# Day 1

PROBABLY THE BEGINNING OF THE COURSE....

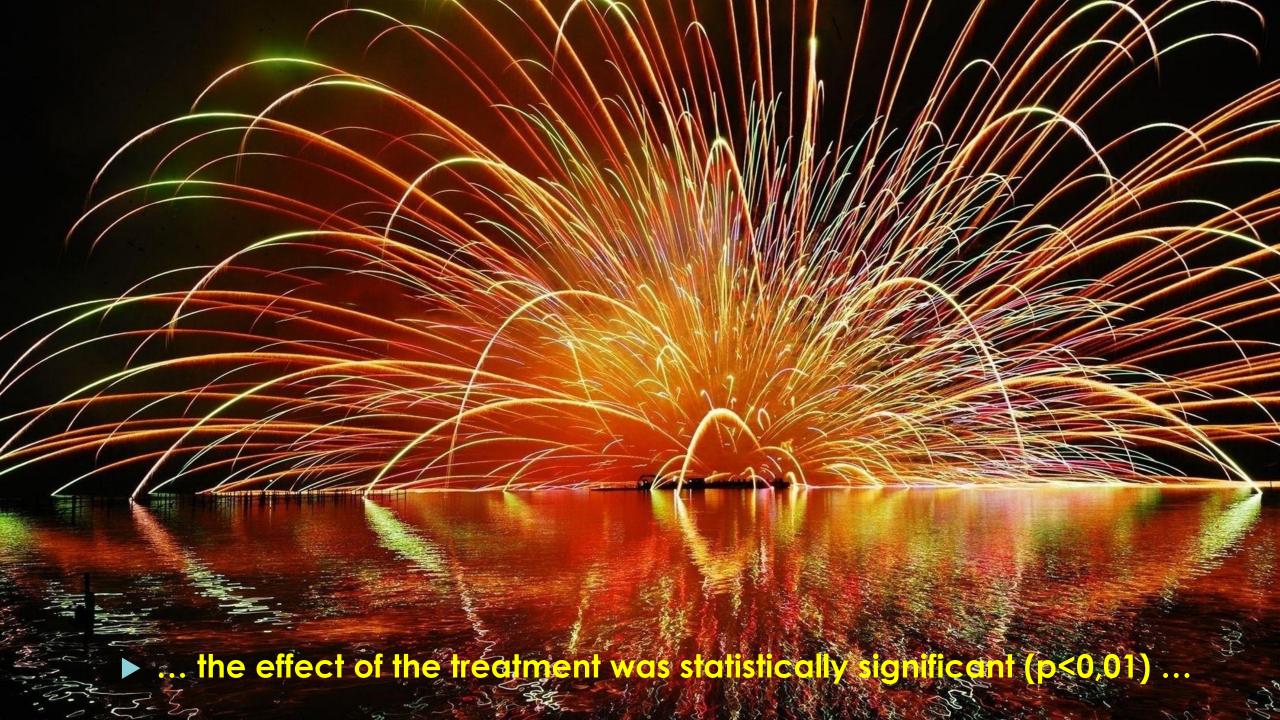
Nicola Bernabò, DVM PhD

## Course introduction

	9.30 – 12.30	14.00 – 16.30
September 18th	Introduction to the course Concept of Probability Frequentist definition Subjective definition Axiomatic definition The Monty Hall problem Biological implications of probability	Practice
September 19th	Descriptive statistics Concept of distribution Gaussian distribution Binomial distribution Exponential distribution Estimation of central tendency and variability	NO
September 20th	Concept of population Sampling strategies Randomization Power analysis	Practice
September 25th	Inferential statistics ANOVA one way ANOVA two ways Non parametrical tests Post hoc tests	Practice
September 26th	Concept of correlation Concept of regression Forecasting models Hints of multivariate statistics (clustering, grouping)	Practice

Before starting, some consideration about the probability.





# Can you give me a definition of the word «probability»?

## **PROBABILITY**

#### Classical definition

The probability of an event is the ratio of the number of cases favorable to it, to the number of all cases possible when nothing leads us to expect that any one of these cases should occur more than any other, which renders them, for us, equally possible.

- 1. the probability of an event is a number between 0 and 1;
- 2. the probability of a certain event is equal to 1;
- 3. the probability of occurrence of one of two incompatible events, that is, of two events that can not occur simultaneously, is equal to the sum of the probabilities of the two events.

