# FLUIDS

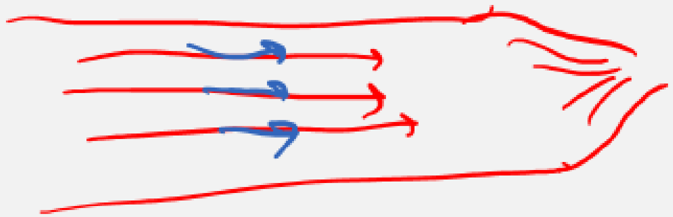
- ☞FLUID STATICS
- ☞FLUID DYNAMICS

# FLUID DYNAMICS

Steady-state flow/motion/regime: the velocity of the fluid at every point remains constant over time; likewise, density and pressure are invariant

LAMINAR FLOW: the fluid moves in ordered layers (streamlines) such that every small portion of fluid traversing a given point follows the same trajectory as any other portion passing through that same point.

IDEAL FLUID: incompressible, exhibits laminar flow, and is inviscid



# CONTINUITY EQUATION


$$x_1 = v_1 \Delta t$$

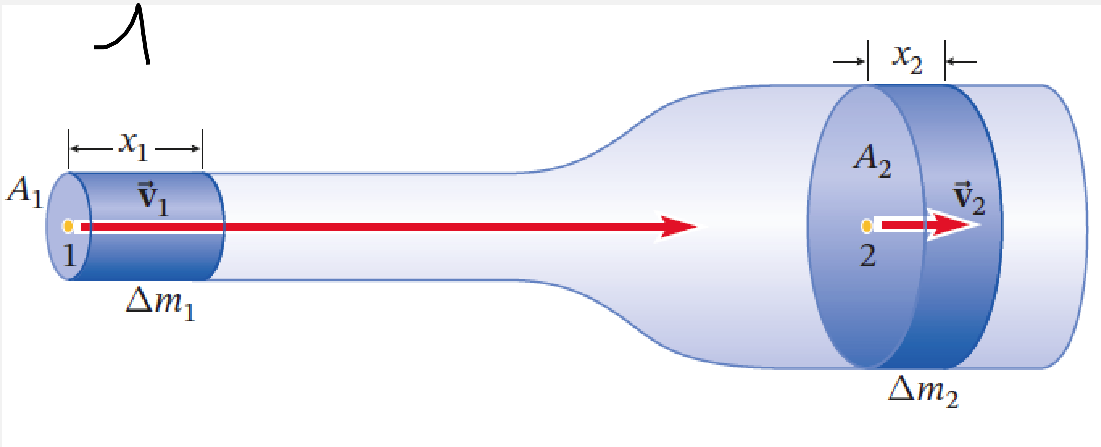
$$\Delta m_1 = \rho V_1 = \rho A_1 x_1 = \rho A_1 v_1 \Delta t$$

$$\Delta m_2 = \rho V_2 = \rho A_2 x_2 = \rho A_2 v_2 \Delta t$$

$$\Delta m_1 = \Delta m_2$$

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

2



# CONTINUITY EQUATION

$$\cancel{\rho}A_1v_1\cancel{\Delta t} = \cancel{\rho}A_2v_2\cancel{\Delta t}$$

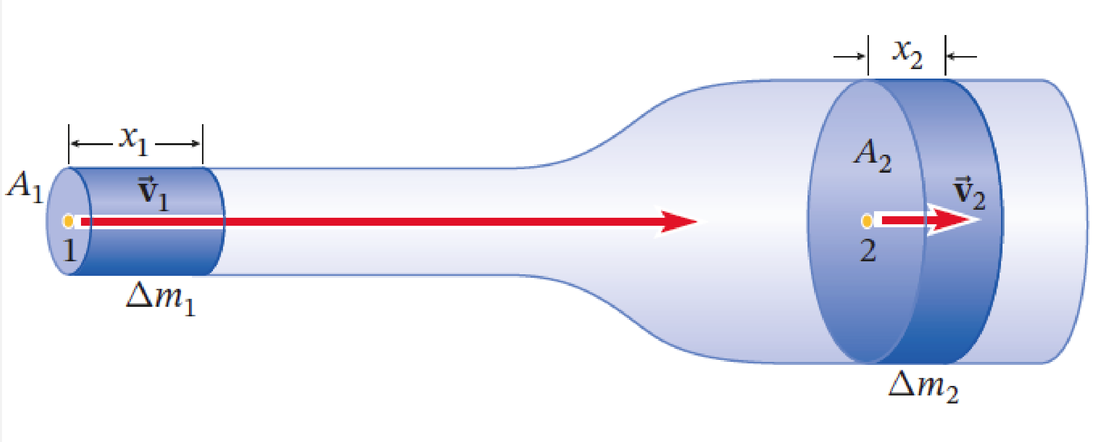
The mass flow rate of the fluid is:

$$\frac{\Delta m}{\Delta t} = \rho A v \qquad (unità\ SI: \frac{kg}{s})$$

The volumetric flow rate is:

$$Q = \frac{\Delta V}{\Delta t} = Av \qquad (unità\ SI: \frac{m^3}{s})$$

$$Av = \frac{\Delta V}{\Delta t} \qquad \rho A v = \frac{\Delta m}{\Delta t}$$



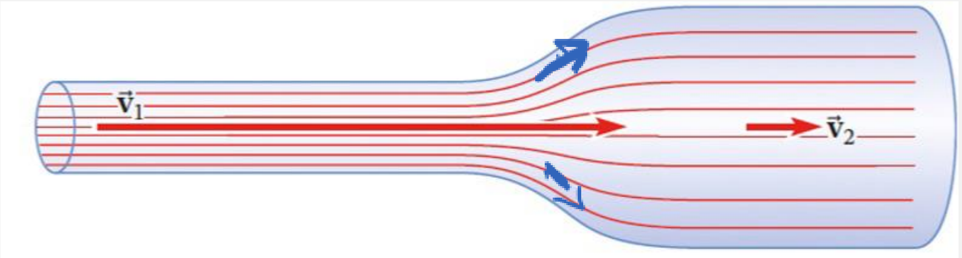

**Continuity equation for an incompressible fluid:**

$$A_1v_1 = A_2v_2 \qquad A \cdot v = cost.$$

# CONTINUITY EQUATION

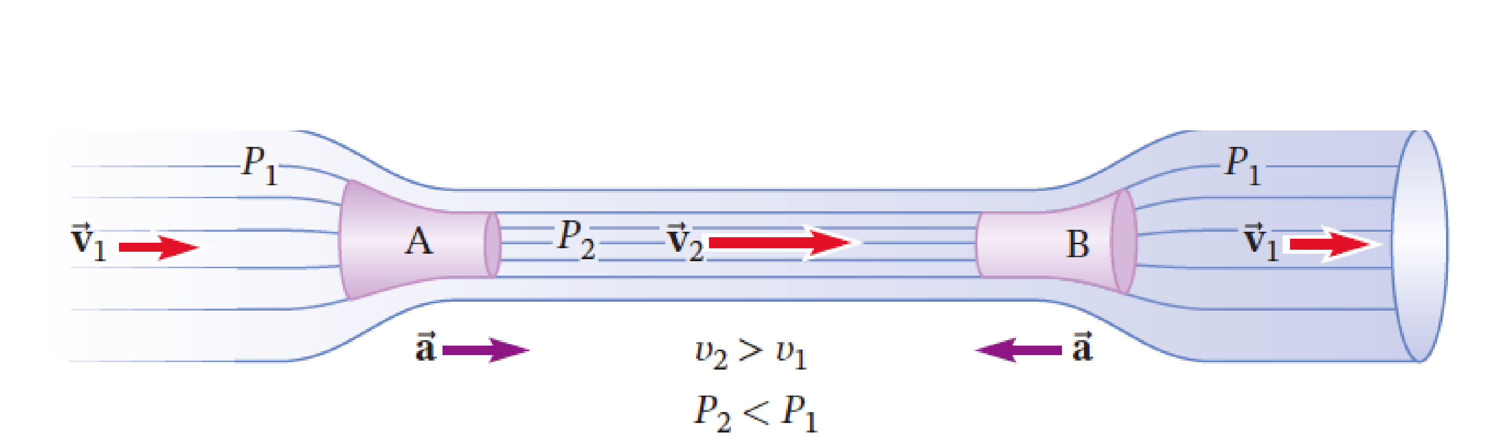
The same volume of fluid entering a conduit within a given $\Delta t$ must exit the conduit within the same time interval.

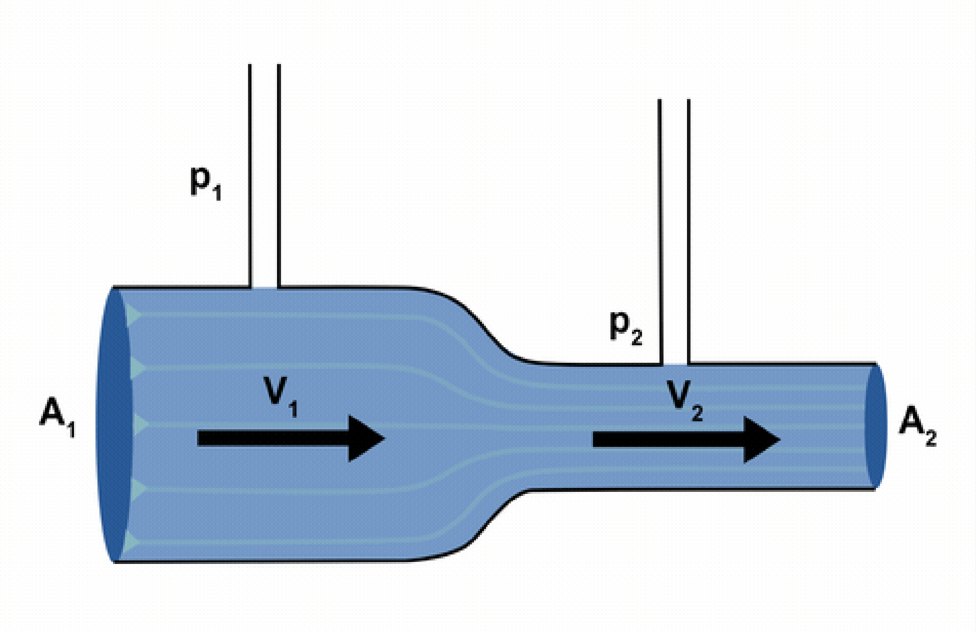
The velocity of the fluid is low when the radius of the conduit is large, and vice versa



