

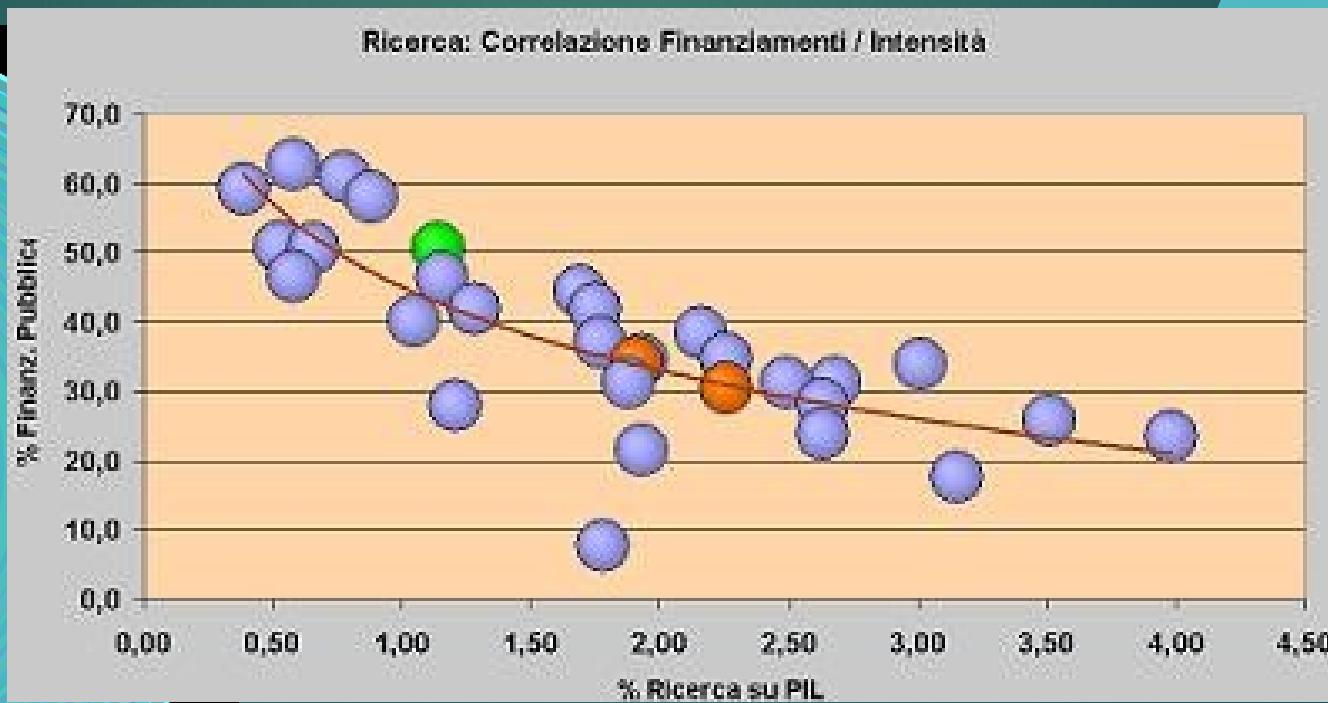


# Day 4

## correlation - regression

# CORRELATION

IN STATISTICS, **DEPENDENCE** OR **ASSOCIATION** IS ANY STATISTICAL RELATIONSHIP, WHETHER CAUSAL OR NOT, BETWEEN TWO RANDOM VARIABLES OR BIVARIATE DATA.



Correlation coefficient =  $r$

$$r = \frac{\sum (\bar{x}-x) (\bar{y}-y)}{\sqrt{\sum (\bar{x}-x)^2 \sum (\bar{y}-y)^2}}$$

# Example 1 ...

weight	height
56	156
53	159
61	
67	163
65	175
71	173
73	178
78	180
78	187
83	190
86	192

Correlation  
Coefficient  
0,960777



# Example 2 ...

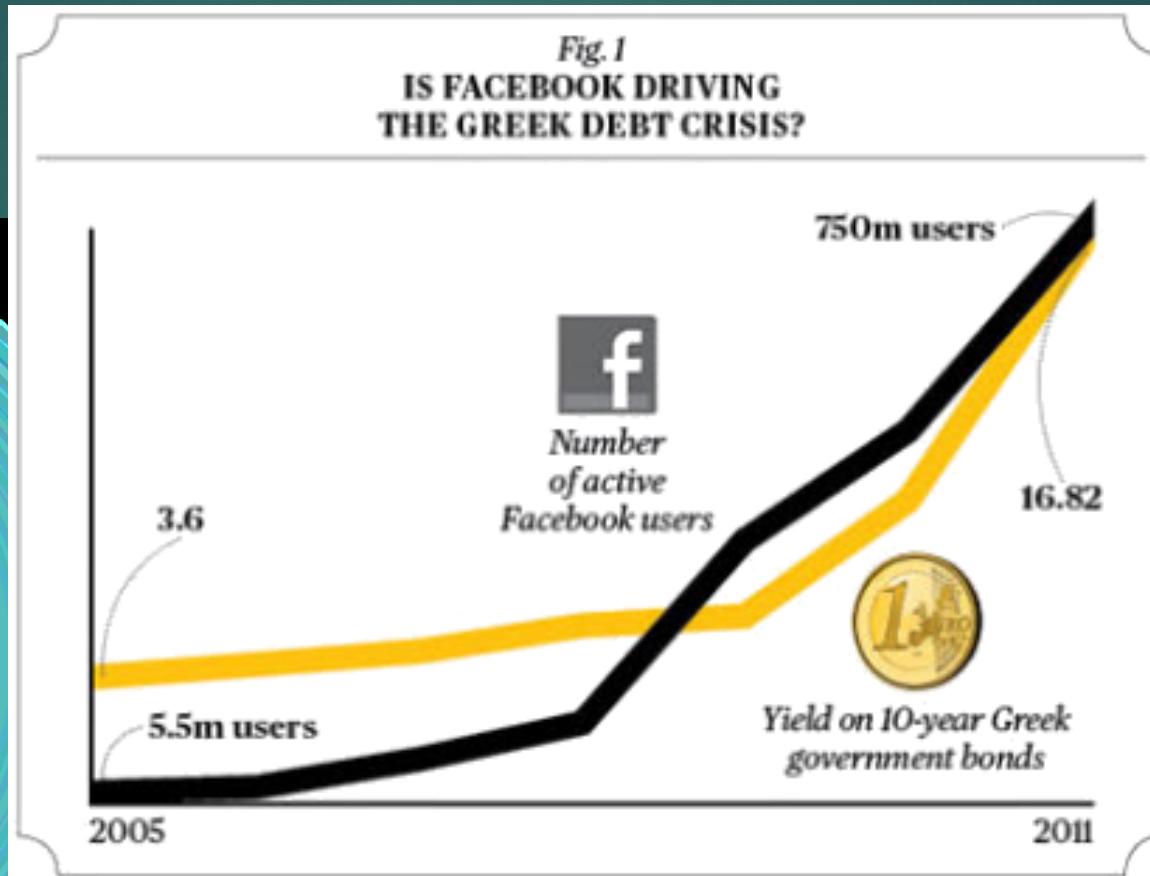
price	age
18.000	0,5
17.000	1
16.500	2
12.000	2,5
11.000	3
9.000	5
6.000	6
6.000	7
3.500	8
1.000	12

