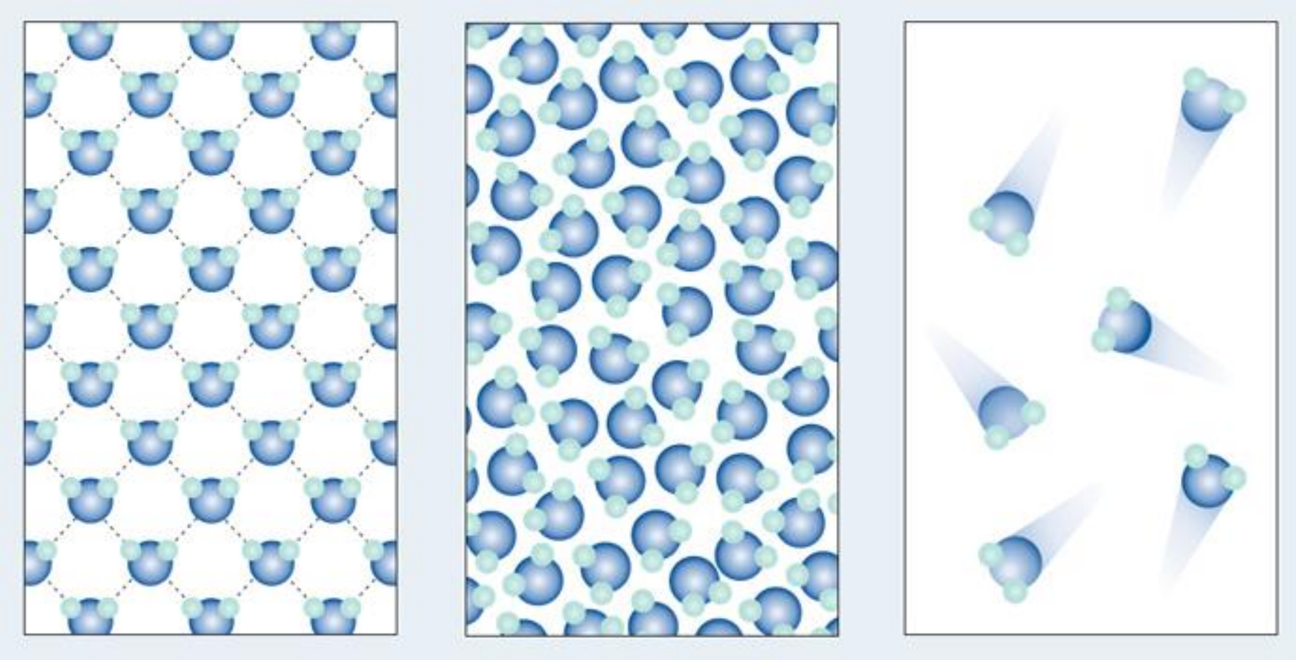
FLUIDS

- ☞ **FLUID STATICS**
- ☞ **FLUID DYNAMICS**

FLUIDS

FLUID: a material that does not have its own shape, but takes the shape of the container that holds it → gases and liquids

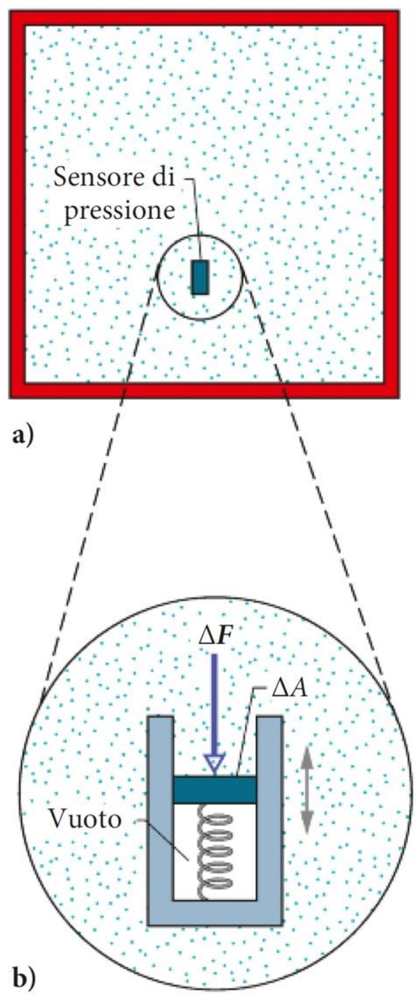


states of matter or **phases of matter**

DENSITY

Table 1. Density of some common substances (kg/m³)

Substance	Density (kg/m³)
Air	1.29
Oxygen	1.43
Expanded polystyrene	100
Balsa wood	120
Cherry wood	800
Ethyl alcohol	806
Olive oil	920
Ice	917
Fresh water	1 000
Sea water	1 025
Ebony wood	1 220
Aluminium	2 700
Iron	7 860
Silver	10 500
Lead	11 300
Mercury	13 600
Gold	19 300

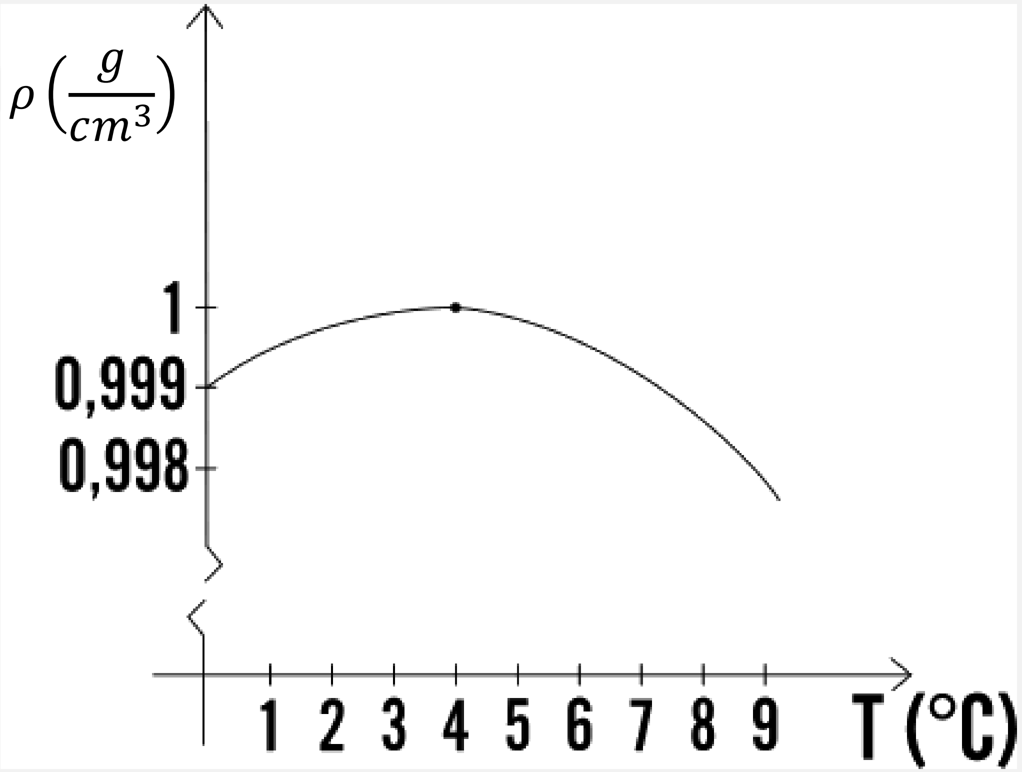


$$\rho = \mathrm{d} = \frac{\Delta m}{\Delta V} = \frac{m}{V}$$

uniform density

$$[\rho] = \frac{Kg}{m^3} \quad o \quad \frac{g}{cm^3}$$

$$\rho_{H_2O} = 1000\frac{Kg}{m^3} = 1000\frac{10^3 g}{(10^2 cm)^3} = 1000\ 10^{-3}\frac{\mathrm{g}}{\mathrm{cm}^3} = 1\frac{\mathrm{g}}{\mathrm{cm}^3}$$