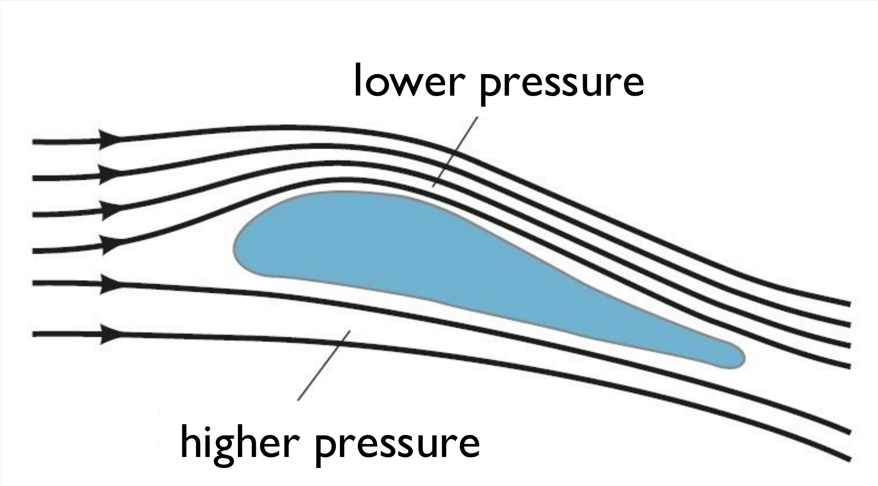

In a conduit of variable cross-section, the streamlines
appear closer together where the fluid flows more rapidly