Correlation  
Coefficient

-0,95892

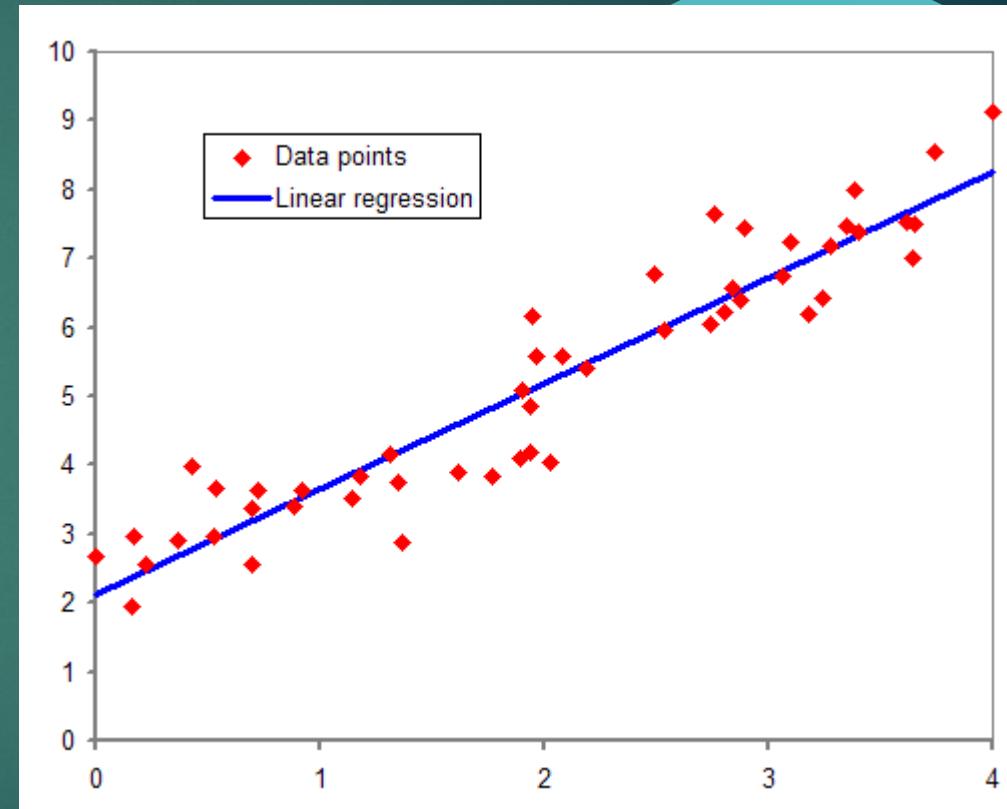


# Example 3 ...



# Linear regression

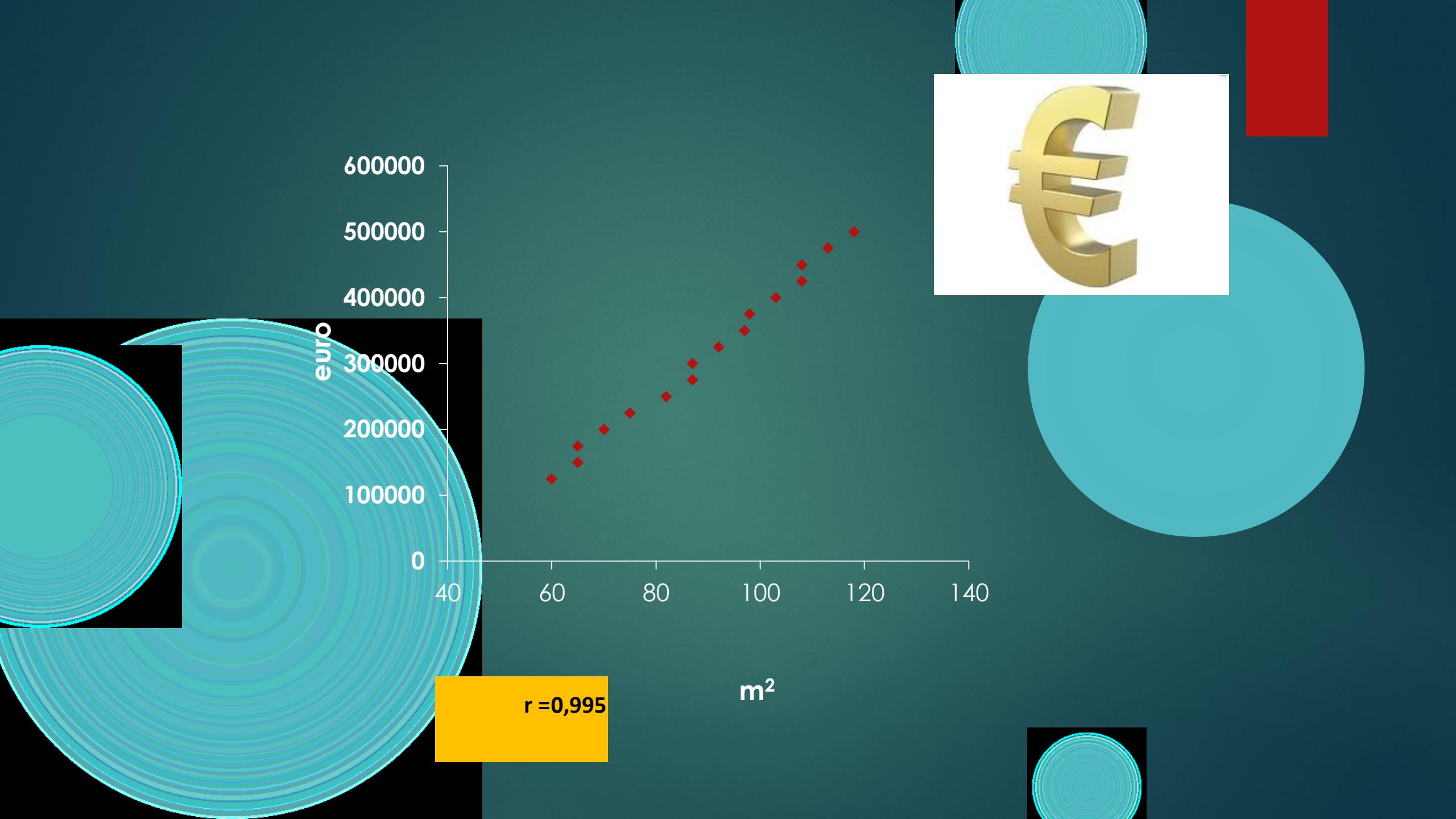
In statistical modeling, **regression analysis** is a set of statistical processes for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables (or 'predictors').