# to play dice



#### Criticisms:

The definition is circular: the probability for a "fair" coin is... A "fair" coin is defined by a probability of...

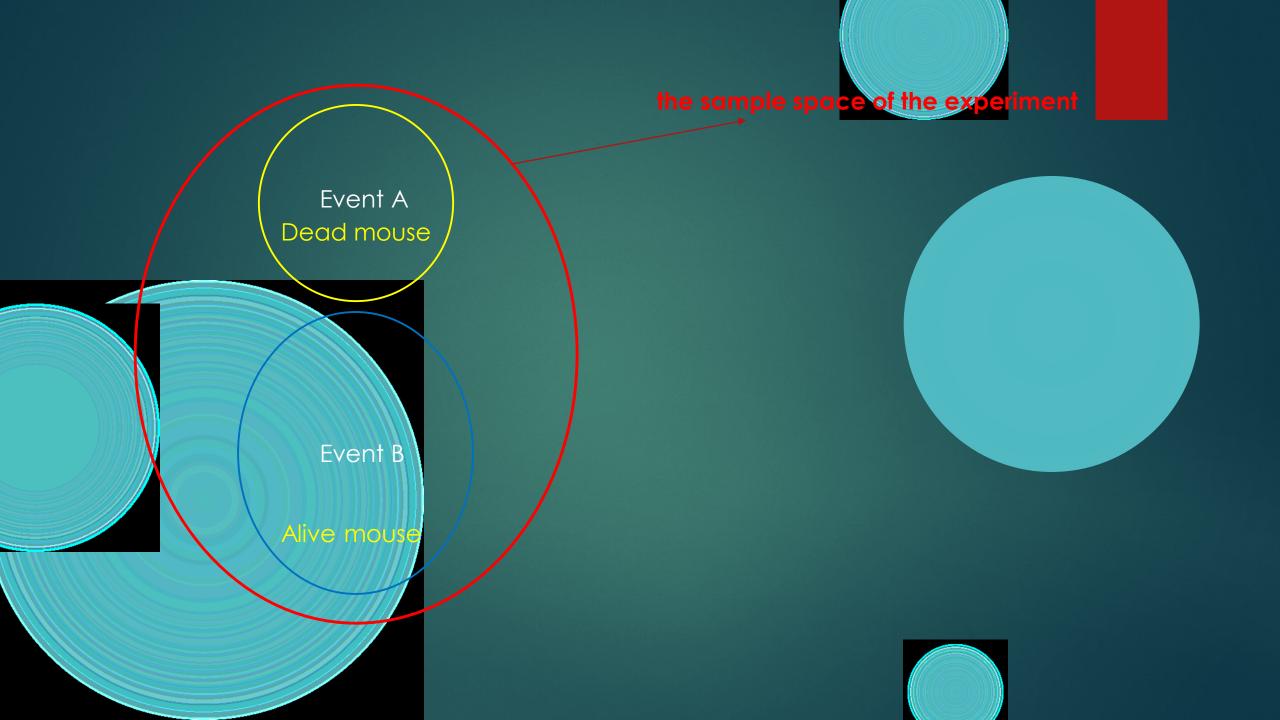
The definition is very limited. It says nothing about cases where no physical symmetry exists. For example coins are not truly symmetric. Can we assign equal probabilities to each side? Can we assign equal probabilities to any real world experience?

It essumes a finite number of possible results and consequently can not be used in the continuum.

# **PROBABILITY**

#### Frequentist defintion

The set of all possible outcomes of a random experiment is called the sample space of the experiment. An event is defined as a particular subset of the sample space to be considered. For any given event, only one of two possibilities may hold: it occurs or it does not. The relative frequency of occurrence of an event, observed in a number of repetitions of the experiment, is a measure of the probability of that event.



#### Criticisms:

► The relative frequency of occurrence of an event, observed in a number of repetitions of the experiment, is a measure of the probability of that event.

- it is impossible to replicate some experiments!
- is it always impossible, in real world, to exactly replicate an experiment!
- how many replications are needed?
  - It could be very hard to consider rare events



## **PROBABILITY**

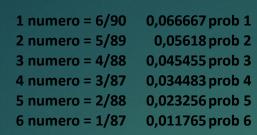
**Subjective defintion** 

the probability of an event is the price that an individual feels is fair to pay to receive 1 if the event occurs, 0 if the event does not occur.



IT is the cost fair?