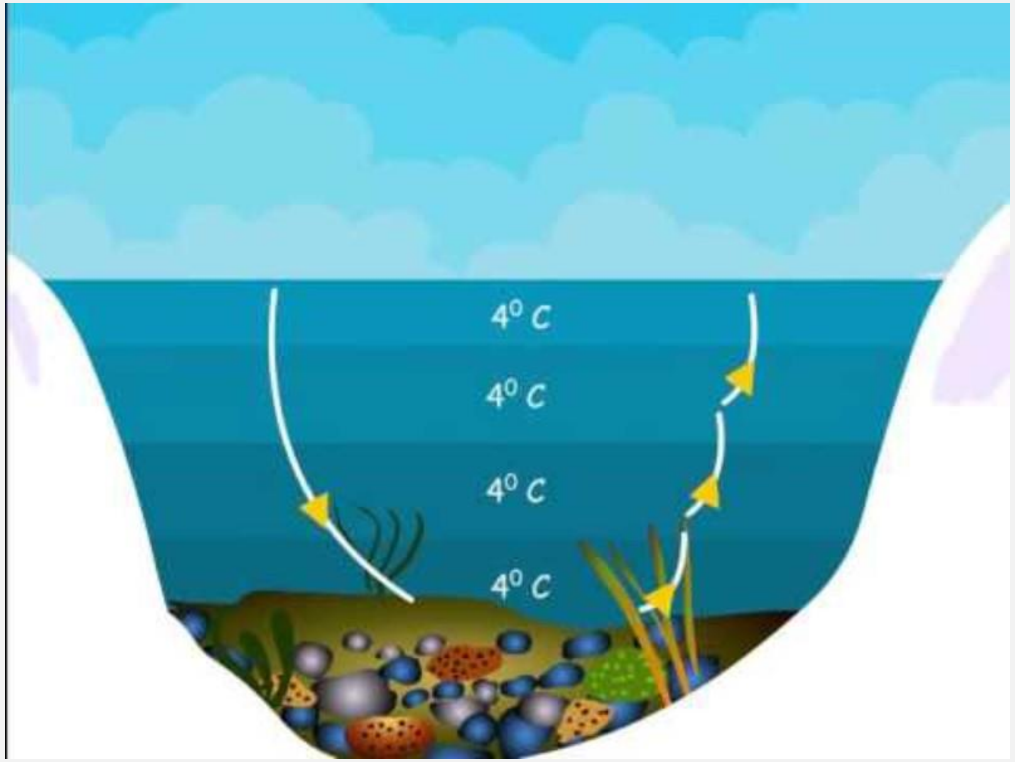


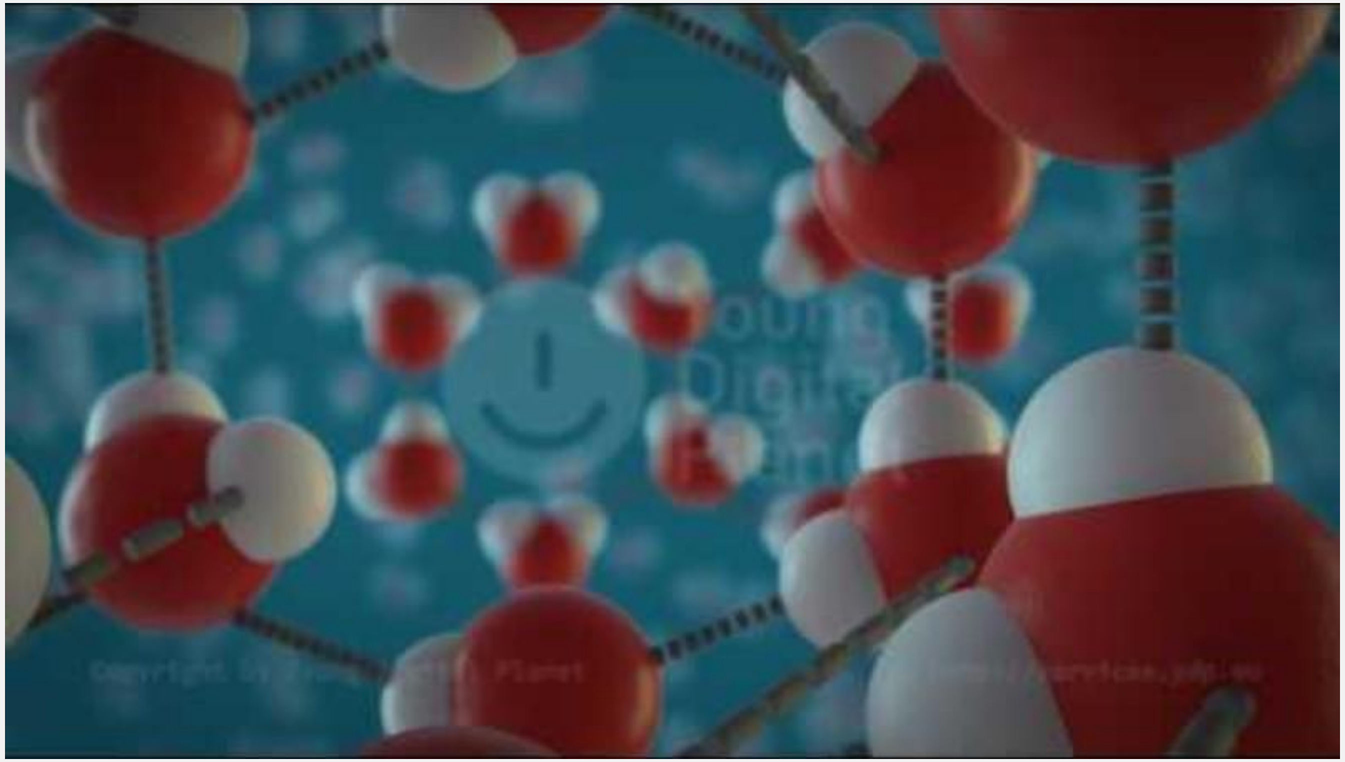
Density in the solid state > density in the liquid state

EXCEPTION: WATER!

Between 0°C and 4°C, water exhibits a peculiar behavior:

- At the freezing point of 0°C, frozen water molecules are farther apart, making ice lighter and more voluminous
- As the temperature increases, the crystal lattice breaks → the lattice no longer exists, but the molecules are organized in more heterogeneous structures
- At 4°C, water molecules are packed as closely together as possible → maximum density
- Above 4°C, the molecules move very rapidly, as in other liquids → density decreases


4⁰ C
4⁰ C
4⁰ C
4⁰ C



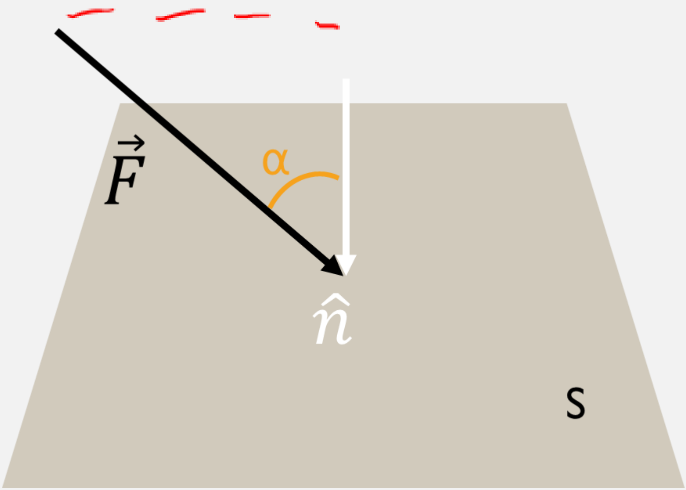
PRESSURE

$$P = \frac{\vec{F} \cdot \hat{n}}{S} = \frac{F cos\alpha}{S}$$

Where α is the angle between $\vec{F}$ e $\hat{n}$

$$[P] = \frac{N}{m^2} = Pa$$

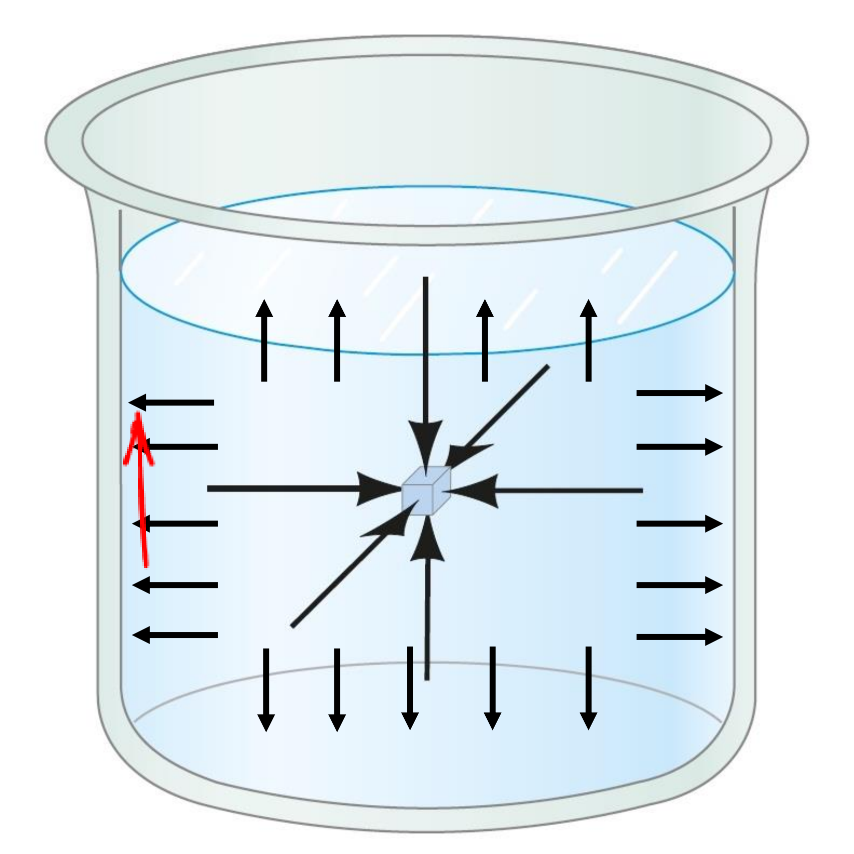
The pressure is a scalar quantity!



$F cos\alpha$ is the component of the force perpendicular to the surface S

The pressure of a fluid will depend on the point where we are located → on the depth at which we are located!

$$[P] = \frac{N}{m^2} = Pa$$

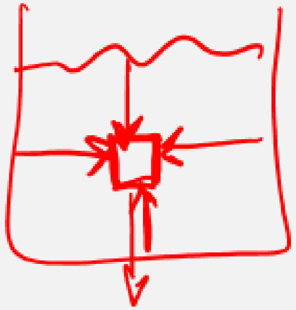


	Atm	Bar	mbar	mmHg	Pa	MPa
Atm	1	1.013	1013	760	101325	0.1013
Bar	1.013	1	10^{-3}	750.062	10^{5}	10
mbar	1013	10^{3}	1	0.75006	10^{2}	10^{-4}
mmHg	760	0.00133	1.3322	1	133.222	7500.62
Pa	101325	10^{5}	10^{2}	0.0075	1	10^{6}
MPa	0.1013	10^{-1}	10^{-4}	7500.6	10^{6}	1

FLUID STATICS

$\Sigma F = ma = 0$

For each small volume of fluid: $$\vec{R} = \sum_i \vec{F}_i = 0$$



The forces acting on a small volume of fluid are:

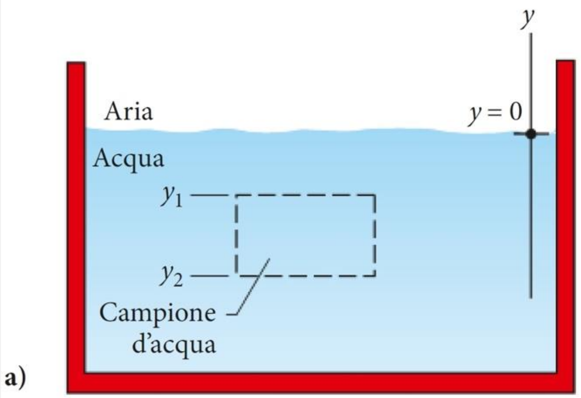
Body forces
gravitational force $mg = \rho V g$,
electric force if the particles are ions immersed in an external electric field

Surface forces
pressure forces $F_S = P \cdot S$

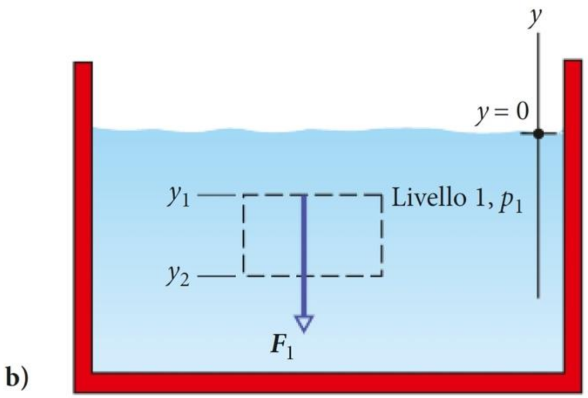
small volume in equilibrium → the sum of all body forces and all surface forces equals 0

FLUID STATICS

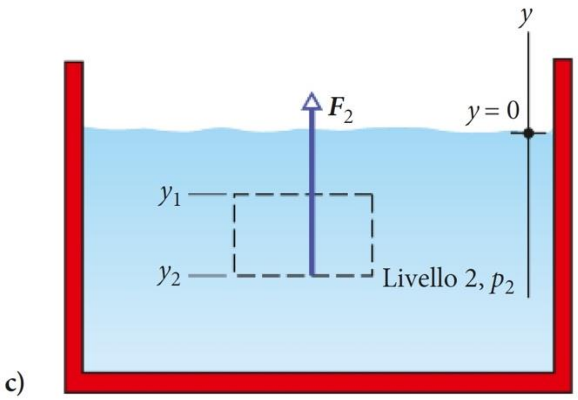
three forces act on this water sample



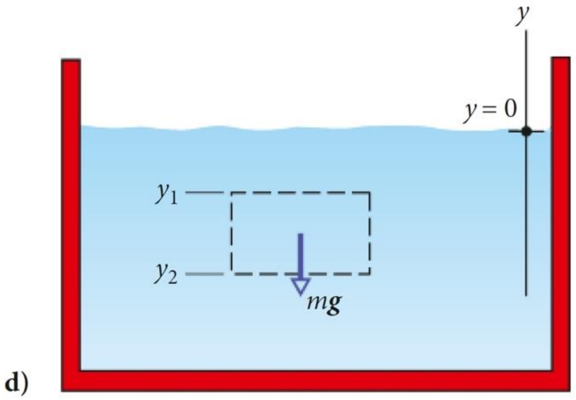
the force downwards is due to the water pressure pushing on the *upper* surface

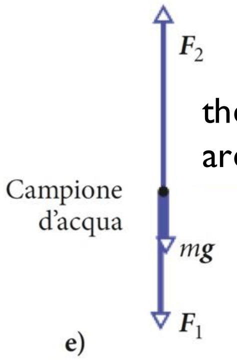


the force upwards is due to the water pressure pushing on the *lower* surface



gravity pulls the water sample downwards



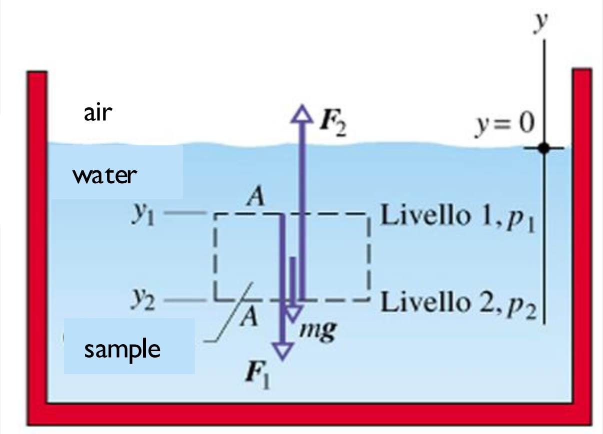


the three forces are in equilibrium

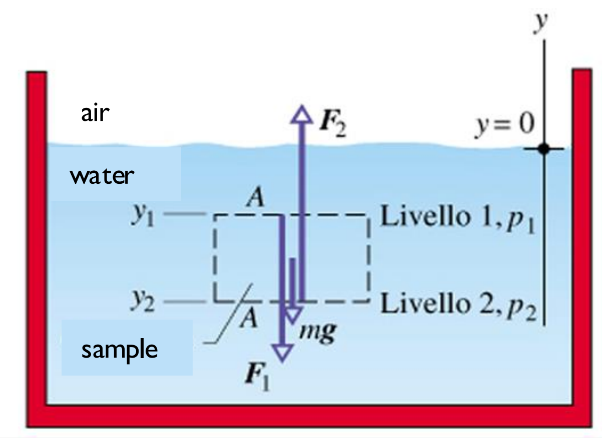
On the upper face (Level 1, depth y_1) of the parallelepiped acts a force $|F_1| = p_1 \cdot A$ directed downward, surface force

On the lower face (Level 2, depth y_2) of the parallelepiped acts a force $|F_2| = p_2 \cdot A$ directed upward, surface force

The gravitational force, body force, mg acts on the center of mass.



STEVIN'S LAW



THE PARALLELEPIPED IS IS AT REST, THEREFORE THE RESULTANT OF THE FORCES APPLIED TO IT MUST EQUAL 0

$$F_w = -mg \qquad F_1 = -p_1 A \qquad F_2 = p_2 A$$

$$0 = \vec{F_P} + \vec{F_1} + \vec{F_2} = -mg - p_1 A + p_2 A$$

$$0 = -\rho V g - p_1 A + p_2 A$$

$$0 = -\rho(y_1 - y_2)Ag - p_1 A + p_2 A$$

$$0 = -\rho(y_1 - y_2)g - p_1 + p_2$$

$$p_2 = p_1 + \rho(y_1 - y_2)g$$

$$p_2 - p_1 = \rho(y_1 - y_2)g$$

$y_1 - y_2 = h = \Delta y$

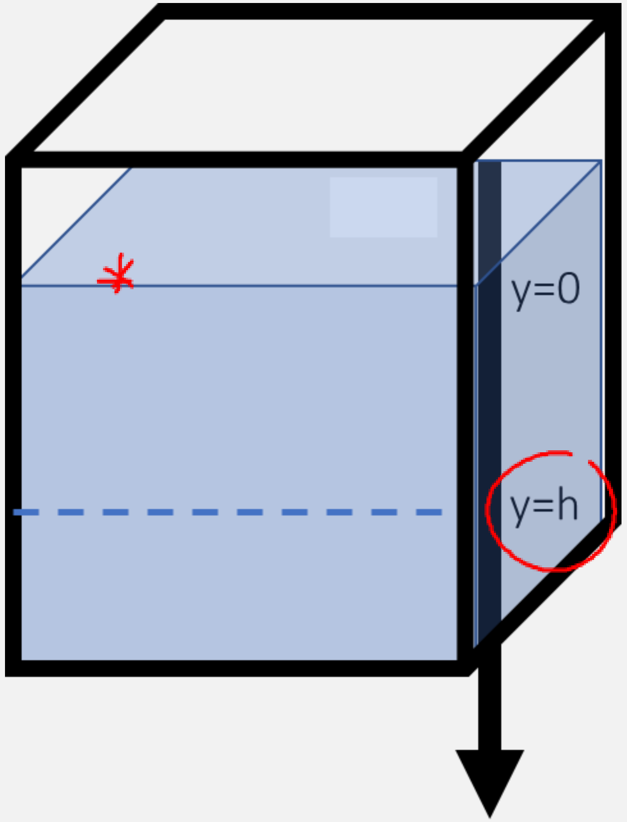
IN A FLUID IN EQUILIBRIUM THE PRESSURE VARIATION IS PROPORTIONAL TO THE VARIATION IN DEPTH AND TO THE DENSITY OF THE FLUID

STEVIN'S LAW

IN A FLUID IN EQUILIBRIUM THE PRESSURE VARIATION IS PROPORTIONAL TO THE VARIATION IN DEPTH AND TO THE DENSITY OF THE FLUID

- This is a general law valid for fluids (including Earth's atmosphere)!
- The pressure at a point in a fluid in static equilibrium depends only on the depth of that point
- The pressure does not depend on any horizontal dimensions of the fluid or its container