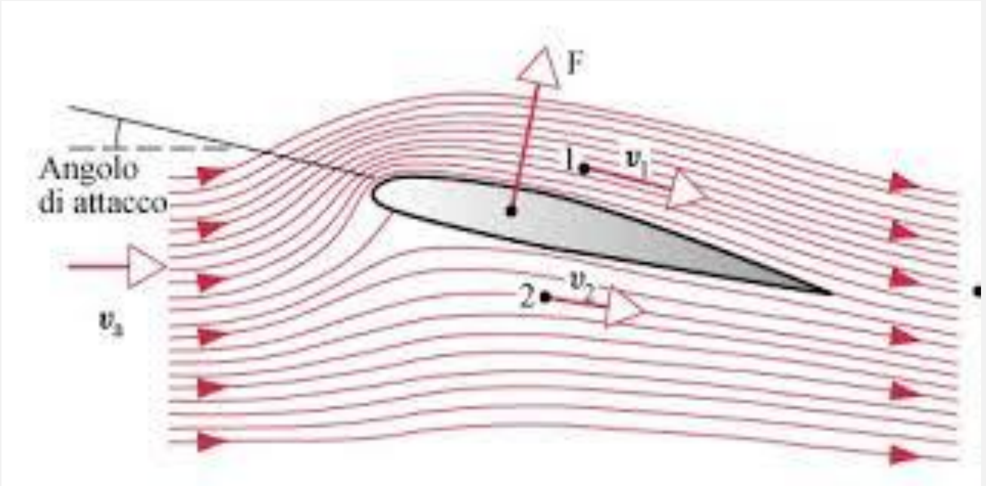
# BERNOULLI'S EQUATION



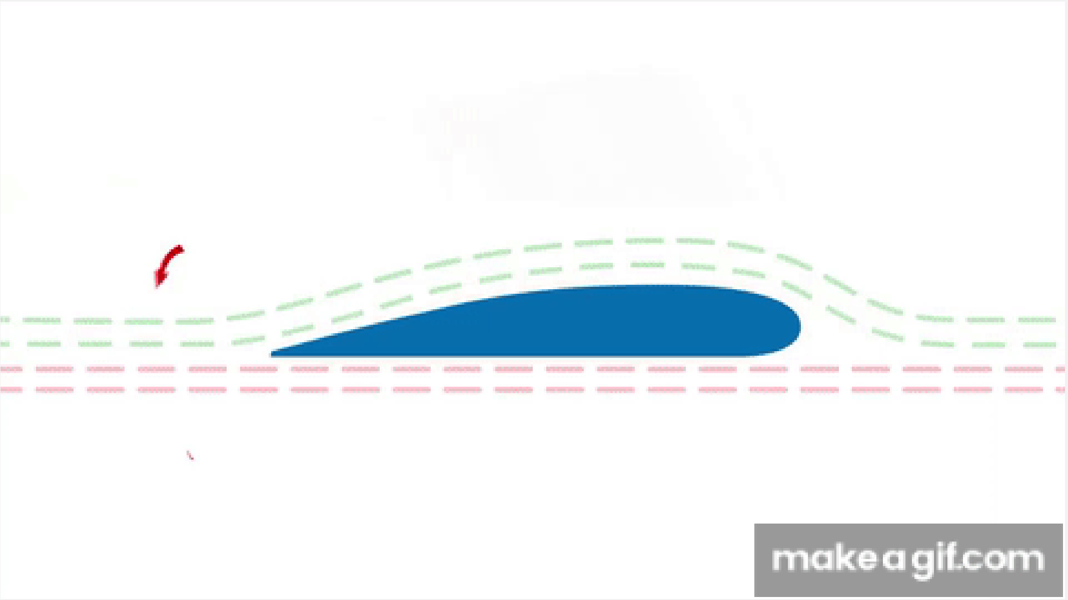




The fluid pressure at the constriction (P2) cannot equal the pressure upstream or downstream of the constriction (P1) → in the case of horizontal flow, the velocity is greater where the pressure is lower → BERNOULLI EFFECT / BERNOULLI'S PRINCIPLE




AERODYNAMIC FORCE / LIFT → resultant force directed upward



# BERNOULLI'S EQUATION

$$A_1\Delta x_1 = A_2\Delta x_2 = V$$

0°　　　　　　　　　　180°

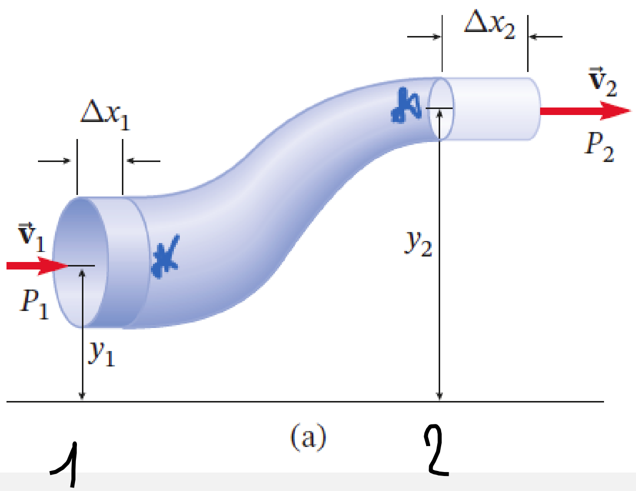
$$W_1 = F_1\Delta x_1 \cos\alpha_1 \text{ and } W_2 = F_2\Delta x_2 \cos\alpha_2$$

$$W = W_1 + W_2 = +P_1A_1\Delta x_1 - P_2A_2\Delta x_2 = (P_1 - P_2)V$$

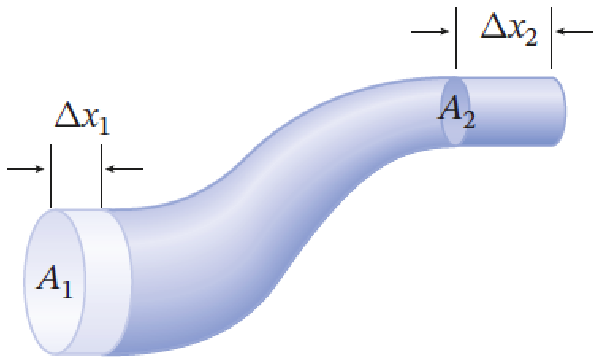
$$\Delta E = \Delta K + \Delta U = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

W

$$(P_1 - P_2)V = \frac{1}{2}\rho V(v_2^2 - v_1^2) + \rho Vg(y_2 - y_1)$$



(a)



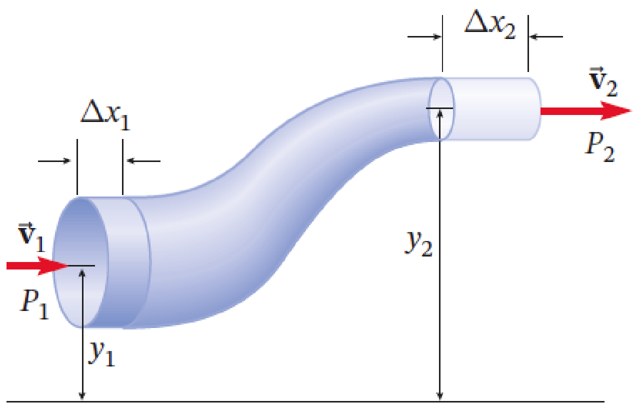
(b)