13



prob TOT (prob 1\*2\*3\*4\*5\*6) =

1,61E-09 = 1/622.614.630

se due possibilità a s. 1/311.307.315 montepremi/probabilità > costo edina

montepremi = 1E+08
valore schedina = 0,321226



#### Criticisms:

Subjective probability is a probability derived from an individual's personal judgment about whether a specific outcome is likely to occur. It contains no formal calculations and only reflects the subject's opinions and past experience. Subjective probabilities differ from person to person, and they contains a high degree of personal bias.



### **PROBABILITY**

#### 2.1.1 Axiomatic definition of probability

Probability is a measure of uncertainty. Once a random experiment is defined, we call probability of the event  $\mathcal E$  the real number  $\operatorname{Prob}\{\mathcal E\}\in[0,1]$  assigned to each event  $\mathcal E$ . The function  $\operatorname{Prob}\{\cdot\}:\Omega\to[0,1]$  is called probability measure or probability distribution and must satisfy the following three axioms:

- 1.  $\operatorname{Prob}\left\{\mathcal{E}\right\} \geq 0$  for any  $\mathcal{E}$ .
- 2. Prob $\{\Omega\}=1$
- 3.  $\operatorname{Prob}\left\{\mathcal{E}_{1}+\mathcal{E}_{2}\right\}=\operatorname{Prob}\left\{\mathcal{E}_{1}\right\}+\operatorname{Prob}\left\{\mathcal{E}_{2}\right\}$  if  $\mathcal{E}_{1}$  and  $\mathcal{E}_{2}$  are mutually exclusive.

These conditions are known as the axioms of the theory of probability [76]. The first axiom states that all the probabilities are nonnegative real numbers. The second axiom attributes a probability of unity to the universal event  $\Omega$ , thus providing a normalization of the probability measure. The third axiom states that the probability function must be additive, consistently with the intuitive idea of how probabilities behave.

#### Criticisms:

how to assign the probability value to events?



# In conclusion ..

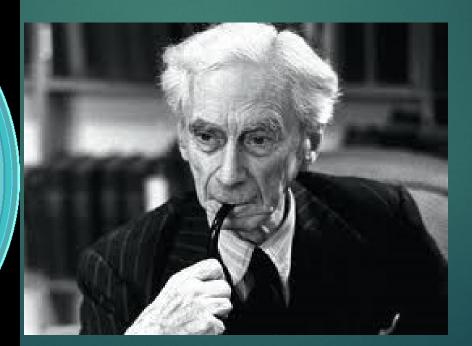
1) lack of a unique definition / measurement criterion;

2) sometimes counterintuitive.



Probability is the most important concept in modern science, especially as nobody has the slightest notion what it means.

Attributed to Bertrand Russell -



# Problem...

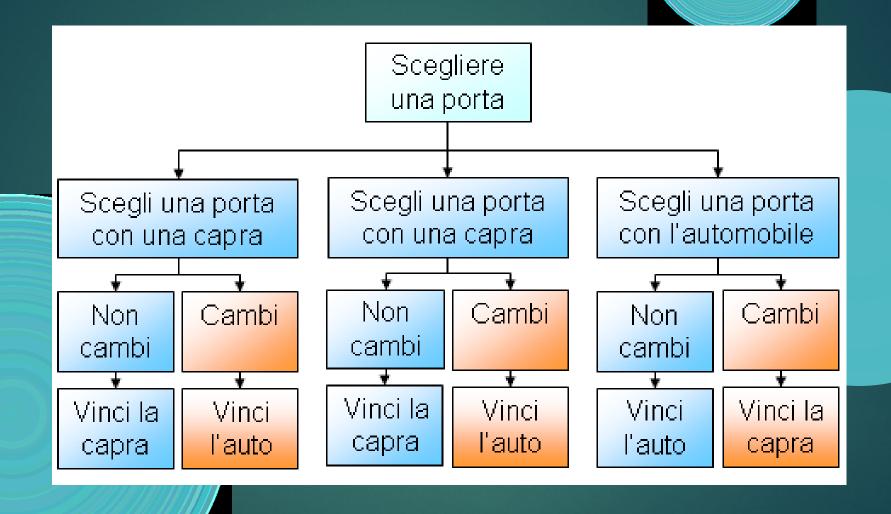
A FRIEND OF MINE HAS TWO NON-TWIN CHILDREN. KNOWING THAT ONE IS MALE, WHAT IS THE PROBABILITY THAT THE OTHER IS FEMALE?