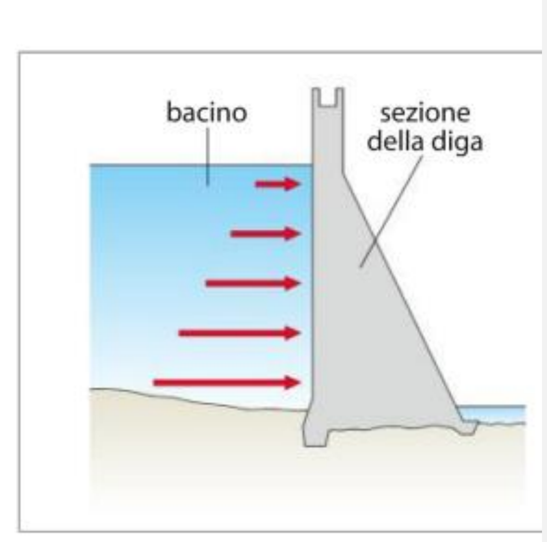
$$p(h) = p_{ext} + \rho g h$$



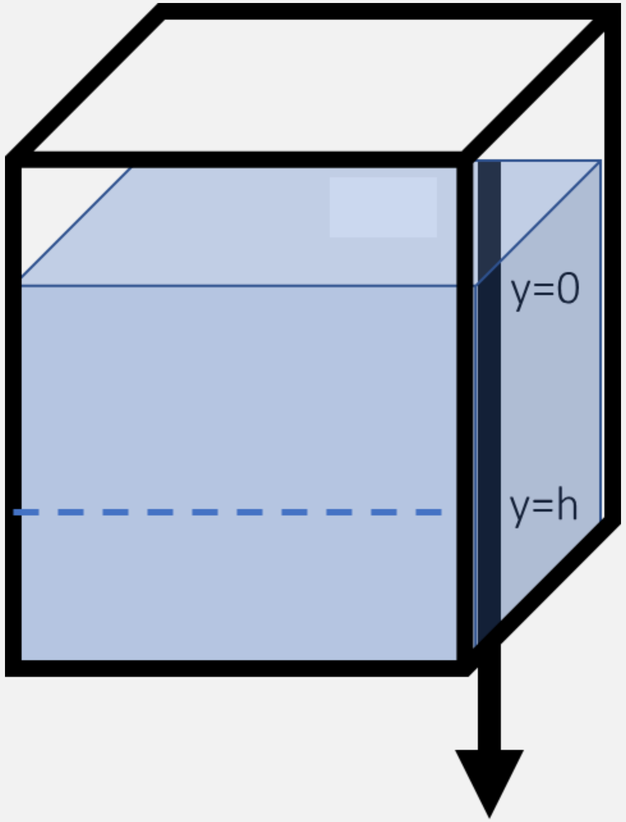
STEVIN'S LAW

IN A FLUID IN EQUILIBRIUM THE PRESSURE VARIATION IS PROPORTIONAL TO THE VARIATION IN DEPTH AND TO THE DENSITY OF THE FLUID

the pressure exerted by a fluid is transmitted to the walls of its container. Pressure and, hence, the force against the walls, increase with depth



on the walls of a dam, the exerted forces are as higher as higher is depth. for this reason, the thickness of the dam must increase towards the bottom.



$$p(h) = p_{ext} + \rho g h$$

PASCAL'S PRINCIPLE

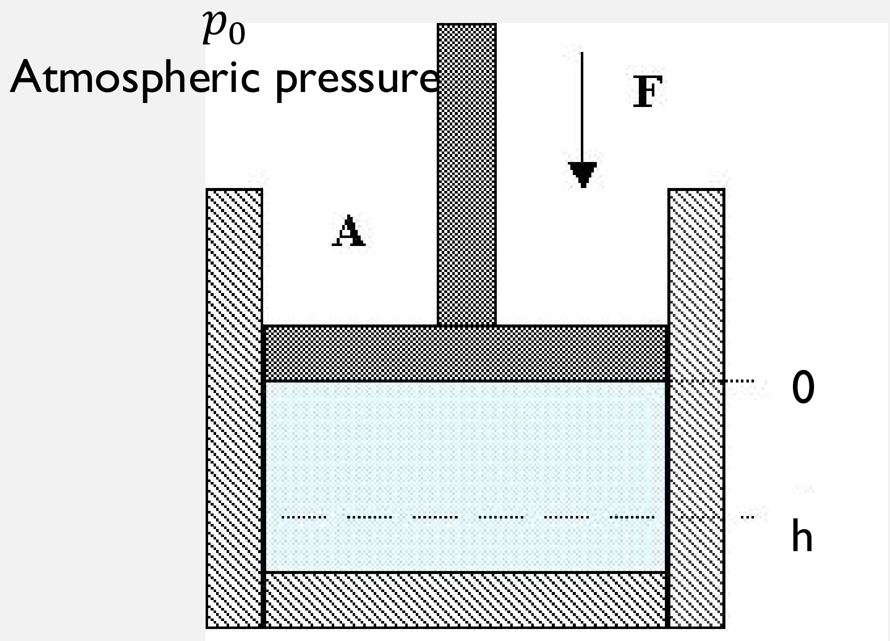
A CHANGE IN PRESSURE APPLIED TO AN INCOMPRESSIBLE CONFINED FLUID IS TRANSMITTED UNDIMINISHED TO EVERY PORTION OF THE FLUID AND TO THE WALLS OF THE CONTAINER

This is an immediate consequence of Stevin's law

Without piston: $$p(h) = p_0 + \rho g h$$

With piston:

$$p(h) = p_{ext} + \rho g h = p_0 + \frac{F}{A} + \rho g h$$



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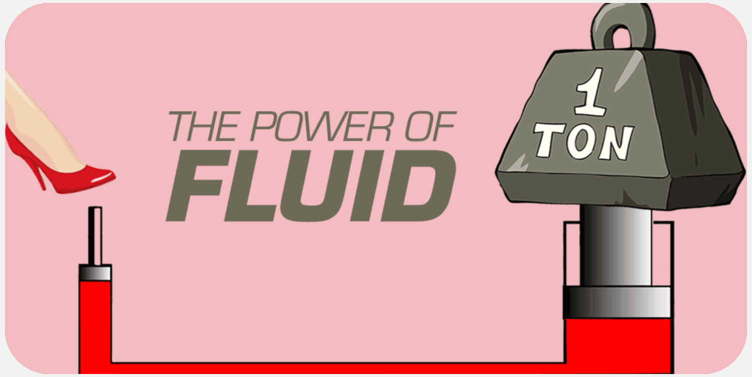
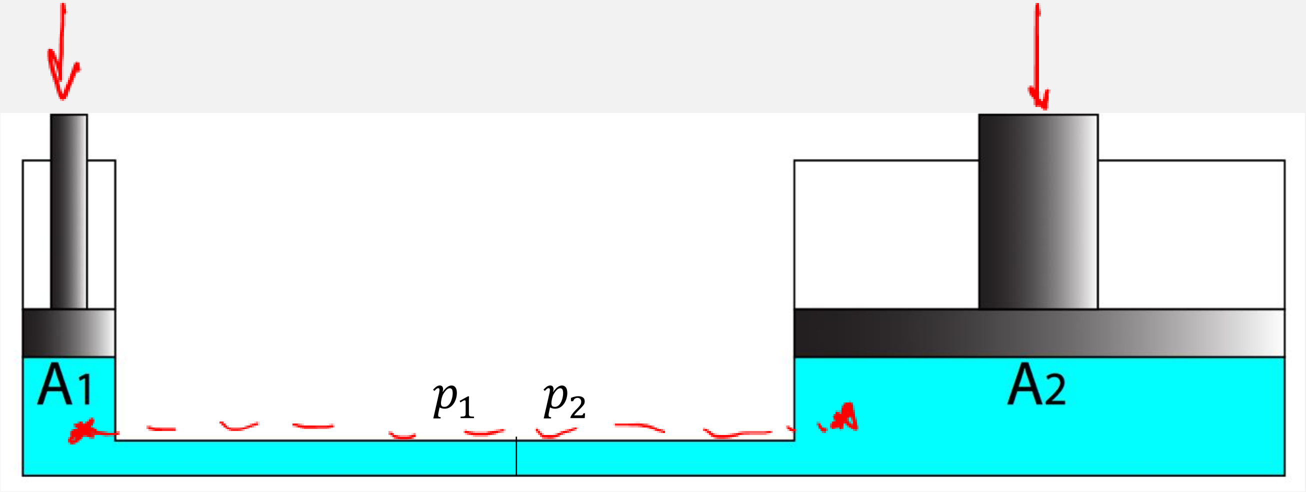




$$p_0 + \frac{F_1}{A_1} + \rho g h_1 = p_0 + \frac{F_2}{A_2} + \rho g h_2$$

In the case where $h_1 = h_2$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad F_1 = \frac{F_2 A_1}{A_2} \qquad \frac{F_2}{F_1} = \frac{A_1}{A_2}$$

TORRICELLI'S EXPERIMENT

kg/m³

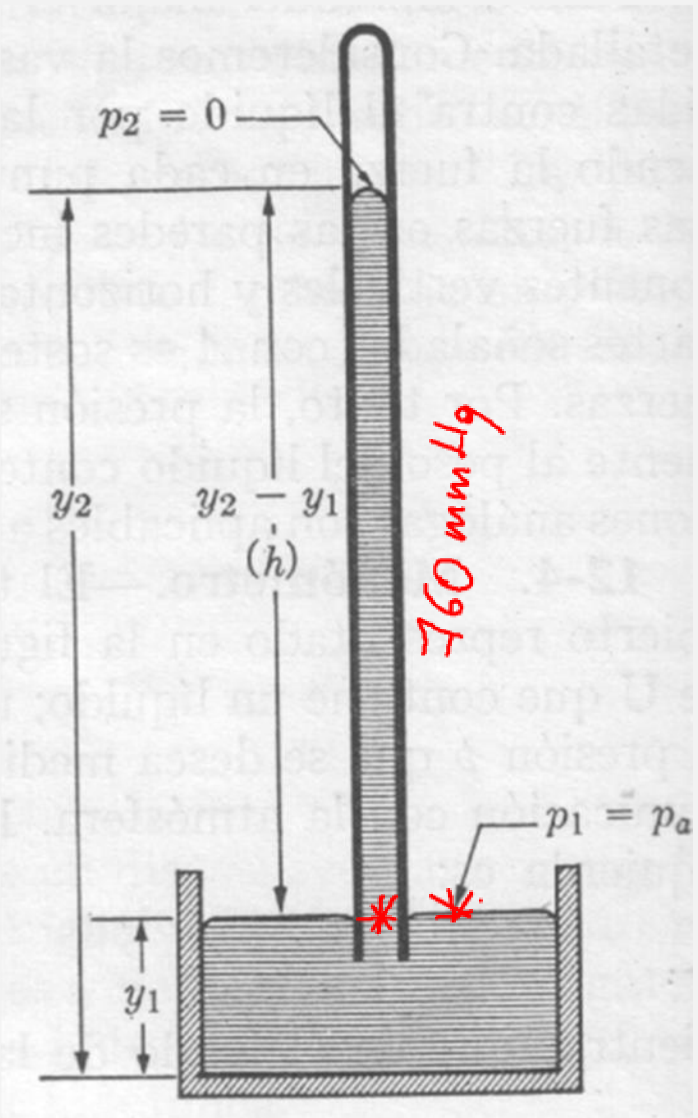
$$p = \rho g h = (13590 \cdot 9.8 \cdot 0.76)\ Pa \approx 101300\ Pa = 1.013 \cdot 10^5\ Pa = 1\ atm$$