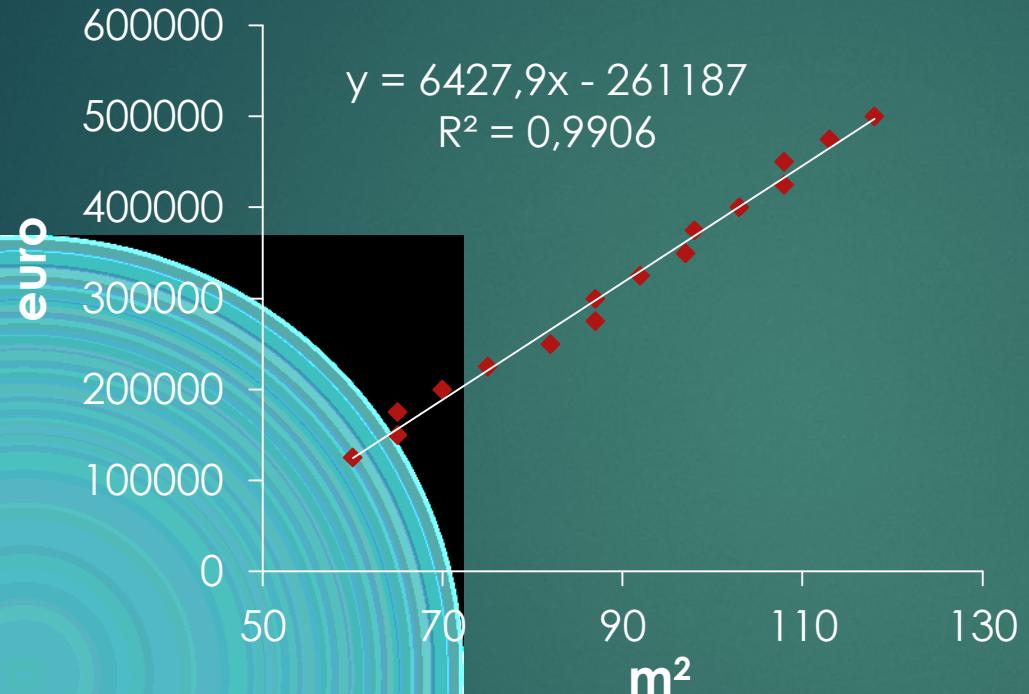


area	price
60	125000
65	150000
65	175000
70	200000
75	225000
82	250000
87	275000
92	300000
97	325000
98	350000
103	375000
108	400000
108	425000
108	450000
113	475000
118	500000



$r = 0,995$

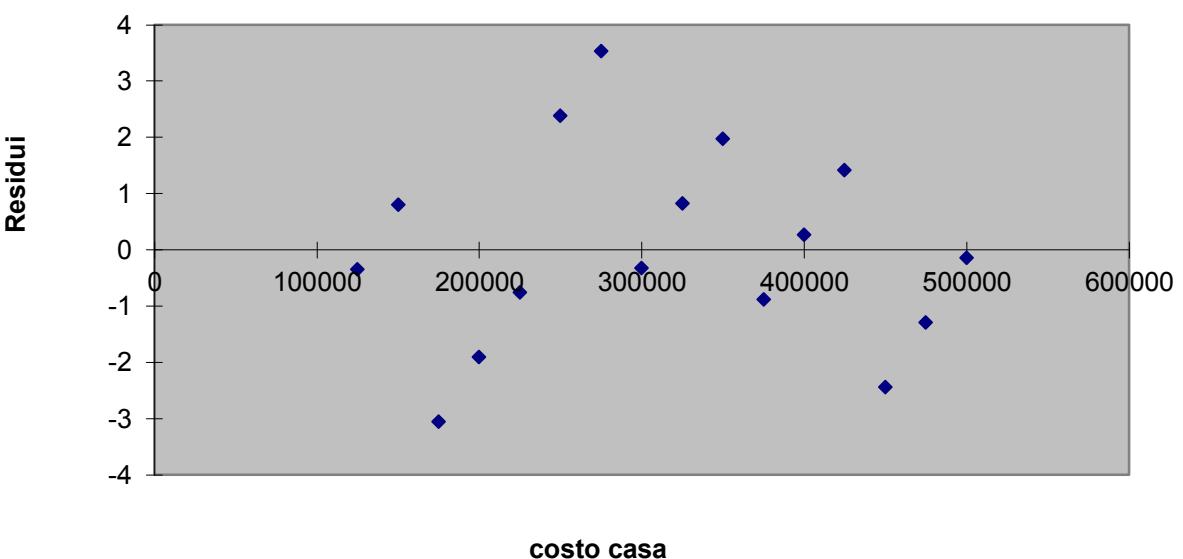




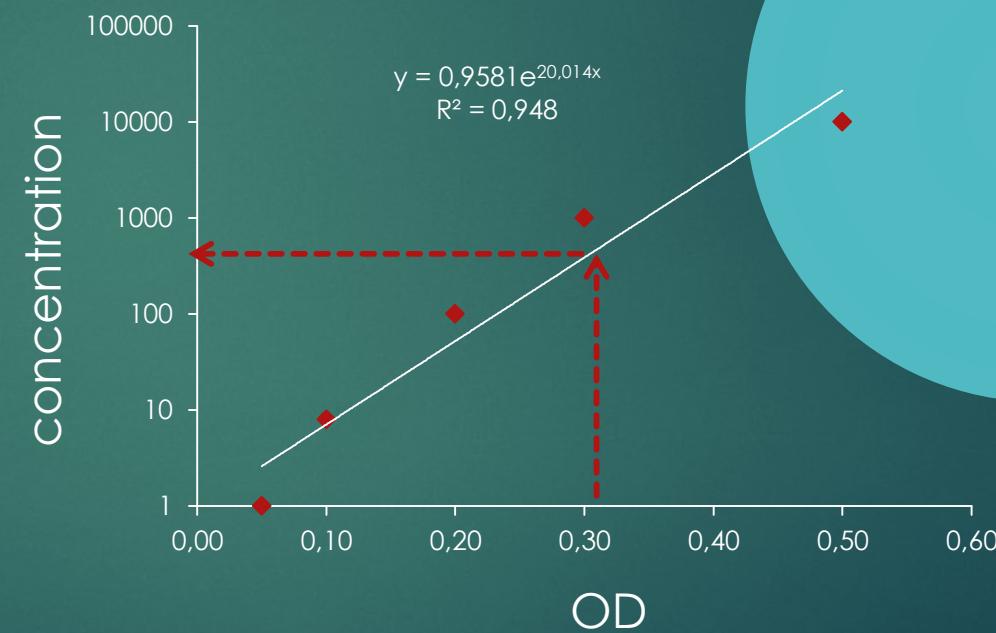
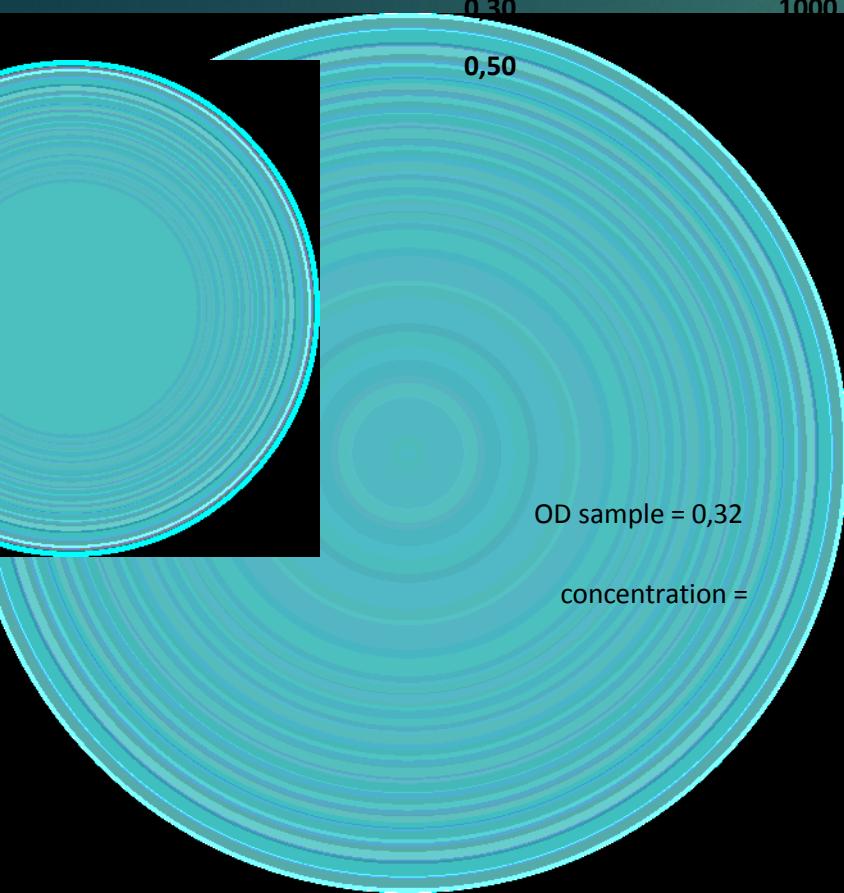
To evaluate the statistical significance of the model as a whole it comes used the test F based on the relationship between variance explained by the model and residual variance.