# The Monty Hall problem





# **Solution**



# Bayes' theorem

P(A) and P(B) = probabilities of observing A and B without regard to each other  $P(A \mid B)$  = conditional probability (the probability of observing A given that B is true)

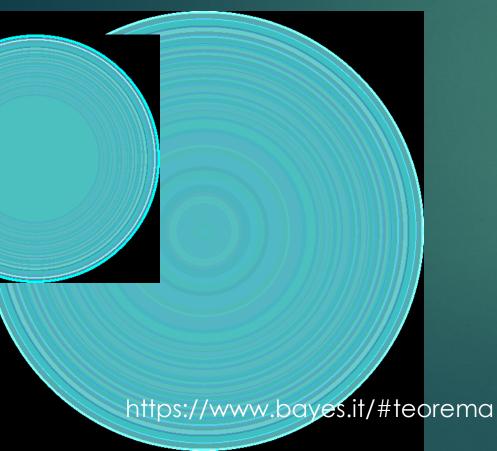
$$P(A \mid B) = rac{P(B \mid A) \, P(A)}{P(B)},$$

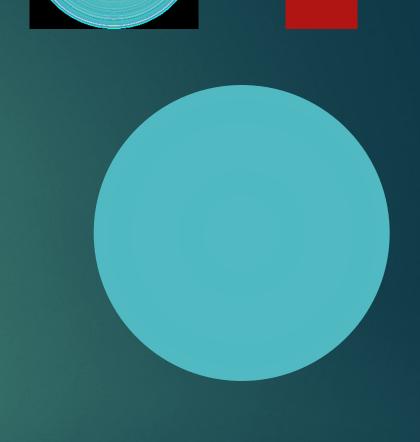
where A and B are events and  $P(B) \neq 0$ .

- ullet P(A) and P(B) are the probabilities of observing A and B without regard to each other.
- ullet  $P(A \mid B)$ , a conditional probability, is the probability of observing event A given that B is true.
- ullet  $P(B \mid A)$  is the probability of observing event B given that A is true.

# example

► Afternoon practical session





#### Rivediamo il problema di Monty Hall...

Mettiamo che la porta 3 sia stata aperta dal conduttore mostrando una capra e che il concorrente abbia selezionato la porta 1.

La probabilità che l'automobile si trovi dietro la porta 2 (ovvero la probabilità di trovare l'auto dopo aver cambiato la scelta iniziale) è :

dove A1 = l'auto si trova dietro alla porta 1;

C3 = il conduttore seleziona una capra dietro la porta 3.

La probabilità a priori che l'automobile si trovi dietro la porta 1, P(A1), = 1/3

La probabilità che il conduttore trovi una capra dietro la porta 3, P(C3), è = 1, (il conduttore sa in anticipo dove è l'automobile)

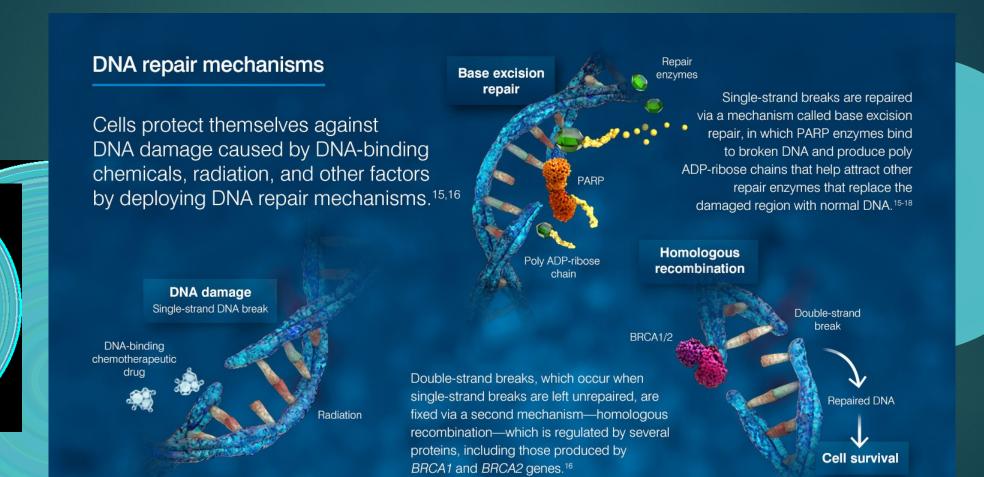
La probabilità che il conduttore selezioni una porta con dietro la capra posto ("a posteriori") che l'automobile sia dietro la porta 1,  $P(C3 \mid A1) = è 1$ .

