREM (or Theorem of Kinetic Energy)

$$W_{TOT} = \Delta K$$

GRAVITATIONAL POTENTIAL ENERGY

Kinetic energy of the stone: $K_i = \frac{1}{2}mv_i^2$

Work done by gravity: $W_P = -mg\Delta y$

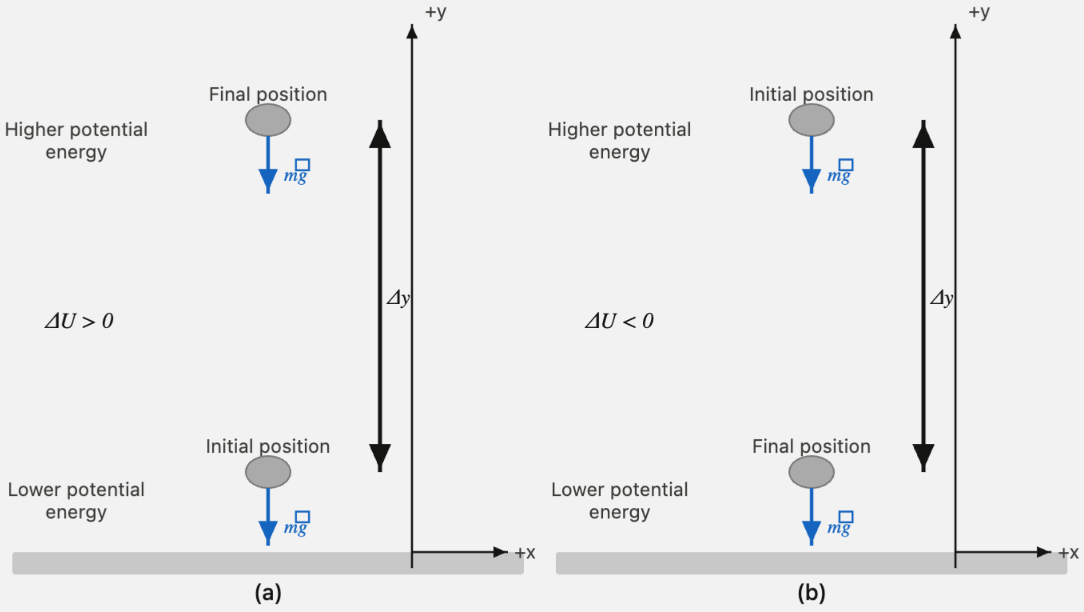
P cos θ Δy
mg cos 180° Δy
- mg Δy

At max height, the stone stops ($K_f = 0$)

$$W_P = K_f - K_i$$

$K_f = 0$

$$-mg\Delta y = -\frac{1}{2}mv_i^2 \rightarrow \Delta y = \frac{v_i^2}{2g}$$



$$v_i^2 = 2gy_{max} \rightarrow t_s = \frac{\sqrt{2gy_{max}}}{g} = \sqrt{\frac{2y_{max}}{g}}$$

Energy stored due to the interaction of a body with something else (here, Earth's gravitational field) that is fully convertible into kinetic energy is called POTENTIAL ENERGY (U)