If the observed p-value is less than the theoretical p-value (usually 0.05) the The model used explains a significant proportion of variance in the phenomenon.

**r = 0,995313  
 p<0.0001**



# Instrument calibration



# Multivariate linear regression

price	nearby pharmacies	Distance to the City centre	floor	year	Distance to the highway	are
125000	6	250	6	1954	200	60
150000	3	300	4	1936	250	60
	4	250	4	1965	180	65
200000	3	220	3	1968	150	70
225000	1	180	1	1973	190	55
250000	4	169	4	2004	80	82
275000	3	300	5	1998	110	87
300000	1	120	3	2000	120	78
325000	5	89	2	1994	90	83
350000	5	24	1	1993	90	67
375000	3	80	1	2001	100	98
400000	2	100	5	2005	200	90
425000	1	60	3	1958	78	95
450000	5	120	2	1935	80	108
475000	6	25	1	2000	34	89
500000	3	10	1	2010	5	94

0,004187254  
NS

-0,855097664  
p<0,0001

-0,57074  
0,0209

0,39940657  
NS

-0,775507084  
0,0004  
p<0,0001



0,827505543  
p<0,0001

## OUTPUT RIEPILOGO

<i>Statistica della regressione</i>	
R multiplo	0,961953367
R al quadrato	0,925354281
R al quadrato corretto	0,875590468
Errore standard	41981,76232
Osservazioni	16

## ANALISI

### NZA

	gdl	SQ	MQ	F	Significatività F
Interazione	6	1,96638E+11	32772964116	18,5949233	0,000133688
o	9	15862215305	1762468367		
	15	2,125E+11			

### Coefficienti

Intercetta	285306,71
n° farmacie vicine	-2867,821061
distanza centro piano	-632,1291229
anno costruzione	-5082,126583
distanza viabilità	-84,46606416
superficie	-44,39216187
	3936,412268

# Example...

age	cholesterol	Glycemia	Pressure (min)	Card freq	Life style (1 - 5)
56	321	188	188	186	1
67	300	200	200	169	2
68	342	156	190	184	1
71	287	189	156	160	1
73	220	177	180	120	2
77	145	138	138	134	3
81	166	100	80	80	4
84	123	77	110	110	5
87	110	80	90	90	2
87	177	86	92	123	5
96	160	80	60	53	5

-0,8878

-0,8862

-0,8845

-0,8647

0,7872

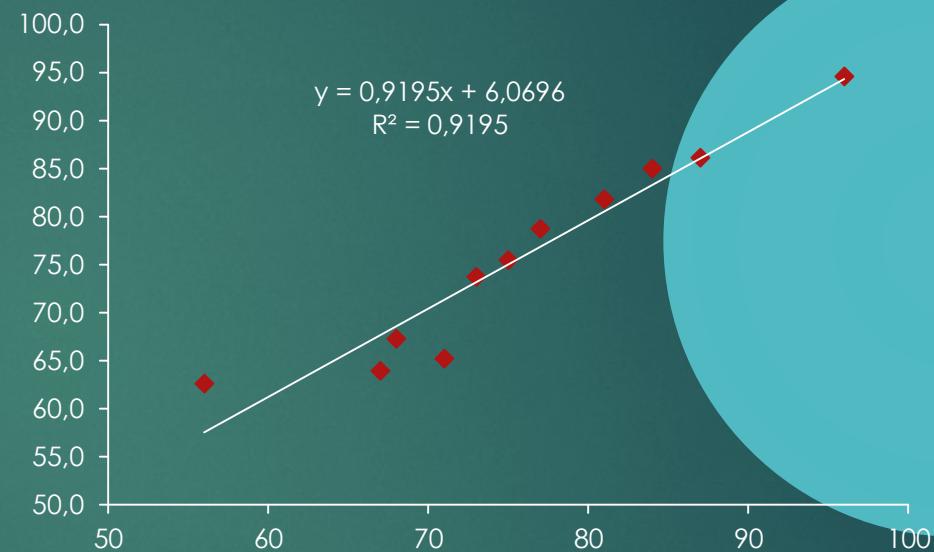
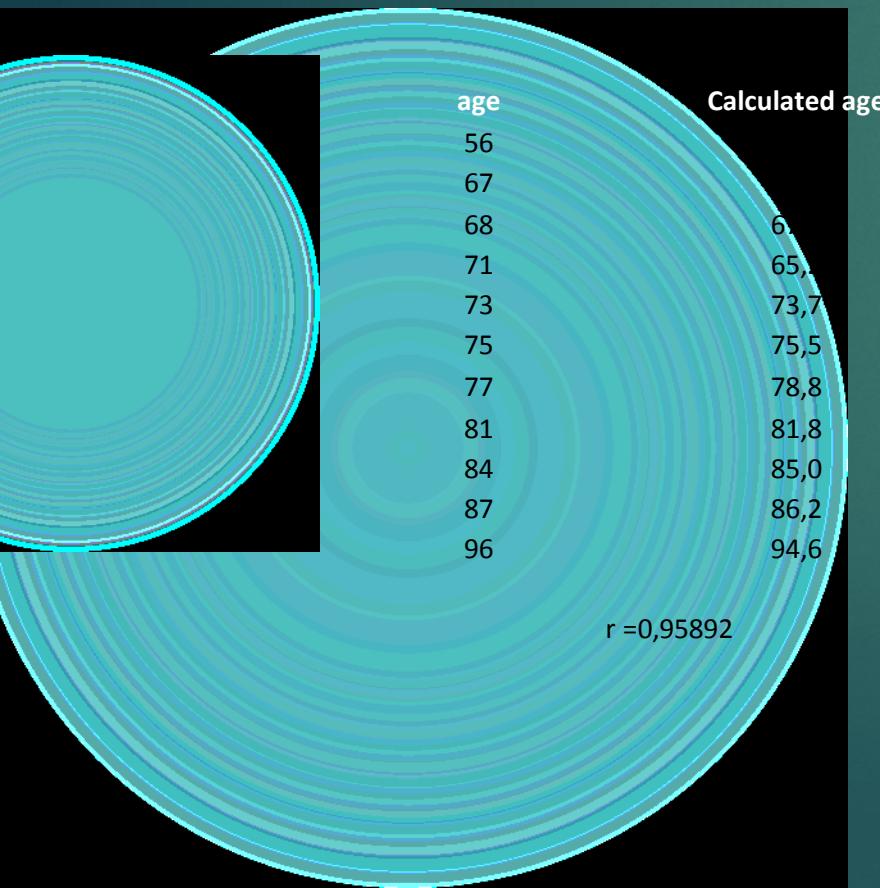
r vs. anni di vita