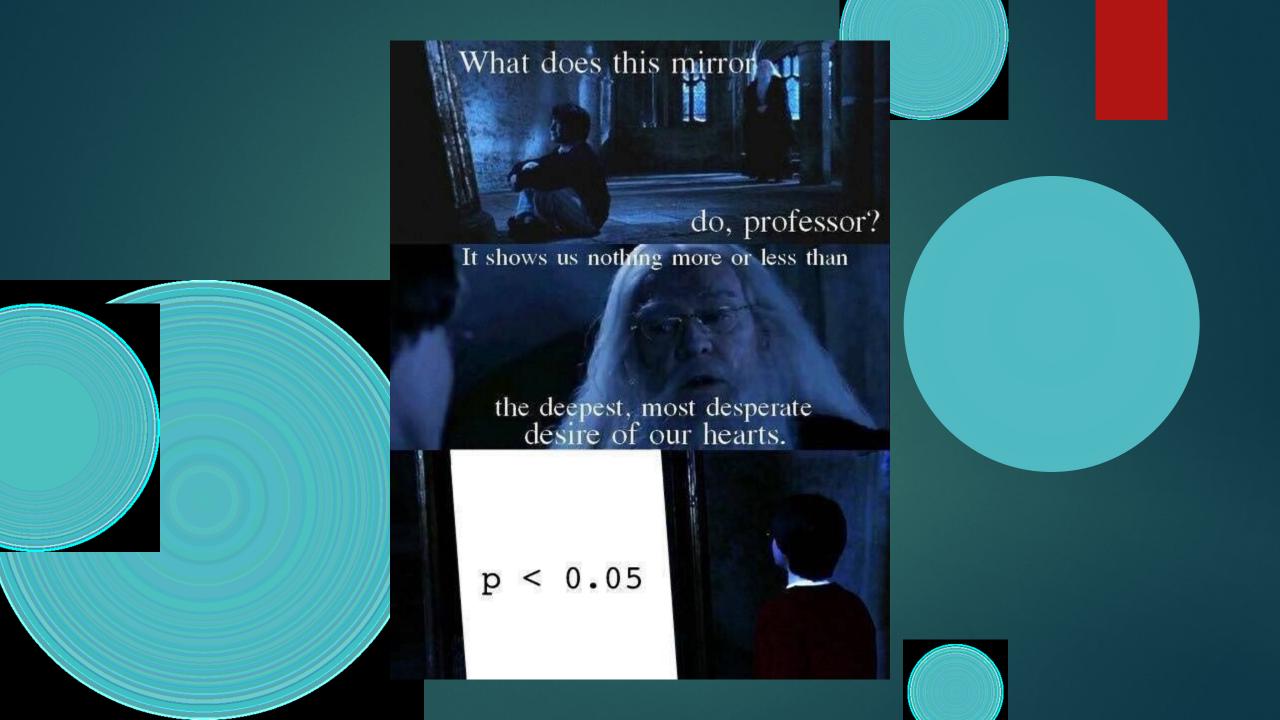
Pertanto, sfruttando il teorema di Bayes la probabilità di trovare l'auto cambiando la scelta iniziale, dopo che il conduttore (onnisciente) ha mostrato una porta con dietro la capra è:

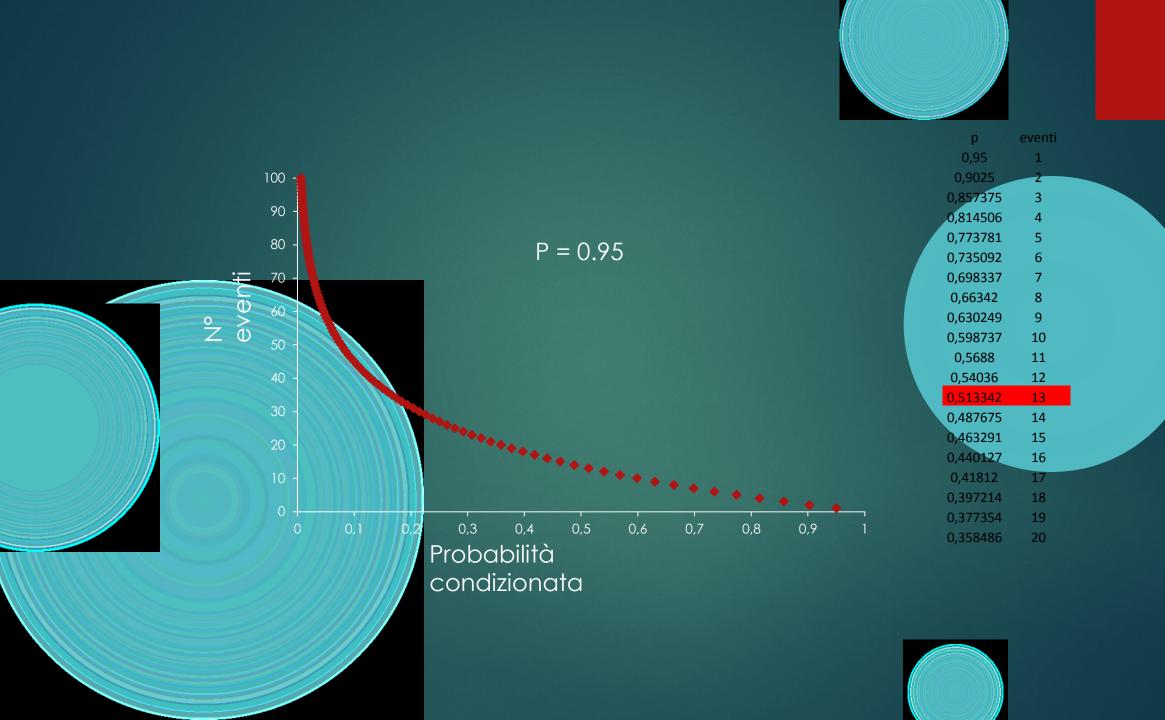
$$1-P(A1-C3) = 1 - P(C3|A1)P(A1) = 1-1*1/3 = 2$$
  