$p_1 = p_a + \rho g h$

$p_1^* = \rho g h + p_{ext}$

1 atm = 760 mmHg

$$h_{H_2O} = \frac{1.013 \cdot 10^5}{9.8 \cdot 10^3} = 10.337\ m$$

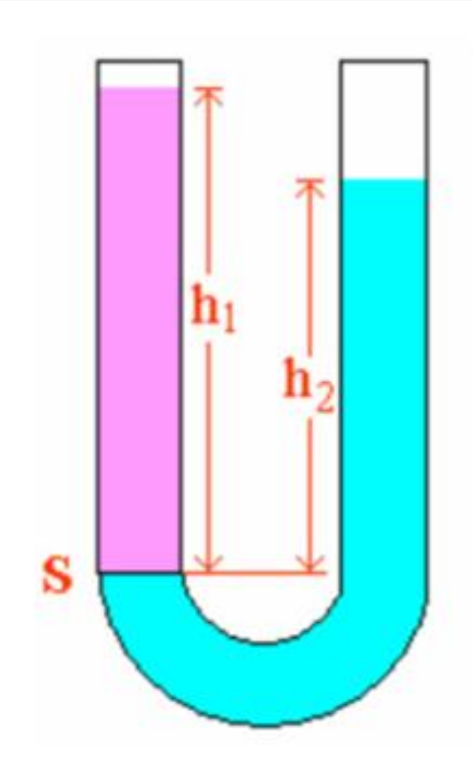


COMMUNICATING VESSELS

TWO IMMISCIBLE LIQUIDS IN COMMUNICATING VESSELS REACH HEIGHTS INVERSELY

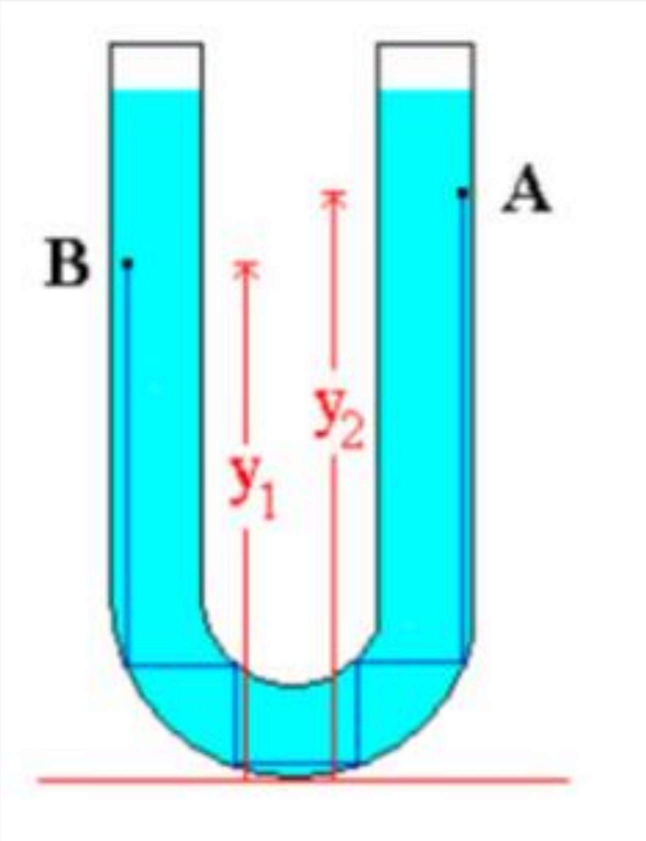
PROPORTIONAL TO THEIR DENSITY

Consequence of Stevin's law



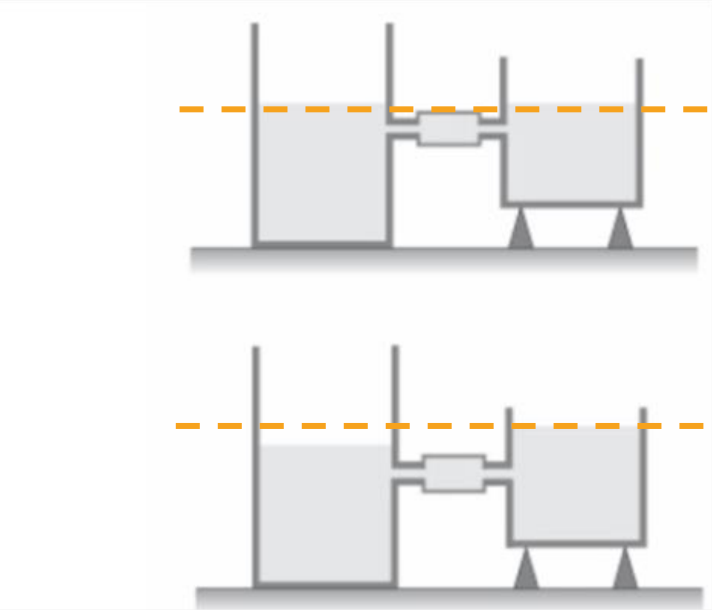
$$\cancel{p_0} + \rho_1 g h_1 = \cancel{p_0} + \rho_2 g h_2 \Rightarrow \frac{\rho_2}{\rho_1} = \frac{h_1}{h_2}$$

COMMUNICATING VESSELS



$$p_2 - p_1 = -\rho g(y_2 - y_1)$$

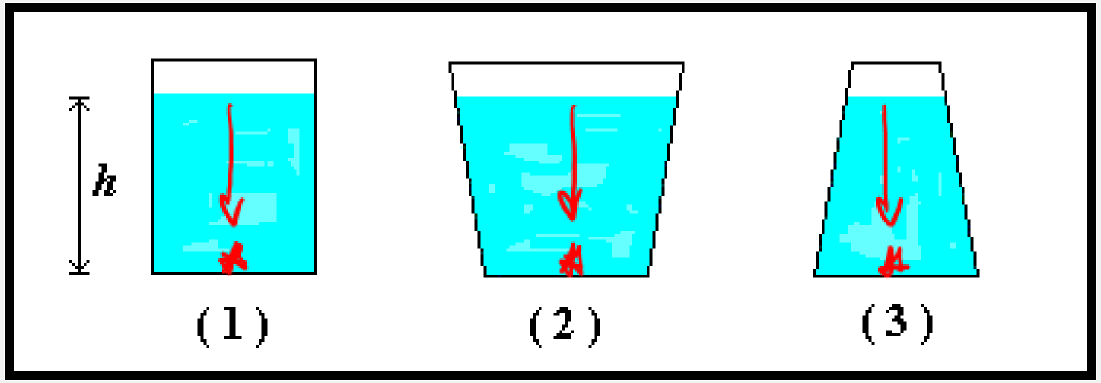
$$p_2 = p_1 \qquad y_2 = y_1$$



Equilibrium

Non-Equilibrium

COMMUNICATING VESSELS AND HYDROSTATIC PARADOX



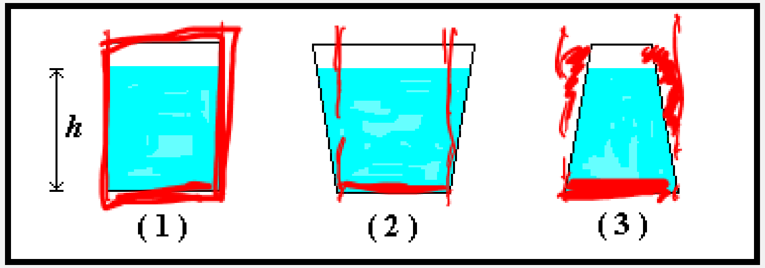
The pressure at a point inside a liquid contained in a vessel does not depend on the shape of the vessel

The three vessels are different, but have the same base and are filled to a height *h*.
The pressure at the bottom of each vessel due to the weight of the liquid, according to Stevin's law, takes the same value $\rho g h$

The force acting on the bottom equals $F = p \cdot A$

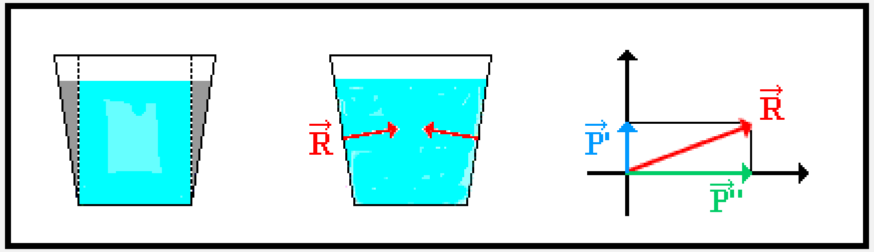
$$F = \rho g h \cdot A = \rho g V = m g = P$$

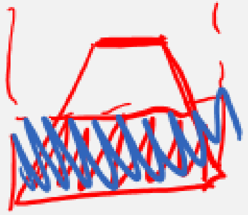
COMMUNICATING VESSELS AND HYDROSTATIC PARADOX

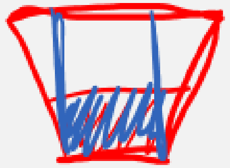


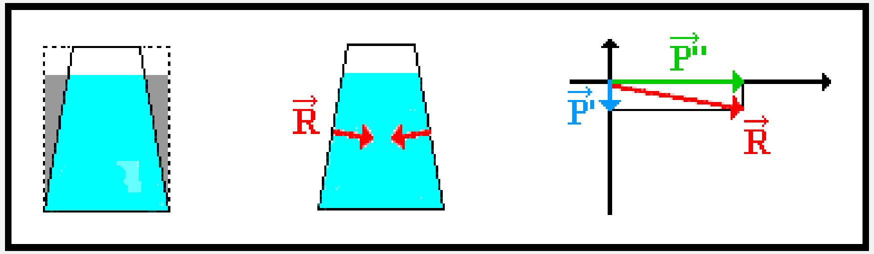
$$F = \rho g h \cdot A = \rho g V = m g = P$$

Hydrostatic paradox: although the weight of the liquid in the various vessels differs from case to case, the force exerted on the bottom is the same in all three cases and equals the weight of the liquid in vessel (1)









HYDROSTATIC PARADOX

Demonstration of the hydrostatic paradox: Pascal's barrel

FIG. 41.—Hydrostatic paradox. Pascal's experiment.