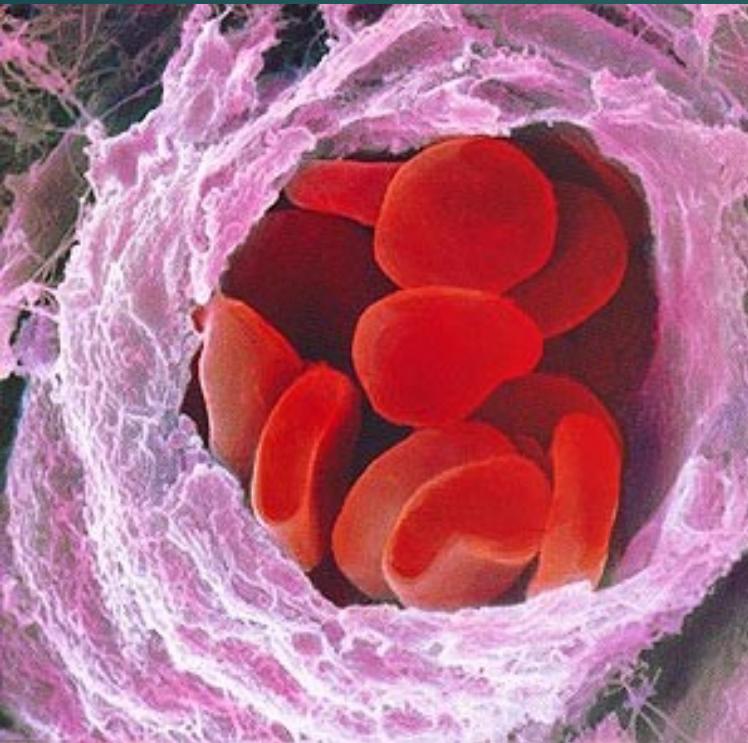
## OUTPUT RIEPILOGO

### Statistica della regressione

R multiplo	0,95892
R al quadrato	0,91952

	<i>Coefficienti</i>
Intercetta	111,6407
colesterolo	-0,01621
glicemia	-0,14676
pressione min	0,027452
freq card	-0,11672
stile di vita (1 - 5)	0,378917





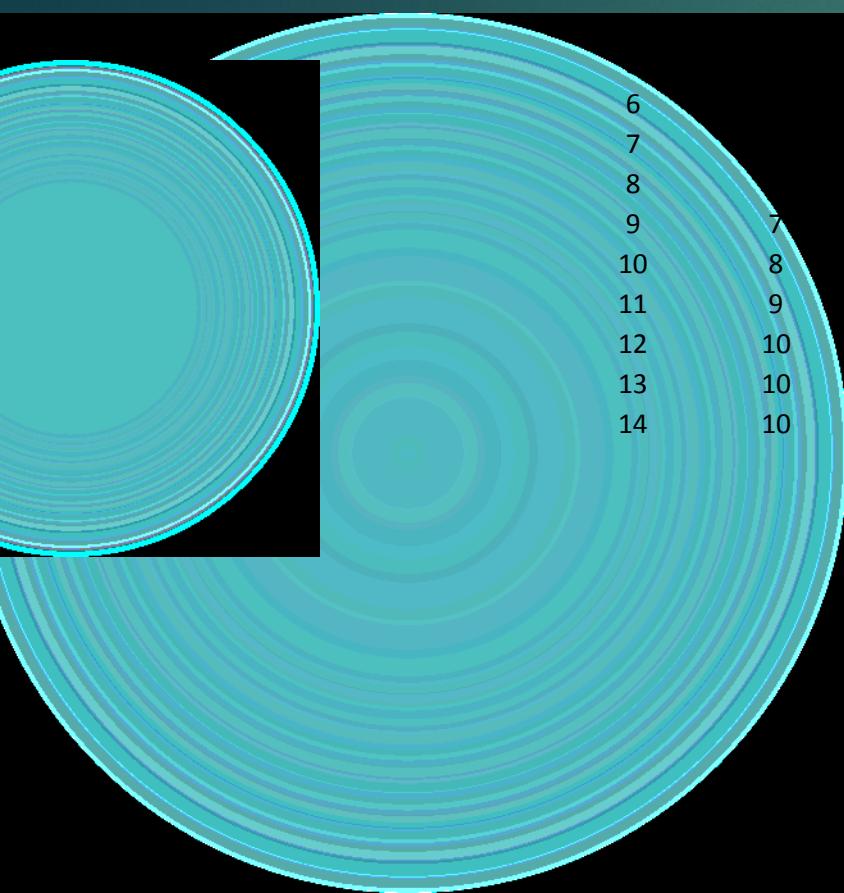
**Table 2** *p* value of correlation of MCV with other hematological parameters in CTR and MICRO group of dogs assessed by a linear multivariate regression model

NS, non significant =  $p > 0.05$

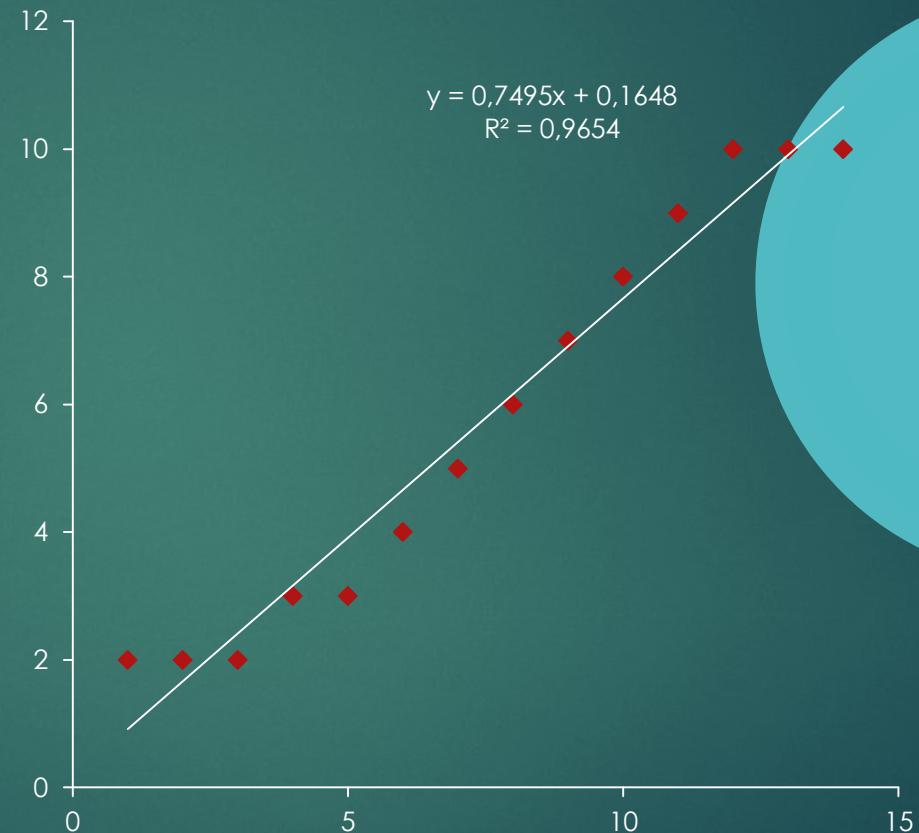
	CTR	MICRO
WBC	NS	$p < 0.05$
RBC	$p < 0.001$	$p < 0.001$
Hgb	$p < 0.05$	NS
Hct	NS	$p < 0.001$
MCH	$p < 0.001$	NS
MCHC	$p < 0.001$	NS
RDW	NS	$p < 0.001$
PLT	NS	NS
MPV	NS	$p < 0.05$