In this specific case, it is called **GRAVITATIONAL POTENTIAL ENERGY**

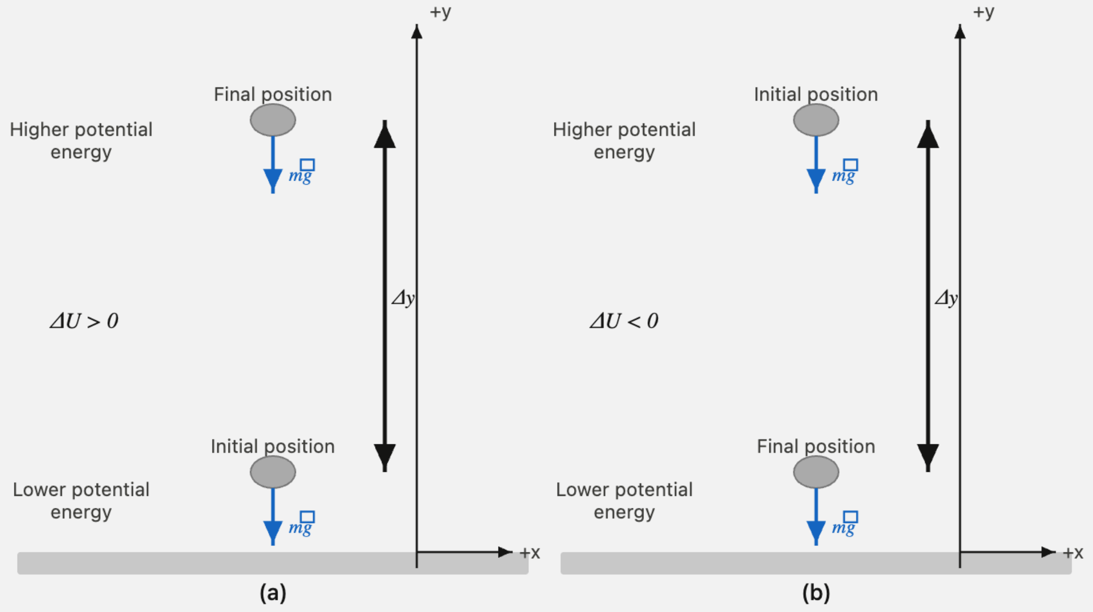
GRAVITATIONAL POTENTIAL ENERGY

Change in gravitational potential energy:

$$\Delta U_{grav} = -W_{grav}$$

$$W_{grav} = \vec{F_g}\Delta\vec{r} = F_g \Delta r \cos\theta = F_{gy}\Delta y = -mg\Delta y$$

$$\Delta U_{grav} = mg\Delta y$$



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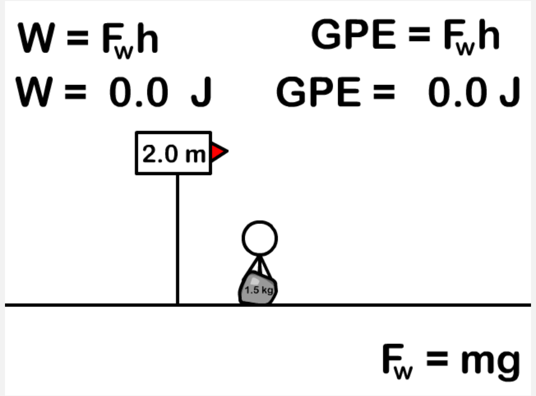
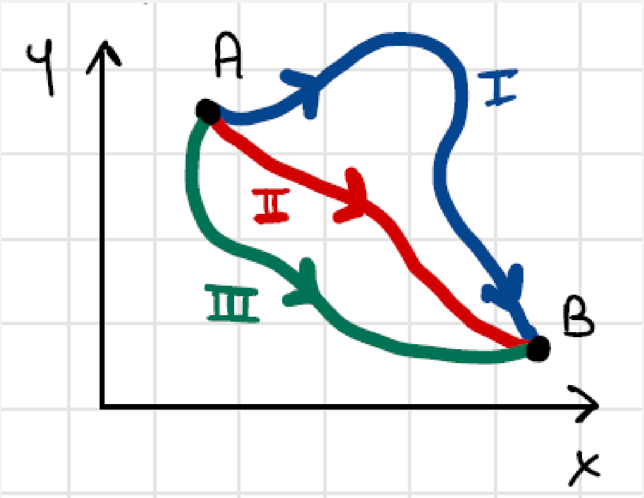
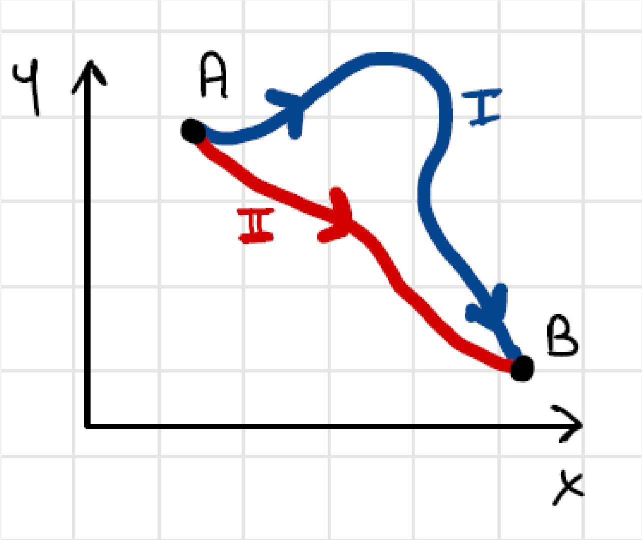