PRESSURE MEASUREMENT

Open-tube manometer

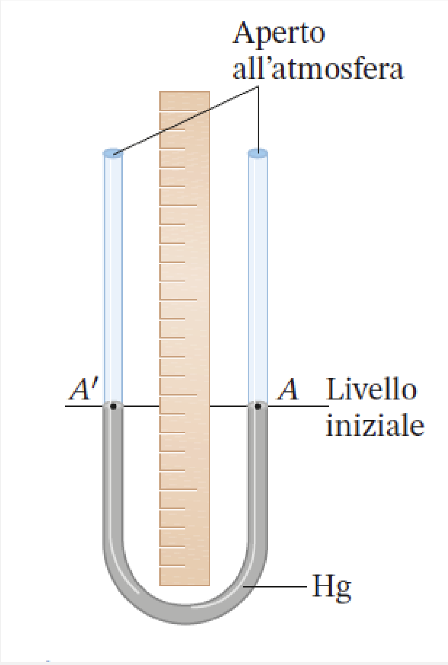
Measurement of the gauge pressure of a gas

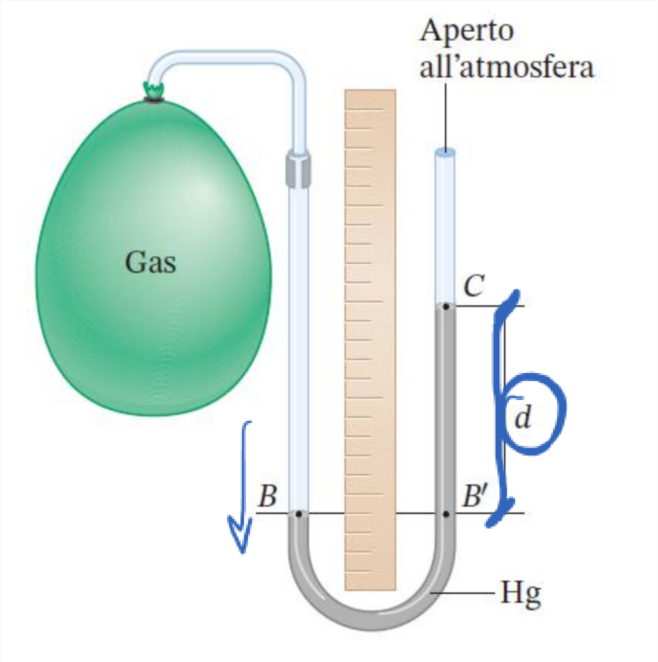
$$P_B = P_{B'} = P_C + \rho g d$$

$$\Delta P = P_B - P_C = \rho g d$$

The difference d between the mercury levels is a measure of the pressure difference

$$P_{rel} = P_{ass} - P_{atm}$$



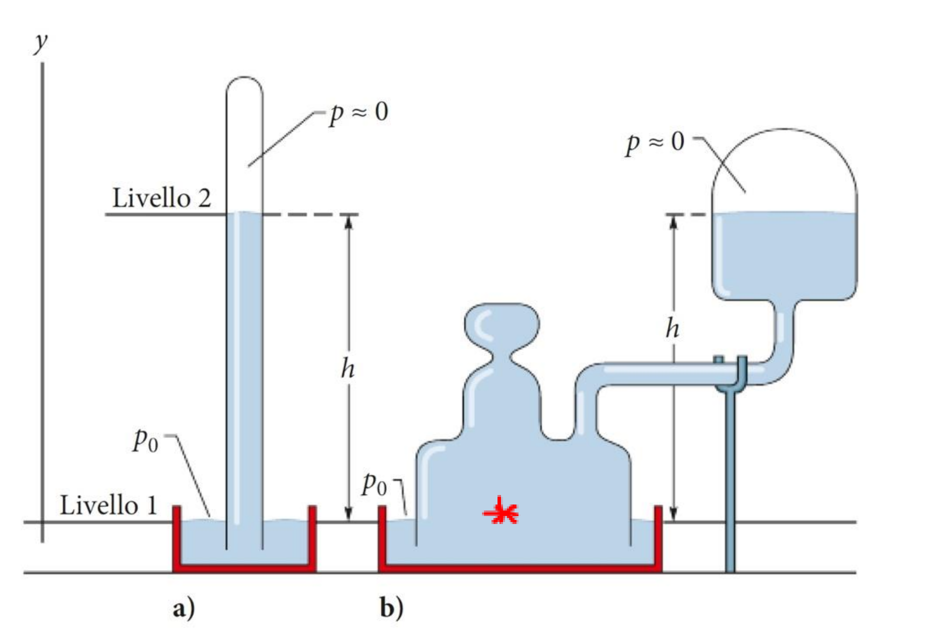


pB = pB'

pallon c.

Patm + ρgd

PRESSURE MEASUREMENT

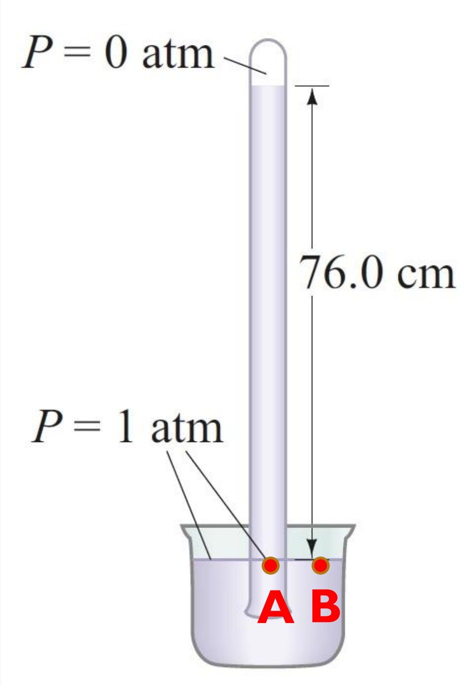


Mercury barometer

$y_1 = 0 \qquad y_2 = h$

$p_1 = p_0 \qquad p_2 = 0$

$$p_0 = \rho g h$$

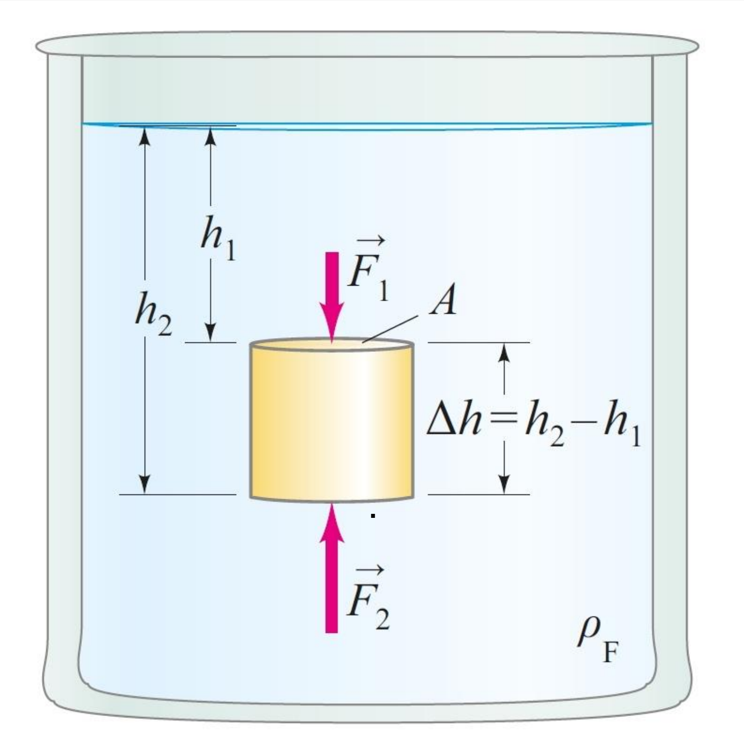


Points A and B are at the same height in the mercury and therefore must have the same pressure. Point B is at atmospheric pressure because the basin is open ($P_B = P_{atm}$). Point A is at a pressure defined by Stevin's law $P_A = \rho g d$. The distance from point A to point B is the atmospheric pressure:

$$p_{atm} = P_B = P_A = \rho g d$$

ARCHIMEDES' PRINCIPLE

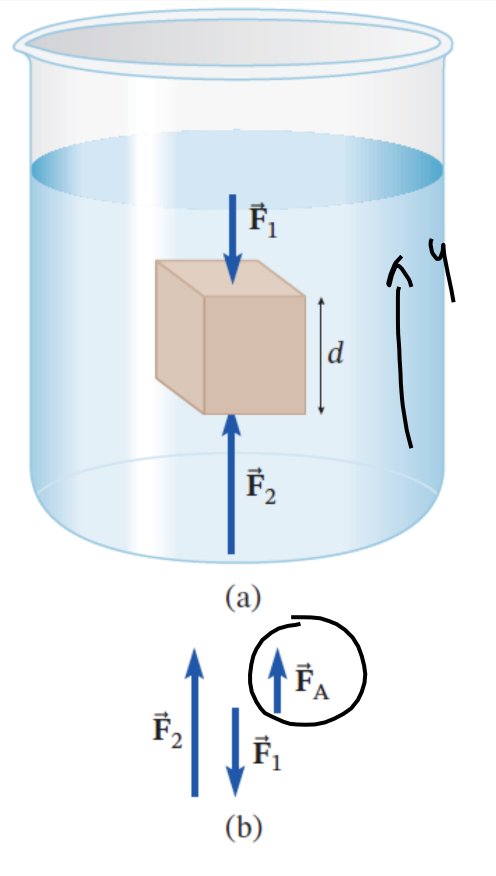
A fluid exerts an upward buoyant force on a submerged object equal in magnitude to the weight of the fluid displaced by the object