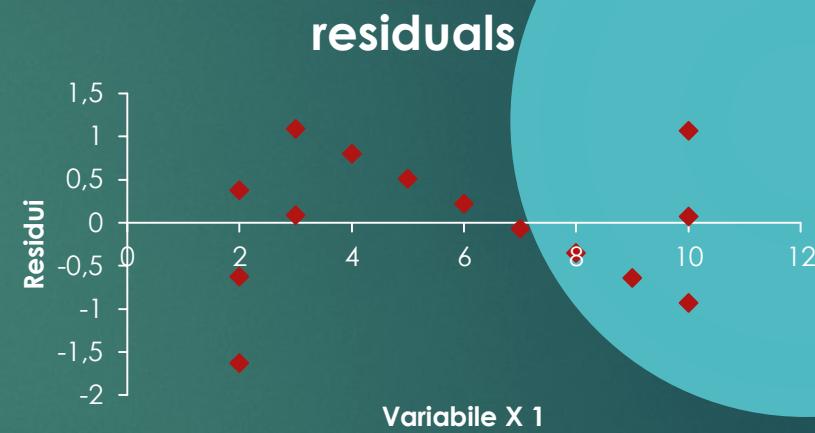
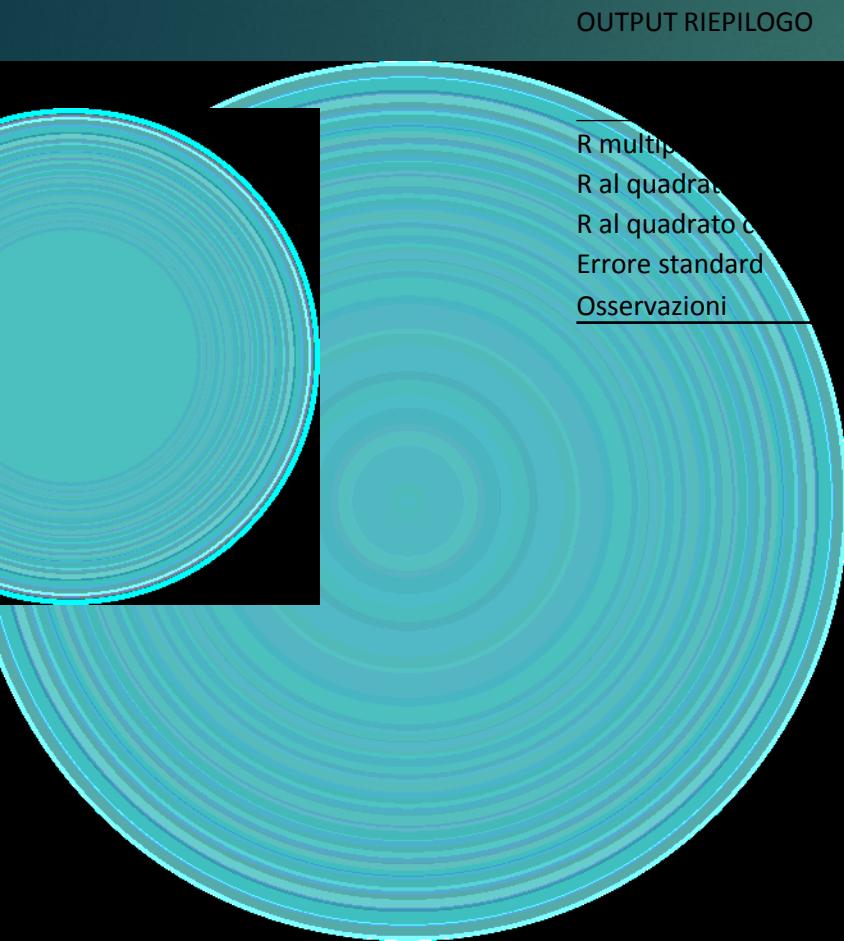
# ANALYTICAL METHODS