CONSERVATIVE FORCES



A force is defined as CONSERVATIVE if the work it does when acting on a body moving along a certain path (or route, C) from A to B does not depend on the path, but only on A and B:

$$W_{I(A,B)} = W_{II(A,B)} = W_{III(A,B)}$$

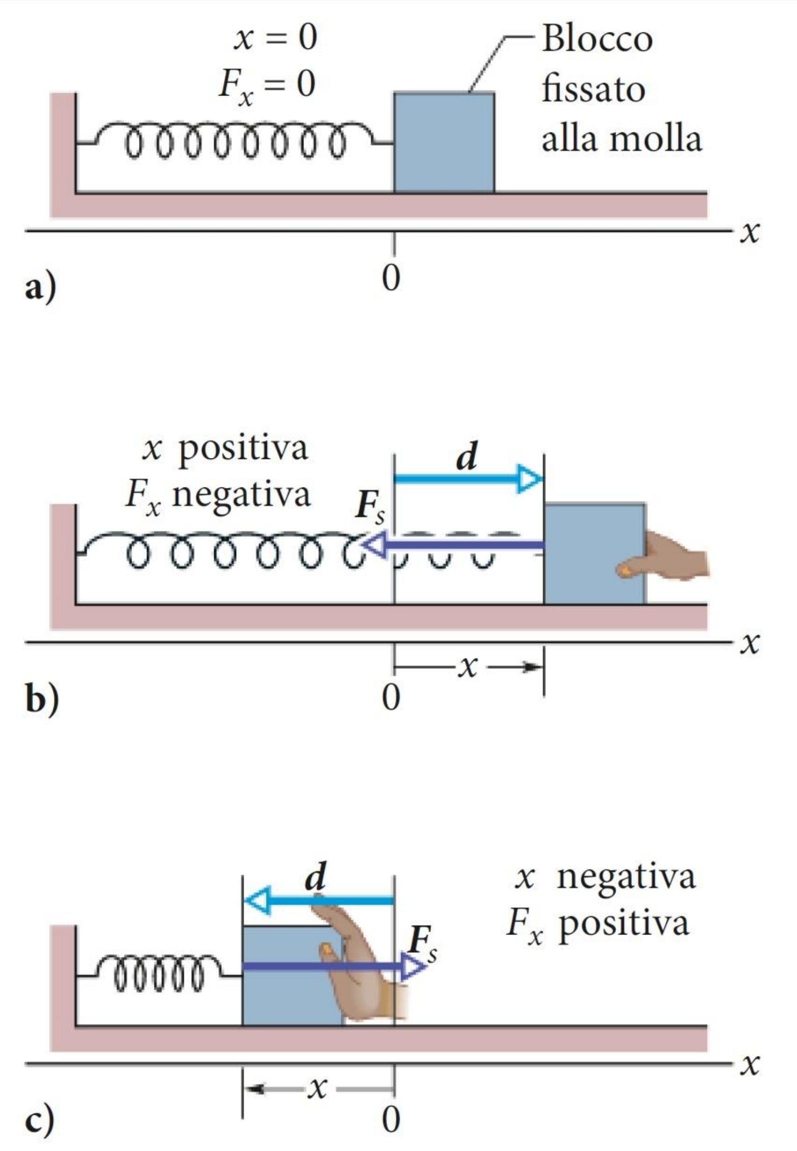


If a force is conservative, the work it does along a closed path (CYCLE) is <u>zero</u>:

$$W_{ABA} = 0$$

A force is defined as CONSERVATIVE if the work it does along a closed path, called a cycle, is zero.