The fluid exerts a pressure $p_1 = \rho_F g h_1$ against the upper surface of the cylinder

The force due to this pressure → $F_1 = p_1 A = \rho_F g h_1 \mathrm{A}$ → downward

The fluid exerts an upward force on the lower surface of the cylinder →$F_2 = p_2 A = \rho_F g h_2 \mathrm{A}$



The RESULTANT FORCE caused by the fluid pressure (HYDROSTATIC BUOYANT FORCE) acts upward:

$$F_A = F_2 - F_1 = \rho_F g h_2 A - \rho_F g h_1 A$$

$$= \rho_F g A(h_2 - h_1)$$

$$= \rho_F g A(\Delta h)$$

$$= \rho_F g V$$

$$= m_F g$$

$V = A\Delta h$
Volume of the cylinder

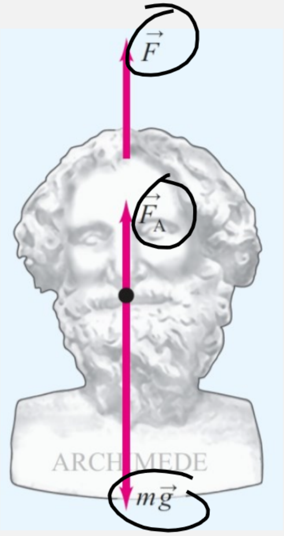
$\rho_F V$
Mass of the displaced fluid

Weight of the fluid occupying a volume equal to that of the cylinder

The hydrostatic buoyant force on the cylinder equals the weight of the fluid displaced by the cylinder

Example

A 70 kg sculpture lies on the sea floor. Its volume is $3.0 \cdot 10^4$ cm^3. What force is needed to lift it (without acceleration)?



$m = 70\ \text{kg}$

$V = 3 \cdot 10^4\ \text{cm}^3$

$\sum F = m \cdot a = 0$

$\vec{F} + \vec{F}_A + \vec{P} = 0$

$F + F_A - P = 0$

$F = P - F_A$

$P = mg = 70\ \text{kg} \cdot 9.8 = 690\ \text{N}$

$F_A = \rho_F V_F g = 1025\ \text{kg/m}^3 \cdot 3 \cdot 10^{-2}\ \text{m}^3 \cdot 9.8 = 300\ \text{N}$

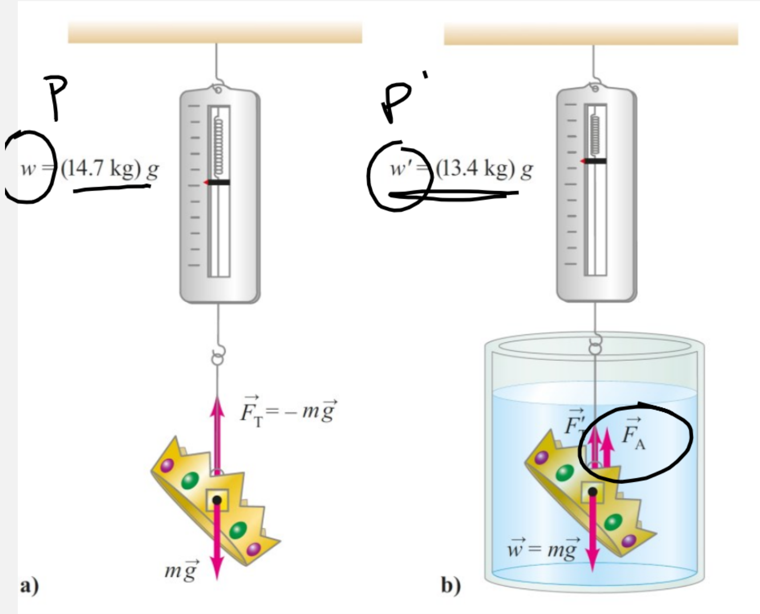
$F = 690\ \text{N} - 300\ \text{N} = 390\ \text{N}$

$\underset{F}{390\ \text{N}} / \underset{g}{9.8\ \text{m/s}^2} = \underset{m}{40\ \text{kg}}$

"è come se"

Example

Is the crown made of gold? When a crown with a mass of 14.7 kg is immersed in water, a precise scale reads only 13.4 kg. Gold has a relative density (compared to water at 4°C) of 19.3.



$$W = F_T = mg$$

$$W' = W - F_A$$

$$W - W' = F_A$$

$$\begin{cases} W = mg = \rho_o V g \\ W - W' = F_A = \rho_F V g \end{cases}$$

$$\frac{W}{W - W'} = \frac{\rho_o V g}{\rho_F V g}$$

$$\frac{W}{W - W'} = \frac{\rho_o}{\rho_F} = 19.3$$

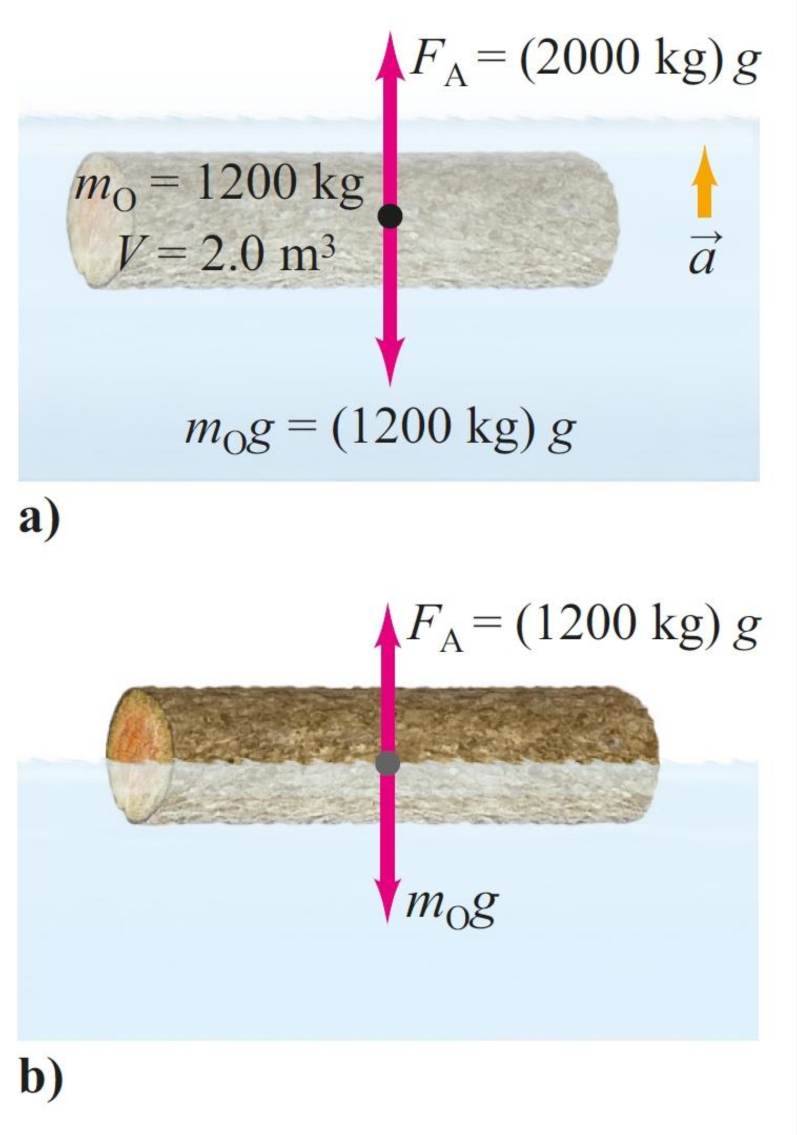
DENSITÀ RELATIVA

$$\frac{14.7\,kg\,g}{14.7\,kg\,g - 13.4\,kg\,g} \neq 19.3$$

$$= 11.3$$

NON È ORO!