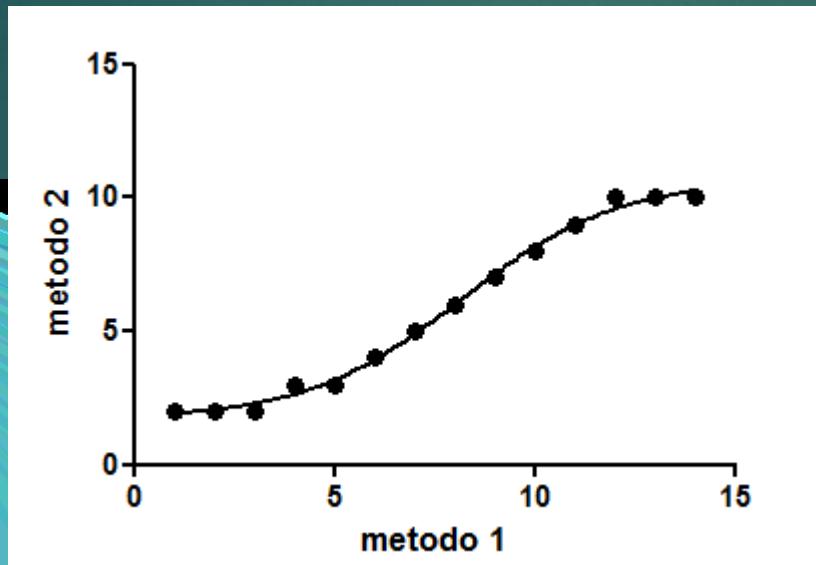
18



method 1	method 2
1	2
2	2
3	2







Boltzmann sigmoidal  
Best-fit values

BOTTOM

TOP

V50

SLOPE

Std. Error

BOTTOM

TOP

V50

SLOPE

95% Confidence Intervals

BOTTOM

TOP

V50

SLOPE

Goodness of Fit

Degrees of Freedom

R<sup>2</sup>

Absolute Sum of Squares

Sy.x

Number of points

Analyzed

1,688

10,68

8,155

1,952

0,2276

0,3054

0,1873

0,2040

1.181 to 2.195

9.999 to 11.36

7.738 to 8.572

1.497 to 2.406

10

0,9958

0,5560

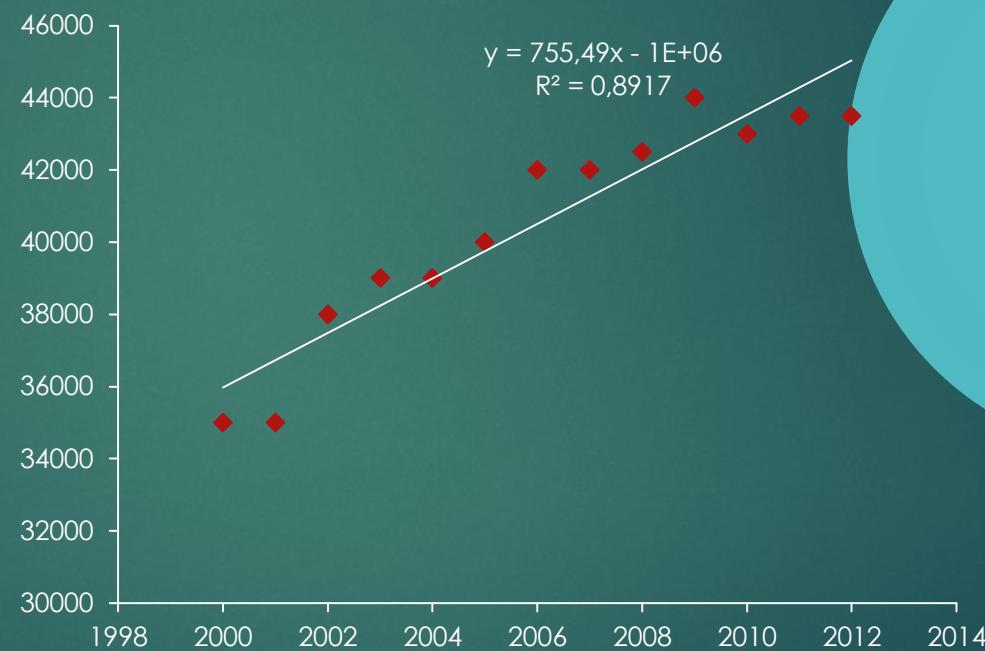
0,2358

14

# REGRESSION AND PREDICTION

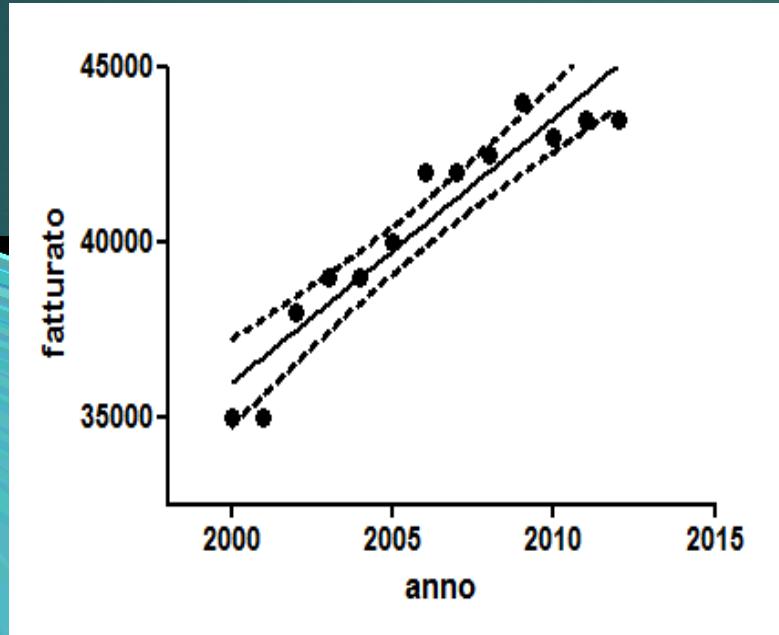
21

anno	fatturato
2000	35000
2001	35000
2002	35000
2003	35000
2004	39000
2005	40000
2006	42000
2007	42000
2008	42500
2009	44000
2010	43000
2011	43500
2012	43500
2012	?
2013	?



# REGRESSION AND PREDICTION

22



Goodness of Fit

$r^2$

Sy.x

Is slope significantly non-zero?

F

DFn, DFd

P value

Deviation from zero?

Data

Number of X values

Maximum number of Y replicates

Total number of values

Number of missing values

0,89

1100

91

1.0, 11

< 0.0001

Significant

13

1

13

0

2010,08	43582,110	967,3312	967,3312
2010,20	43672,760	982,8868	982,8868
2010,32	43763,420	998,6403	998,6403
2010,44	43854,070	1014,583	1014,583
2010,56	43944,730	1030,705	1030,705
2010,68	44035,380	1046,999	1046,999
2010,80	44126,040	1063,457	1063,457
2010,92	44216,700	1080,071	1080,071
2011,04	44307,350	1096,835	1096,835
2011,16	44398,010	1113,740	1113,740
2011,28	44488,660	1130,782	1130,782
2011,40	44579,320	1147,955	1147,955
2011,52	44669,970	1165,251	1165,251
2011,64	44760,630	1182,666	1182,666
2011,76	44851,290	1200,195	1200,195
2011,88	44941,940	1217,832	1217,832
2012,00	45032,600	1235,574	1235,574

# Non linear models: sigmoid

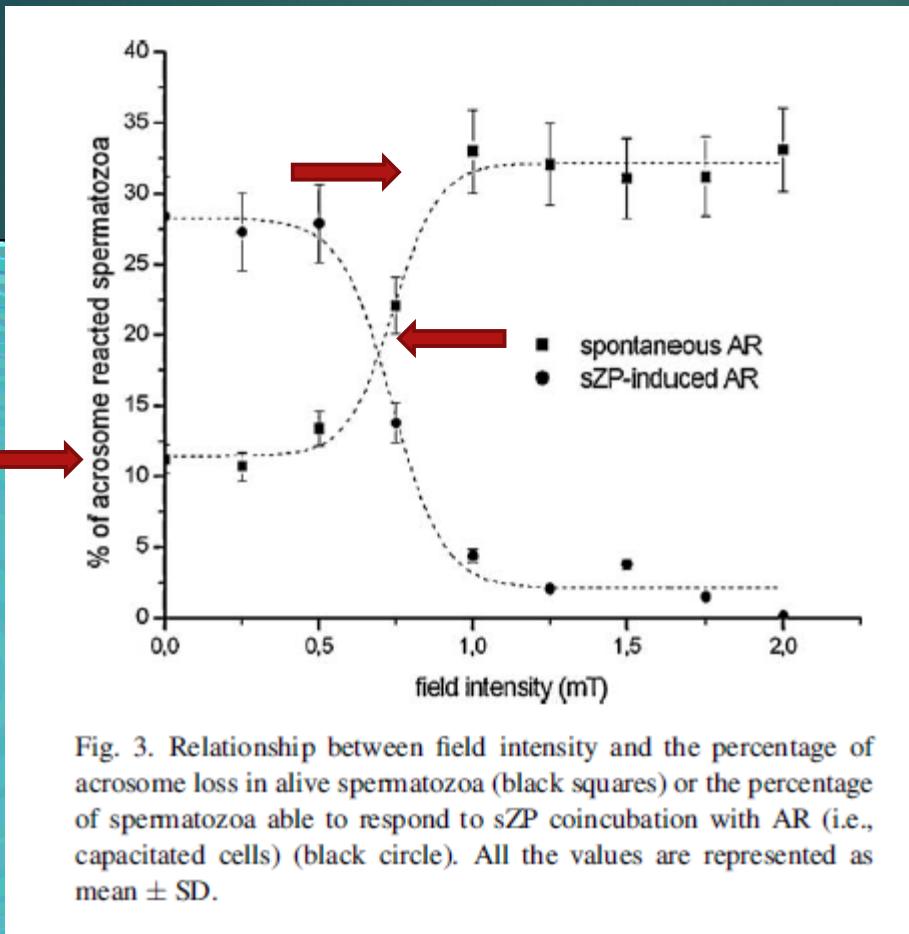
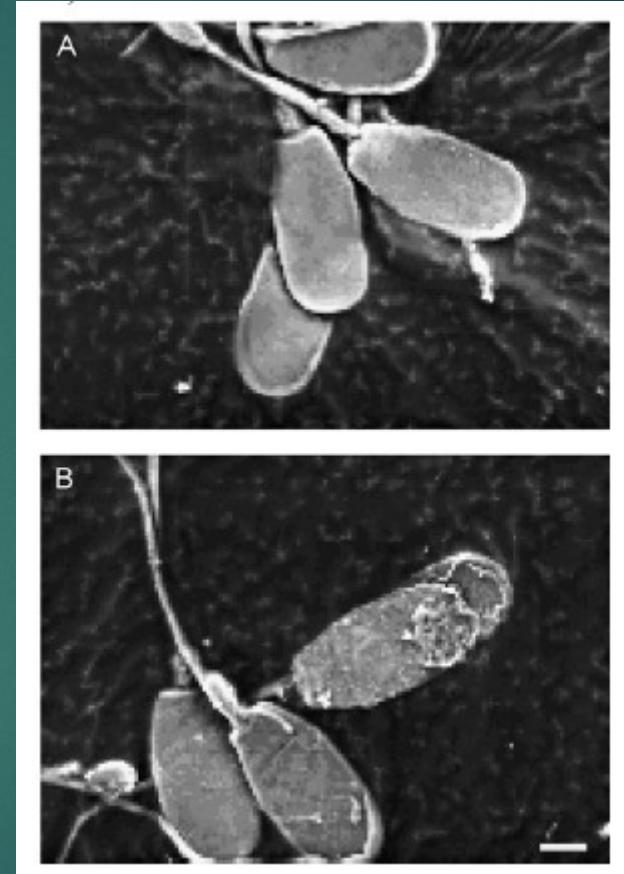