ELASTIC FORCE



To keep a spring stretched or compressed by an amount x from its equilibrium position, a force must be applied $\overrightarrow{F_P} \propto x$

$$F \quad = kx$$

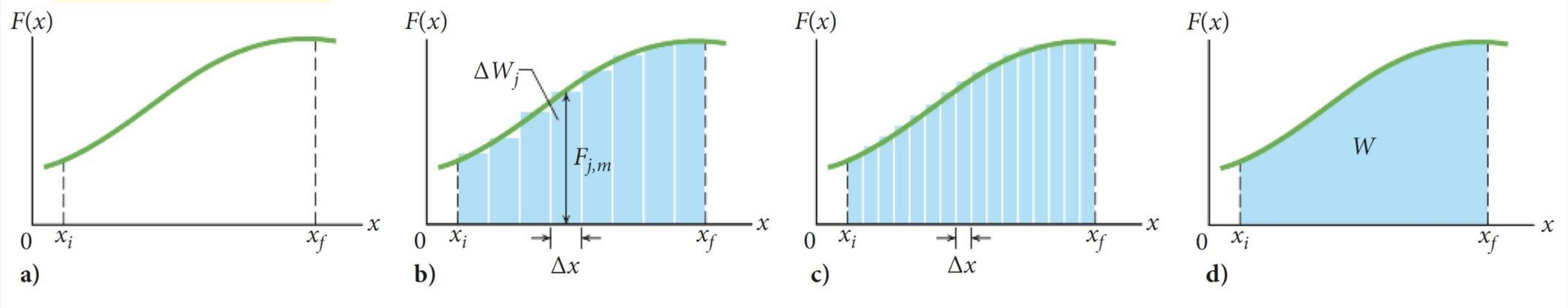
The spring exerts a force in the opposite direction (RESTORING FORCE, or ELASTIC FORCE), which tends to return the spring to its equilibrium length:

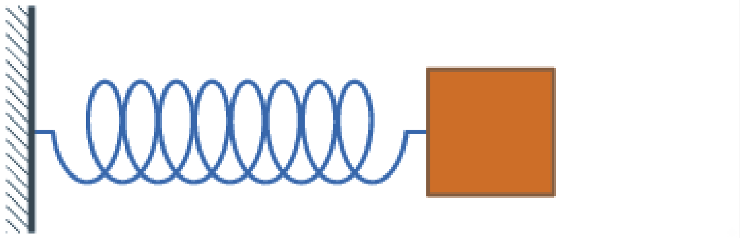
Hooke's law → $F_S = -kx$

ELASTIC FORCE

The force to compress/stretch the spring <u>is not constant</u>:

$$\Delta W_i = F_{i,x}\Delta x_i$$

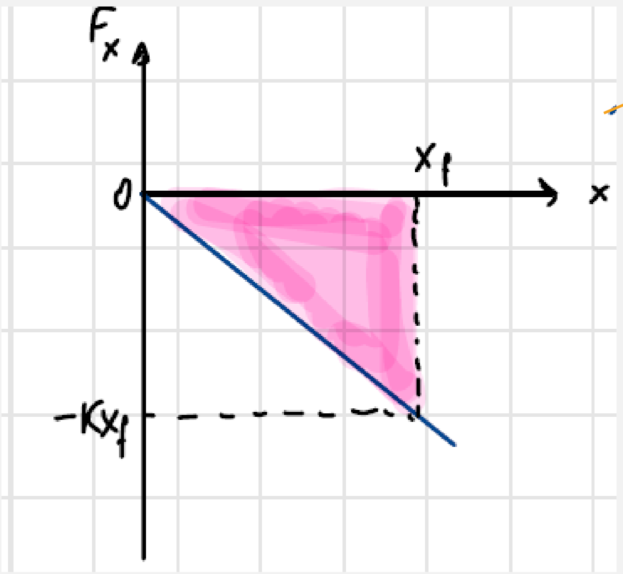




The work done by a non-constant force $\vec{F}$ acting on a body whose displacement is $\Delta\vec{x}$:

$$W = \lim_{\Delta x \to 0} \sum F_{x,i}\Delta x_i = \int F_x dx$$

ELASTIC FORCE



The work equals the area under the curve between x_i and x_f
triangle area $base = x_f = x, height = -kx_f = -kx$:

$$W = \frac{1}{2} base \times height = -\frac{1}{2} kx^2$$

$$W = \int F_x dx \longrightarrow W = \int_{x_i}^{x_f} F_x dx = \int_{x_i}^{x_f} \overbrace{-kx}^{F} dx = -k \int_{x_i}^{x_f} x dx$$