BUOYANCY



Log completely submerged:

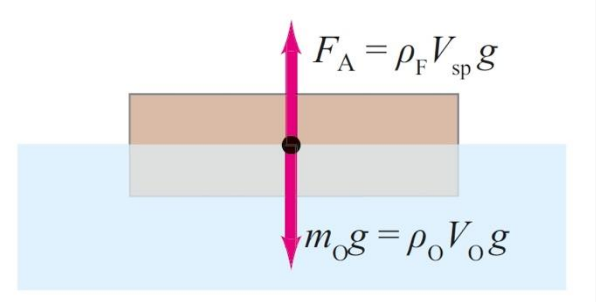
$$F_A > m_o g \;\rightarrow\; \rho_F V g > \rho_o V g \;\rightarrow\; \rho_F > \rho_o$$

Floating log:

$$F_A = m_o g \;\rightarrow\; \rho_F V_{f.\mathrm{disp}} g = \rho_o V_o g \;\rightarrow\; \frac{V_{f.\mathrm{disp}}}{V_o} = \frac{\rho_o}{\rho_F}$$

The percentage of an object submerged is given by the ratio of the object's density to the fluid's density

ARCHIMEDES' FORCE AND DENSITY



$$F_w = m_{obj} \cdot g = \rho_{obj} \cdot V_{obj\ tot} \cdot g$$

$$F_A = \rho_{fluid} \cdot V_{obj\ imm} \cdot g$$

I. If $F_P > F_A$ → $\rho_{obj} \cdot V_{obj\ tot} \cdot g > \rho_{fluid} \cdot V_{obj\ imm} \cdot g$

$$\frac{\rho_{obj}}{\rho_{fluid}} > \frac{V_{obj\ imm}}{V_{obj\ tot}}$$

The body sinks! → $V_{obj\ imm} = V_{obj\ tot}$

$$\frac{\rho_{obj}}{\rho_{fluid}} > \frac{V_{obj\ imm}}{V_{obj\ tot}} = 1$$

$a \neq 0$

$$\rho_{obj} > \rho_{fluid}$$

The body sinks

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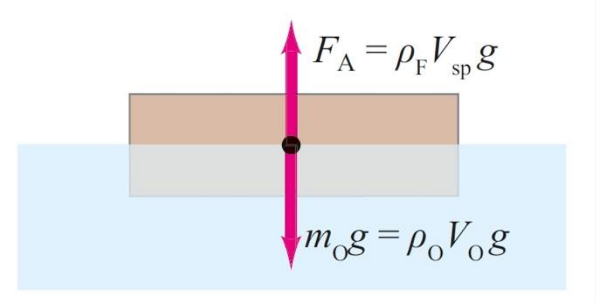


$$F_P = m_{obj} \cdot g = \rho_{obj} \cdot V_{obj\ tot} \cdot g$$

$$F_A = \rho_{fluid} \cdot V_{obj\ imm} \cdot g$$

2. If $F_P = F_A$ →

$$\frac{\rho_{obj}}{\rho_{fluid}} = \frac{V_{obj\ imm}}{V_{obj\ tot}}$$

Equilibrium and fully submerged:

$$\frac{\rho_{obj}}{\rho_{fluid}} = \frac{V_{obj\ imm}}{V_{obj\ tot}} = 1$$

$$\rho_{body} = \rho_{fluid}$$

Submerged body in equilibrium

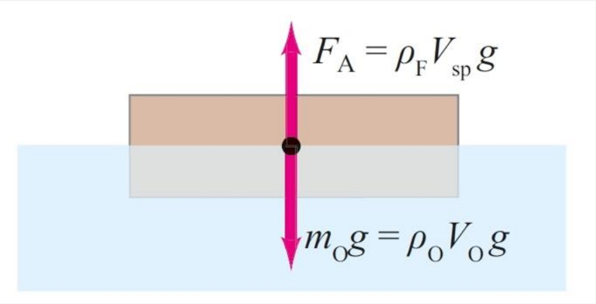
Equilibrium and partially submerged:

$$\frac{\rho_{obj}}{\rho_{fluid}} = \frac{V_{obj\ imm}}{V_{obj\ tot}} < 1$$

$$\rho_{obj} < \rho_{fluid}$$

Body floats in equilibrium

ARCHIMEDES' FORCE AND DENSITY



$$F_P = m_{\text{obj}} \cdot g = \rho_{\text{obj}} \cdot V_{\text{obj}\,tot} \cdot g$$

$$F_A = \rho_{fluid} \cdot V_{\text{obj}\,imm} \cdot g$$

3. If $F_P < F_A$ → $\dfrac{\rho_{\text{obj}}}{\rho_{fluid}} < \dfrac{V_{\text{obj}\,imm}}{V_{\text{obj}\,tot}}$

Since the submerged body tends to rise, we will have $V_{\text{obj}\,imm} = V_{\text{body}\,tot}$ until it emerges:

$$\frac{\rho_{\text{obj}}}{\rho_{fluid}} < \frac{V_{\text{obj}\,imm}}{V_{\text{obj}\,tot}} = 1$$

$$\rho_{\text{obj}} \leqslant \rho_{fluid}$$

Submerged body rises

Example

A body is fully immersed in fresh water (ρH_2O = 1000 kg/m^3) and its weight in water is measured as 80N. If its weight out of water is 120N, find: 1) the buoyant force when submerged; 2) the volume of the body; 3) the density of the body.

$F_A = P - P'$ → P = peso in aria
P' = peso in acqua

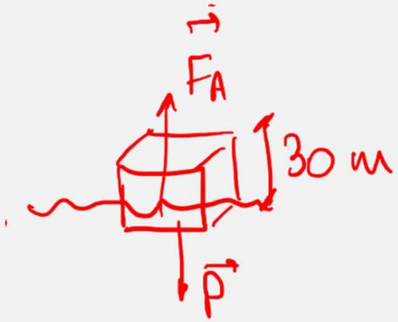
(1) $F_A = 120N - 80N = 40N$

(2) $F_A = \rho_F V g \rightarrow V = \dfrac{F_A}{\rho_F g} = \dfrac{40N}{1000\ kg/m^3 \cdot 9.8} = 0.004\ m^3$

(3) $P = \rho_c V g \rightarrow \rho_c = \dfrac{P}{Vg} = \dfrac{120N}{0.004\ m^3 \cdot 9.8} = 3061\ kg/m^3$

Example

A block of ice (ρ_{ice} = 920 kg/m^3) in the shape of a parallelepiped with a height of 30 m floats in seawater (ρ_{H2O} = 1030 kg/m^3). How long is the part of the parallelepiped above the water?



→ equilibrio → $P = F_A$

$$\rho_c V_c \cancel{g} = \rho_F V_{imm} \cancel{g} \rightarrow V_{imm} = \frac{\rho_c V_c}{\rho_F}$$

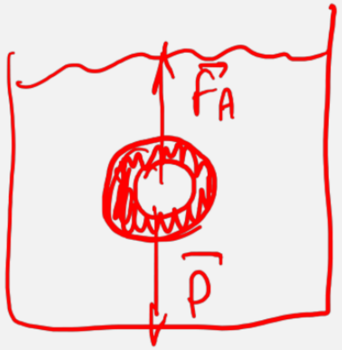
V_c? → parallelepipedo → $A_b \cdot h$

$$\cancel{A_b} \cdot h_{imm} = \frac{\rho_c \cdot \cancel{A_b} \cdot h_{TOT}}{\rho_F}$$

$$h_{imm} = \frac{\rho_c h_{TOT}}{\rho_F} = \frac{920\ kg/m^3 \cdot 30m}{1030\ kg/m^3} = 26.8m$$

Example

An iron body (ρ_{Fe} = 7800 kg/m^3) has a cavity inside. Knowing that the body's mass is 780 g and that once immersed in seawater a weight reduction of 1.56 N is measured, determine the volume of the internal cavity.



$m = 780\,g$

$$P_{ARIA} - P_{ACQUA} = 1.56\,N = F_A \rightsquigarrow F_A = \rho_F V g$$

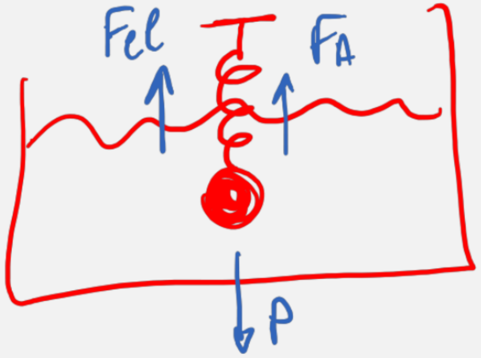
$$\hookrightarrow V = \frac{F_A}{\rho_F g} = 1.56 \cdot 10^{4}\,m^3$$

$$\rho_{Fe} = \frac{m_{Fe}}{V_{Fe}} \longrightarrow V_{Fe} = \frac{m_{Fe}}{\rho_{Fe}} = \frac{0.78\,Kg}{7800\,Kg/m^3} = 10^{-4}\,m^3$$

$$V_{CAV} = V - V_{Fe} = 1.56 \cdot 10^{-4}\,m^3 - 10^{-4}\,m^3 = 0.56 \cdot 10^{-4}\,m^3 = \underline{56\,cm^3}$$

Example

A copper body (ρ_{Cu} = 8900 kg/m^3) with a mass of 3 kg is completely immersed in water (ρ_{H2O} = 1000 kg/m^3) and hung from a spring of negligible mass that is deformed by 3 cm. Calculate the spring constant.



$$\vec{F}_A + \vec{P} + \vec{F}_{EL} = 0 \quad \text{(equilibrio)}$$

$$K\Delta x = mg - \rho_F V g, \qquad V = \frac{m}{\rho}$$

$$K\Delta x = mg - \rho_F \frac{m}{\rho} g \rightarrow K = \frac{mg - \rho_F \frac{m}{\rho} g}{\Delta x} = 870.2 \text{ N/m}$$

Example

An 85 kg canoe made of thin aluminum has the shape of a half-hollowed log with a radius of 0.475 m and a length of 3.23 m. A) When the canoe is placed in water, what percentage of the canoe's volume is below the waterline? B) How much mass can be added to this canoe before it starts to sink?

$$\frac{\pi r^2 \cdot h}{2}$$

$$V = 1.145\ m^3$$

equilibrio $\rightarrow$ $P = F_A \rightarrow mg = \rho_F g V_{IMM}$

$$V_{IMM} = \frac{mg}{\rho_F g} = \frac{85\ Kg}{1000\ Kg/m^3} = 0.085\ m^3$$

$$\frac{V_{IMM}}{V} \cdot 100 = 7.4\% \quad (A)$$

$V_{IMM} = V$

$$(B) \quad \rho_{H_2O} V g = m' g \rightarrow m' = \frac{\rho_{H_2O} V g}{g} = 1145\ Kg$$

$$\Delta m = m' - m = (1145 - 85)\ Kg = 1060\ Kg$$