# BERNOULLI'S EQUATION

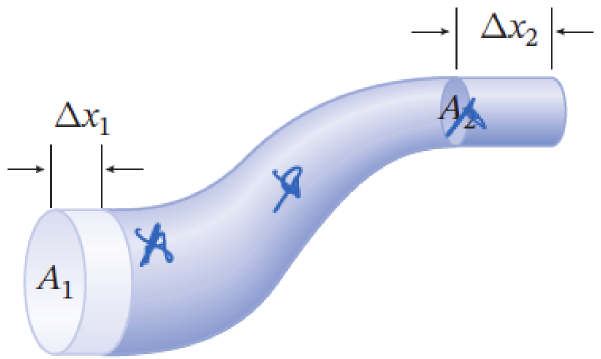
$$(P_1 - P_2)V = \frac{1}{2}\rho V(v_2^2 - v_1^2) + \rho V g(y_2 - y_1)$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

that is $\quad P + \rho g y + \frac{1}{2}\rho v^2 = constant$

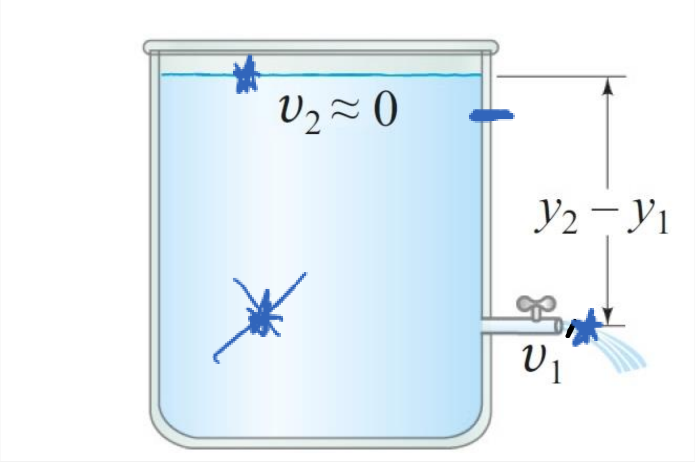


(a)



(b)

# TORRICELLI'S THEOREM




$$\cancel{P_2} + \rho g y_2 + \cancel{\frac{1}{2}\rho v_2^2} = \cancel{P_1} + \rho g y_1 + \frac{1}{2}\rho v_1^2$$