Since: $\int x dx = \frac{1}{2} kx^2,$

$W = -\frac{1}{2} k(x_f^2 - x_i^2)$ con $x_i = 0, x_f = x$

$$W = -\frac{1}{2} kx^2$$

In general: $W_{elast} = \left(-\frac{1}{2} kx_f^2\right) - \left(-\frac{1}{2} kx_i^2\right) = -\frac{1}{2} kx_f^2 + \frac{1}{2} kx_i^2$

ELASTIC POTENTIAL ENERGY

The change in ELASTIC POTENTIAL ENERGY equals the work done by the spring with opposite sign:

$$\Delta U_{elast} = -W_{elast}$$

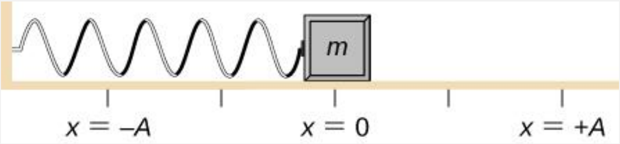
$$\Delta U_{elast} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

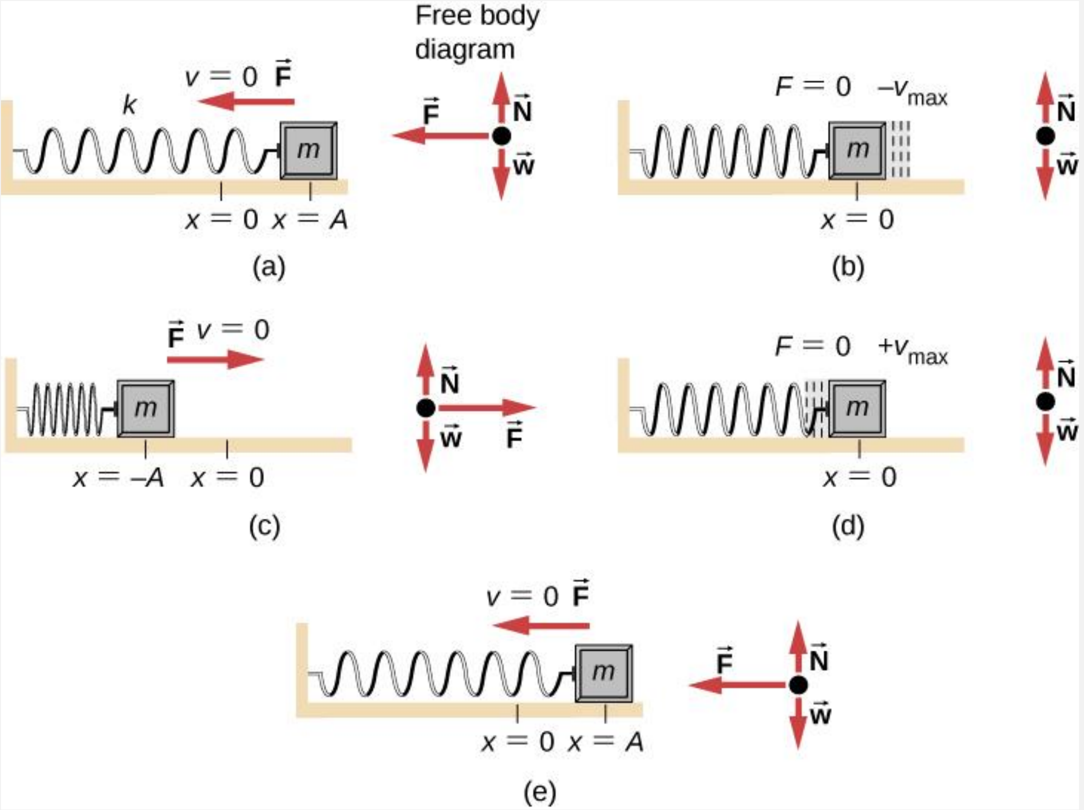
Considering $U = 0$ at the equilibrium position ($x = 0$):