 $P(C3)$  1 3

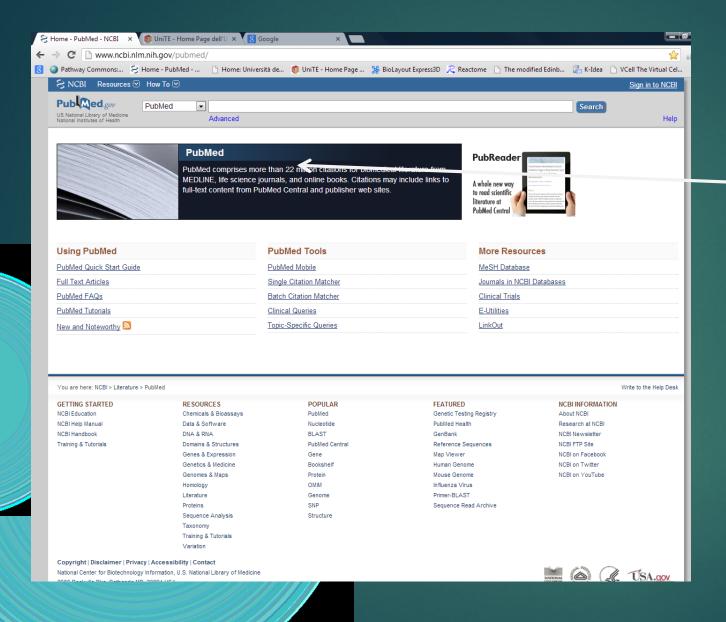


# Probability and biology







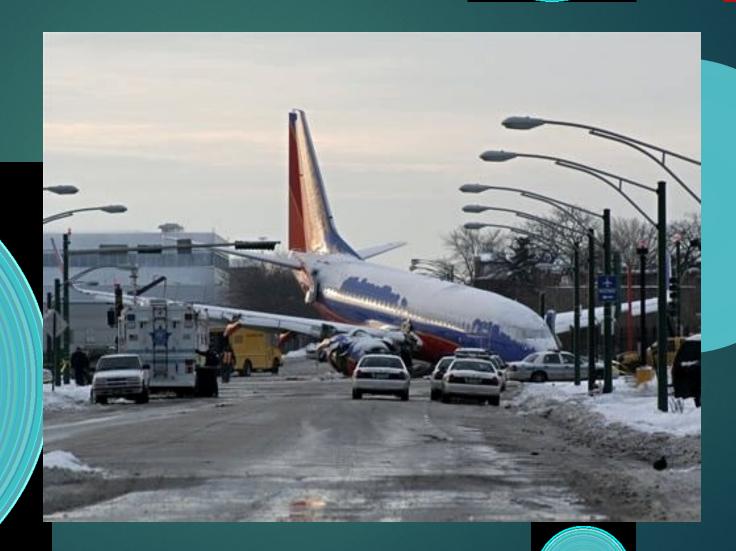


5% = 1.100.000

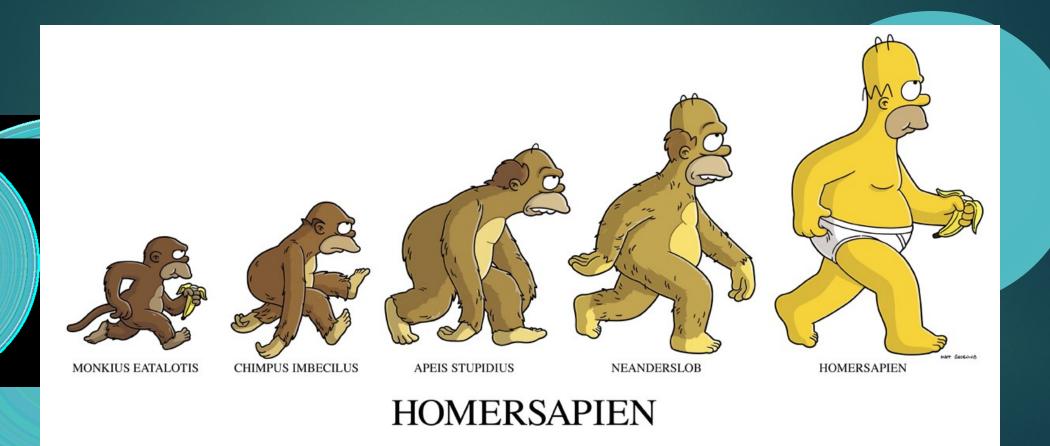
P evento raro (1/10.000)

events	р
1	0,0001
2	0,0000001
2	1F-12

3 1E-12 4 1E-16 5 1E-20 6 1E-24 7 1E-28 8 1E-32 9 1E-36 10 1E-40

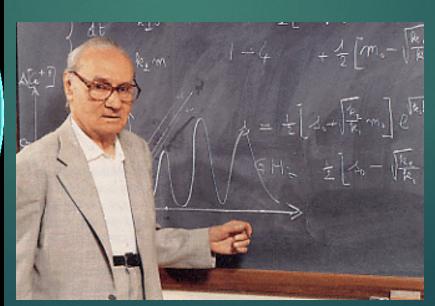


# ... but ...

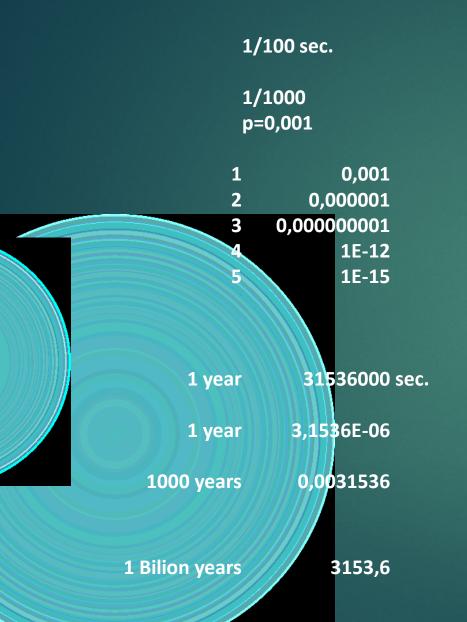


# Probabilità per unità di tempo

«Il concetto di probabilità per unità dì tempo risulta essenziale quando si tratti di spiegare come mai sia possibile il verificarsi di un certo esito, che di fatto richiede la concorrenza concertata di un gran numero di eventi casuali diversi. È la difficoltà di fronte alla quale molti capitolano, quando si tratta di giustificare in qualche modo il formarsi spontaneo di un sistema chimico appropriato di una prima cellula vivente. Sembra loro che il numero di eventi casuali concorrenti che debbono verificarsi sia talmente elevato, da rendere la probabilità della comparsa spontanea della vita un evento di probabilità evanescente» (Mario Ageno)



## Some calculations...





# Randomization