$A_2 \gg A_1 \;\rightarrow\; v_2 = \text{negligible}\ (A_1 v_1 = A_2 v_2)$

$P_1 = P_2 = P_{atm}$

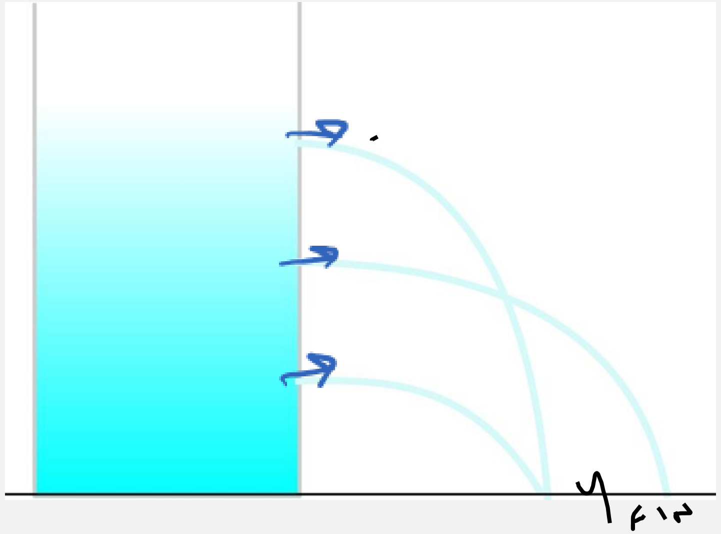
$v_2 \approx 0$

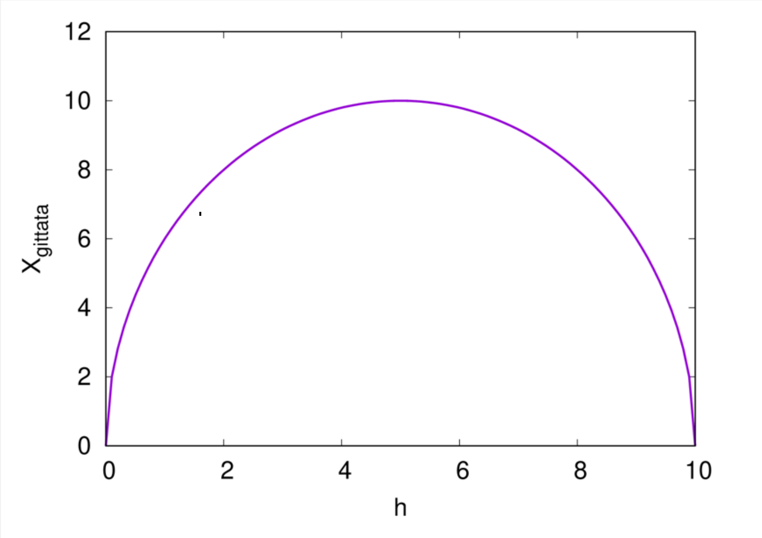
$$\cancel{P_2} + \rho g y_2 + \cancel{\frac{1}{2}\rho v_2^2} = \cancel{P_1} + \rho g y_1 + \frac{1}{2}\rho v_1^2$$

$$\cancel{\rho} g y_2 = \cancel{\rho} g y_1 + \frac{1}{2}\cancel{\rho} v_1^2$$

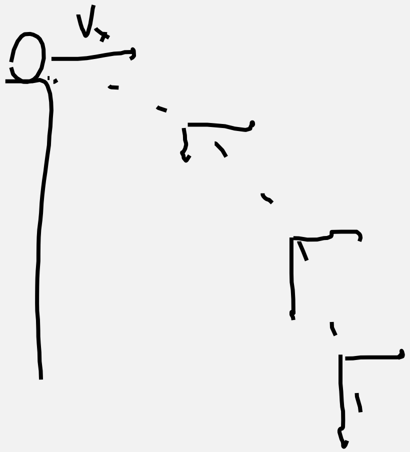
$$\boxed{v_1 = \sqrt{2g(y_2 - y_1)} = \sqrt{2gh}}$$

# TORRICELLI'S THEOREM

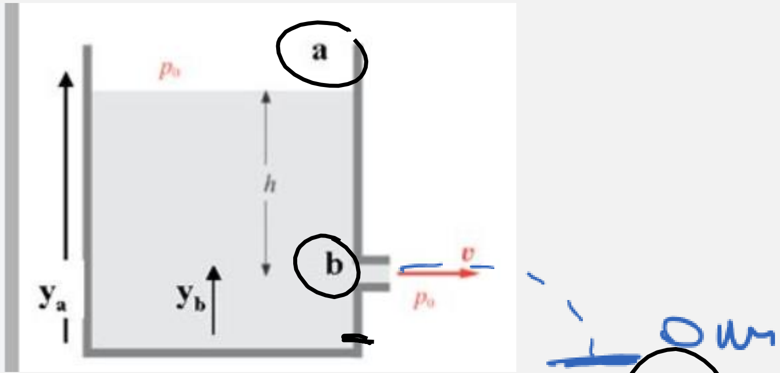





$$v_x = \sqrt{2gh}$$




The fluid exits with horizontal velocity $v_b$ and subsequently follows a parabolic trajectory:



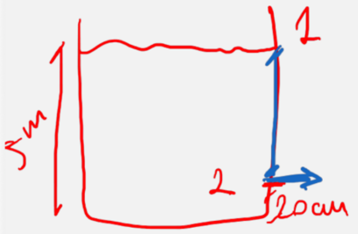

$$v_x = v_b ; x = v_b \Delta t$$

$$y_f = y_b - \frac{1}{2} g \Delta t^2 \rightarrow 0 = y_b - \frac{1}{2} g \Delta t^2 \rightarrow \Delta t = \sqrt{\frac{2y_b}{g}}$$

$$x = v_b \Delta t \rightarrow \sqrt{2gh}\sqrt{\frac{2y_b}{g}} = \sqrt{4hy_b} = \sqrt{4(y_a - y_b)y_b} = \boxed{2\sqrt{(y_a - y_b)y_b}}$$

# Example

Consider a tank filled with water in which the free surface is located at a height of 5 m, while the lateral orifice is situated 20 cm above the base. Determine the horizontal distance from the tank at which the water jet emerging from the orifice lands.




$$V_2 = \sqrt{2 \cdot 9.8\,m/s^2 \cdot 4.8\,m} = 9.7\,m/s$$

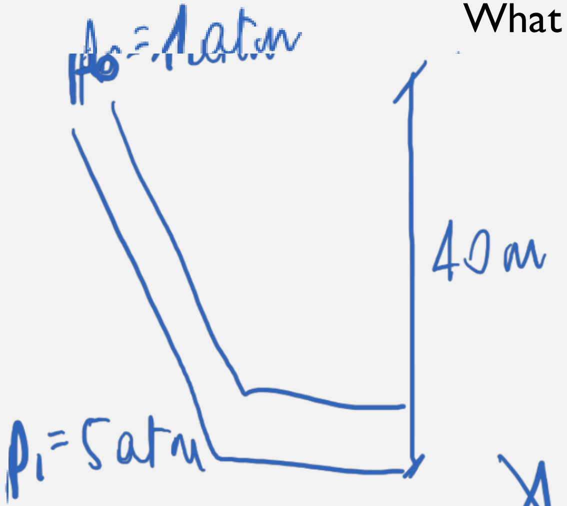
$$\begin{cases} x = \cancel{x_0} + V_x t \\ \cancel{y} = y_0 + \cancel{V_y t} - \frac{1}{2} g t^2 \end{cases} \qquad \begin{cases} x = V_x t \\ 0 = y_0 - \frac{1}{2} g t^2 \end{cases}$$