$$U_{elast} = \frac{1}{2}kx^2$$

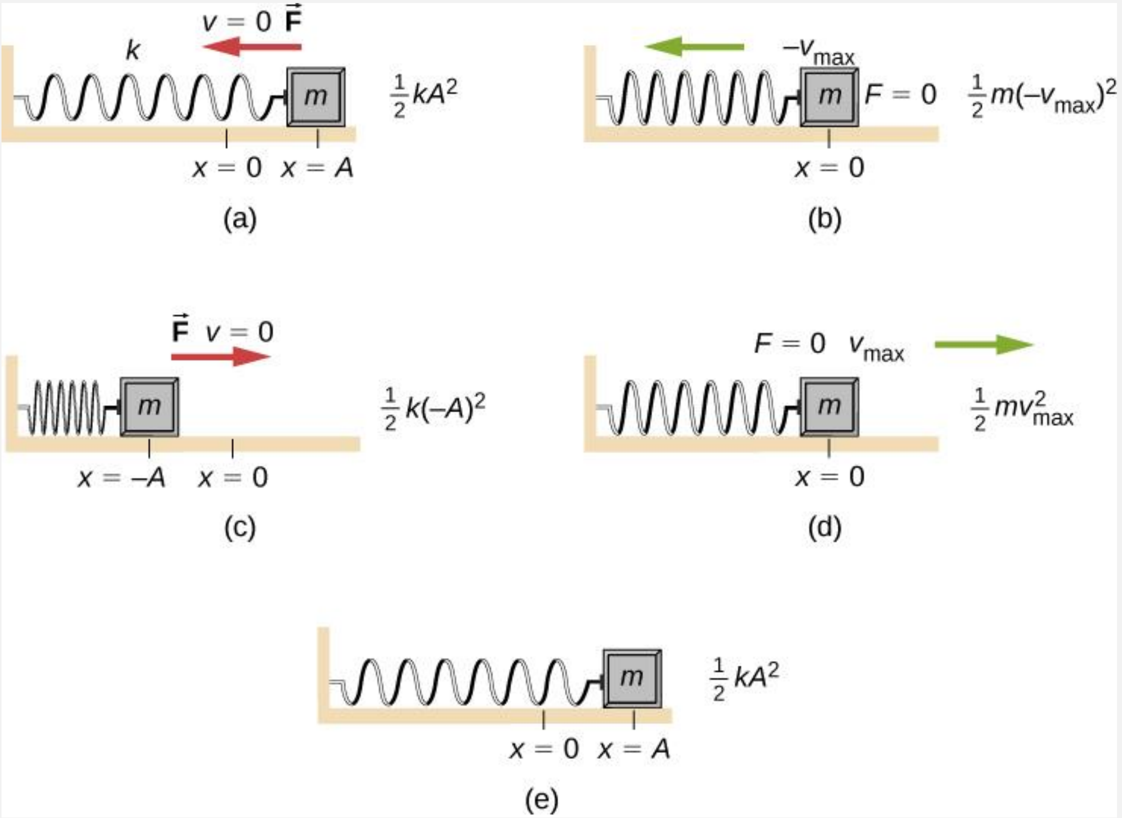
Elastic potential energy stored in an ideal spring

ELASTIC FORCE AND ELASTIC POTENTIAL ENERGY



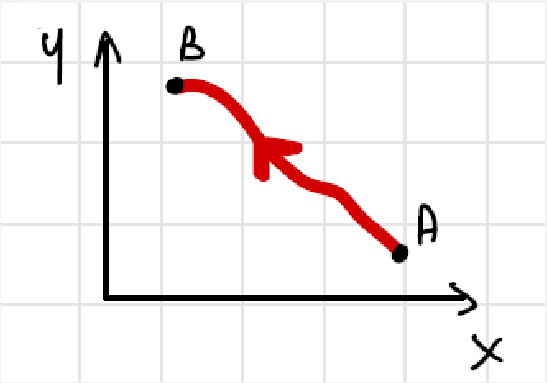


Analysis of the elastic force



Analysis of elastic potential energy

MECHANICAL ENERGY AND ITS CONSERVATION



For both conservative and non-conservative forces:

$$W_{A,B} = K(B) - K(A)$$

Only for conservative forces:

$$\begin{cases} W_{A,B} = U(A) - U(B) \\ W_{A,B} = K(B) - K(A) \end{cases}$$

← W

← T. forze vive

$$U(A) - U(B) = K(B) - K(A)$$

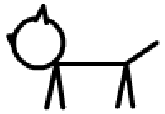
$$U(A) + K(A) = U(B) + K(B)$$

The sum of potential energy and kinetic energy is called MECHANICAL ENERGY

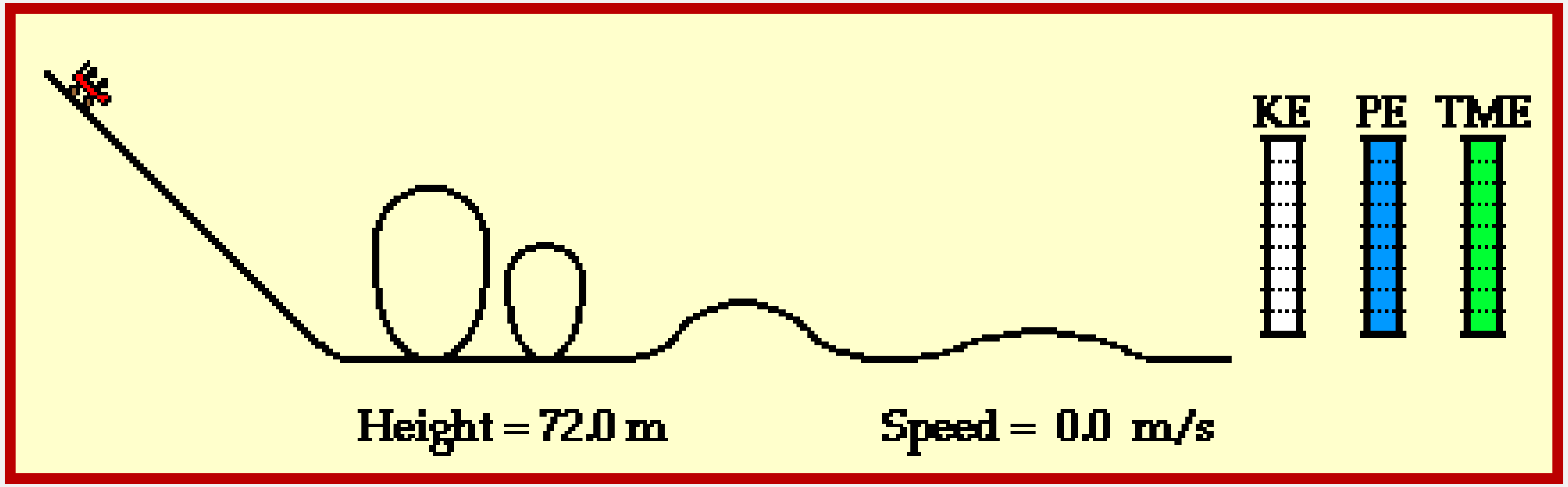
MECHANICAL ENERGY is conserved when we are in a conservative force field

Mechanical Energy:
20,000J

Kinetic Energy:
0J



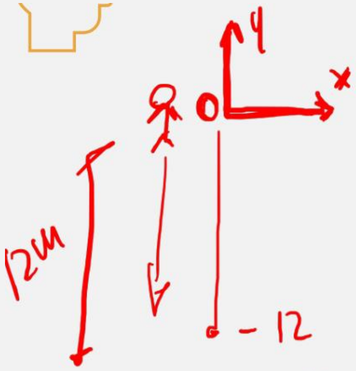
Potential Energy:
20,000J



Example

A climber descends a cliff of $12m$. The climber has mass $60kg$ and descends from rest, sliding along a vertical rope. Reaches the ground with a speed of $2m/s$: determine the energy dissipated by friction with the rope (the local value of g is $9.78\ N/kg$; ignore air resistance).

una fune verticale. Arriva a terra con una velocità di $2m/s$: determinare l'energia dissipata dall'attrito nel contatto con la fune (il valore locale di g è $9.78\ N/kg$; ignorare la resistenza dell'aria).



$m = 60 kg$

$v_i = 0 m/s$

$v_f = 2 m/s$

$g = 9.78\ m/s^2 = 9.78\ N/kg$

$\Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$= \frac{1}{2} \cdot 60 kg \cdot (2 m/s)^2 - 0 = +120 J$

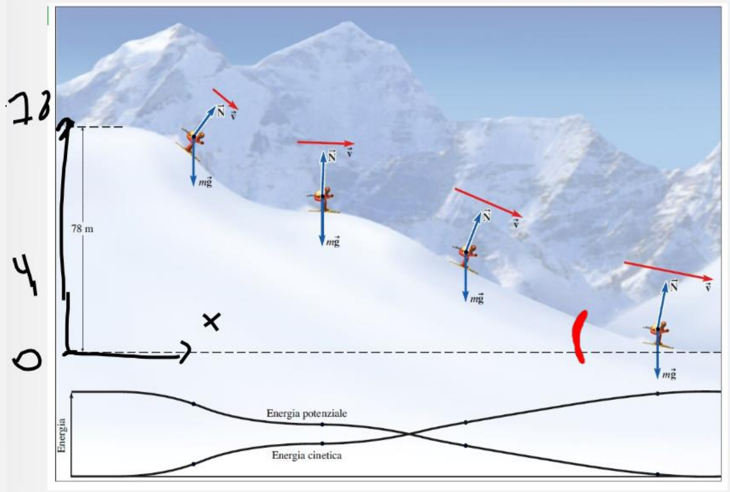
$\Delta U = U_f - U_i = mg\Delta y = 60 kg \cdot 9.78\ N/kg \cdot (-12 m) = -7040 J$

$W_{DISSIP.} = W_{FA} = \Delta E_{MECC} = \Delta K + \Delta U = +120 J + (-7040 J) = -6920 J$

e. dissip. → attrito

Example

A ski slope has a vertical drop of $78\ m$. An inexperienced skier, unable to control their speed, descends this slope. Neglecting friction and air resistance, what will their speed be at the bottom of the slope?



$$U_i + K_i = U_f + K_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$v_i = 0$ $\quad h_f = 0$

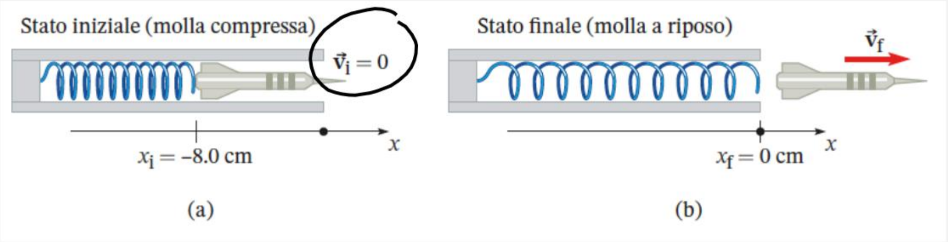
$$mgh_i = \frac{1}{2}mv_f^2$$

$$gh_i = \frac{1}{2}v_f^2 \rightarrow v_f = \sqrt{2gh_i} = \sqrt{2 \cdot 9.8\ m/s^2 \cdot 78\ m}$$

$$= 39\ m/s$$

Example

In a spring-loaded dart gun, the spring with $k = 400.0\ N/m$ is compressed by $8.0\ cm$ when the dart (of mass $20.0\ g$) is inserted. What is the launch speed of the dart when the spring is released? Neglect friction forces.



$U \leftrightarrow K$

el. $\rightarrow \frac{1}{2}kx^2$; $\rightarrow \frac{1}{2}mv^2$

$$U_i + K_i = U_f + K_f$$

$$\frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2 \Rightarrow \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2$$

$v_i = 0$, $x_f = 0$

$$v_f = \sqrt{\frac{\frac{1}{2}kx_i^2}{\frac{1}{2}m}} = \sqrt{\frac{\frac{1}{2}\cdot 400\cdot(0.08)^2}{\frac{1}{2}\cdot 0.2\ kg}} =$$

$$v_f = \sqrt{\frac{k}{m}\cdot x_i^2} = \sqrt{\frac{400\ N/m}{0.02\ kg}\cdot 0.0064\ m^2} = 11\ m/s$$