$$\begin{cases} x = V_2 t \\ 0 = y_0 - \frac{1}{2} g t^2 \end{cases} \rightarrow t = \sqrt{\frac{2y_0}{g}} \Rightarrow x = V_2 \sqrt{\frac{2y_0}{g}} = 1.96\,m$$

$$\boxed{x = \sqrt{2(y_1 - y_2)\underset{y_0}{y_2}} = 1.96\,m}$$

## Example

A pipe of constant cross-section descends from a mountain with an elevation difference of 40 m. The fluid pressure at the summit equals atmospheric pressure, while the downstream pressure is p1=5atm. The upstream fluid velocity is 5m/s. Assume the fluid in the pipe is ideal. A) Calculate the fluid velocity throughout the sections of the pipe. B) What is the density of the fluid? What fluid might it be?




$$P_0 + \rho g h_0 + \cancel{\tfrac{1}{2}\rho v_0^2} = P_1 + \rho \cancel{g h_1} + \cancel{\tfrac{1}{2}\rho v_1^2} \rightarrow ?$$

1atm    40m    5m/s    5atm    0m

A

$$A_0 v_0 = A_1 v_1 \rightarrow v_0 = v_1 = v$$

B

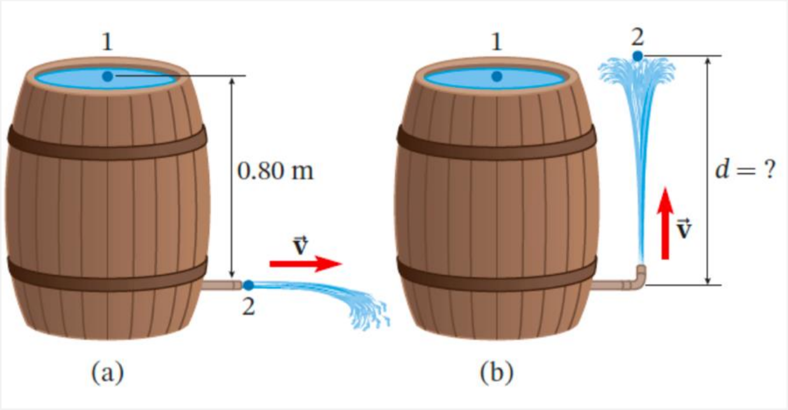
$$P_0 + \rho g h_0 = P_1 \rightsquigarrow \rho = \frac{P_1 - P_0}{gh} = \frac{5atm - 1atm}{9.8 \cdot 40} = 1033\ kg/m^3$$

acqua salata

# Example

A barrel filled with water has a tap near the bottom, at a depth of 0.80 m below the free surface of the water. A) At what velocity does the water exit if the tap is oriented horizontally? B) What height does the jet reach if the opening is oriented upward?

altezza raggiunge lo zampillo, se l'apertura è orientata verso l'alto?




$$\cancel{P_1} + \rho g h_1 + \cancel{\frac{1}{2}\rho v_1^2} = \cancel{P_2} + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$P_1 = P_2$

$v_1 \approx 0$

$$\cancel{\rho} g h_1 = \cancel{\rho} g h_2 + \frac{1}{2}\cancel{\rho} v_2^2$$

$$\boxed{v_2 = \sqrt{2gh} = 4\ m/s}$$

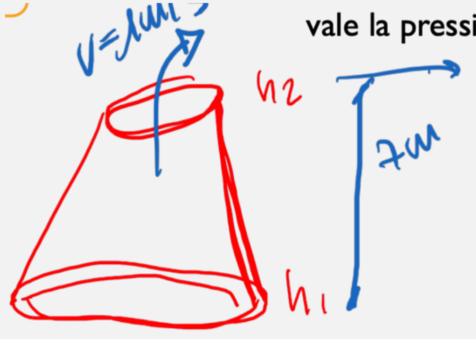
$$\rho g h_1 = \rho g h_2 + \cancel{\frac{1}{2}\rho v_2^2}$$

$$\cancel{\rho g} h_1 = \cancel{\rho g} h_2 \longrightarrow h_1 = h_2$$

# Example

Consider a vertical truncated-cone-shaped tube, 7m tall, with a cross-sectional area of $30 cm^2$ at the lower end and $10 cm^2$ at the upper end. Water exits from the top at a velocity of 1m/s and a pressure of $10^5$Pa. Determine the pressure at the base of the tube.

vale la pressione alla base del tubo?




$P_2 = 10^5$ Pa

$P_1$ ?

$$P_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

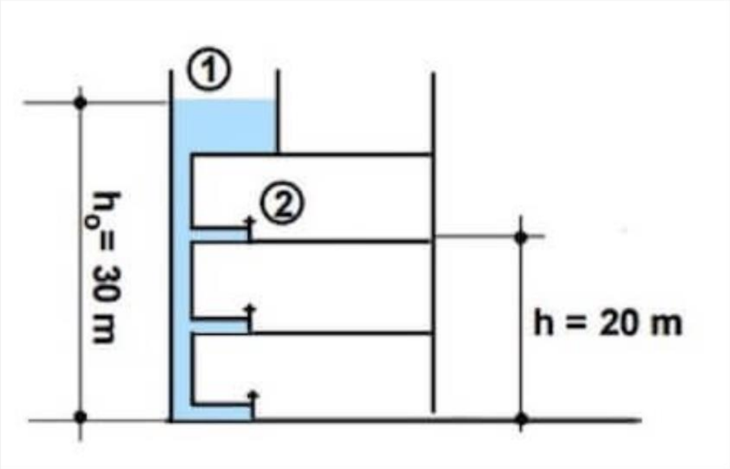
$$A_1 v_1 = A_2 v_2 \rightarrow v_1 = \frac{A_2 v_2}{A_1} = \frac{10 cm^2 \cdot 100 cm/s}{30 cm^2} = 33.33 cm/s$$

$0.33 m/s$

$$P_1 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = 1.69 \cdot 10^5 Pa$$

# Example

A water tank is located at the top of a building and is filled such that the upper water level is 30m above ground. Pipes of much smaller cross-section extend from the tank. Each apartment has faucets with a cross-sectional area of 10 $cm^2$. Calculate the time required to fill a container with a capacity of 30 $dm^3$ located in an apartment situated 20 m above ground.




$$\cancel{P_1} + \rho g h_1 + \frac{1}{2}\cancel{\rho v_1^2} = \cancel{P_2} + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$g h_1 = g h_2 + \frac{1}{2} v_2^2 \rightarrow v_2 = 14\, m/s$$

$$Q = Av = \frac{\Delta V}{\Delta t}$$

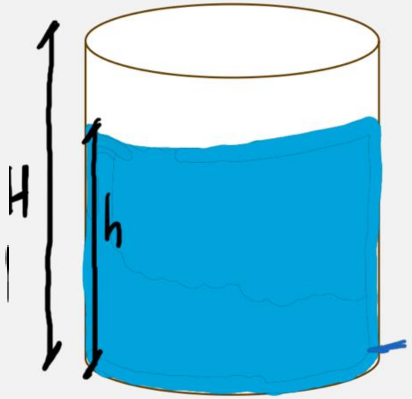
$$Q = 10\, cm^2 \cdot 14\, m/s = 14\, dm^3/s$$

volume

$$Q = \frac{V}{t} \rightarrow t = \frac{V}{Q} = \frac{30\, dm^3}{14\, dm^3/s} = \boxed{2.14 s}$$

# Example

Consider a cylindrical water container with a diameter of 0.1m and a height of 0.2m. An orifice with a surface area of 1cm$^2$ is made laterally at the level of the base. Water exits through this orifice at a flow rate of 1.4x10$^{-4}$ m$^3$/s. Determine the height of the water level in the container and the time required for complete drainage.




$H = 0.2\,m$

$d = 0.1\,m$

$Q = 1.4 \cdot 10^{-4}\,m^3/s$

$h?\quad \Delta t?$

$Q = A \cdot v \rightarrow \begin{cases} v = \dfrac{Q}{A} \\ v = \sqrt{2gh} \end{cases}$

$Q = \dfrac{\Delta V}{t} \rightarrow V_f - V_i$

$\begin{cases} v = \dfrac{Q}{A} \\ v = \sqrt{2gh} \end{cases} \rightarrow \dfrac{Q}{A} = \sqrt{2gh} \rightarrow h = \dfrac{Q^2}{2gA^2} = \dfrac{(1.4 \cdot 10^{-4}\,m^3/s)^2}{2 \cdot 9.8\,m/s^2 \cdot (10^{-4}\,m^2)^2}$

$= 0.1\,m$

$h$ (acqua)

$V_{ACQUA} \rightarrow A_b \cdot h = \pi \left(\dfrac{d}{2}\right)^2 \cdot h = 7.85 \cdot 10^{-4}\,m^3$

$Q = \dfrac{V}{t} \rightarrow t = \dfrac{V}{Q} = \dfrac{7.85 \cdot 10^{-4}\,m^3}{1.4 \cdot 10^{-4}\,m^3/s} = 5.6\,s$