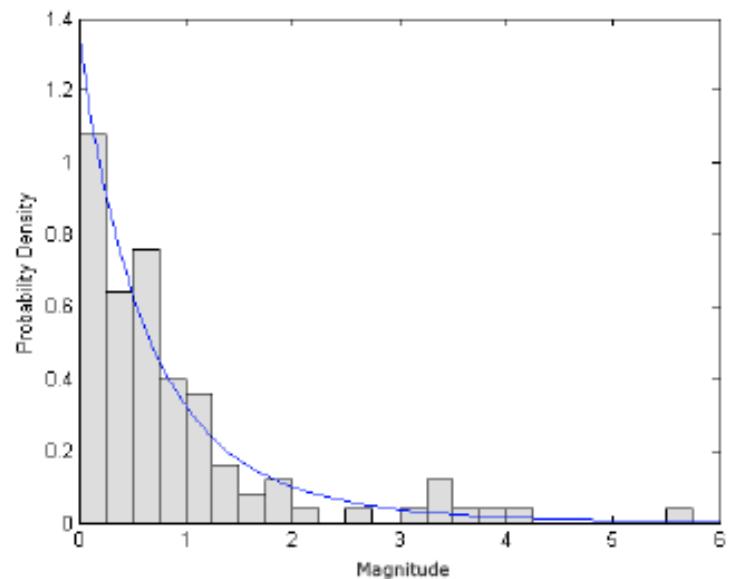
Fig. 3. Relationship between field intensity and the percentage of acrosome loss in alive spermatozoa (black squares) or the percentage of spermatozoa able to respond to sZP coincubation with AR (i.e., capacitated cells) (black circle). All the values are represented as mean  $\pm$  SD.



# Non linear model: power law

$$y = ax^{-b}$$

Chart 2: The Power Law Distribution



# Logistic model

- ▶ La **regressione logistica** è un caso particolare di modello lineare generalizzato avente come funzione link la funzione logit. Si tratta di un modello di regressione applicato nei casi in cui la variabile dipendente  $y$  sia di tipo dicotomico riconducibile ai valori 0 e 1, come **lo sono tutte le variabili che possono assumere esclusivamente due valori: vero o falso, maschio o femmina, vince o perde, sano o ammalato, ecc.**

## Logit

Da Wikipedia, l'enciclopedia libera.

Il logit è una funzione, che si applica a valori compresi nell'intervallo (0,1), tipicamente valori rappresentanti probabilità. Viene definito come

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = \ln(p) - \ln(1-p)$$

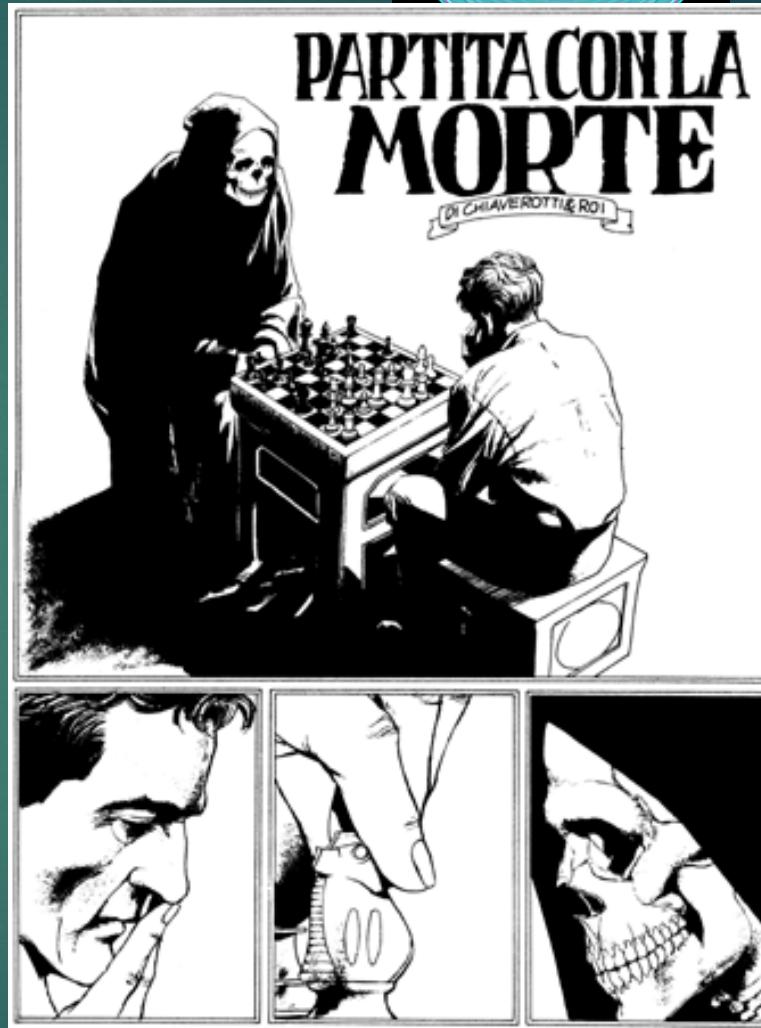
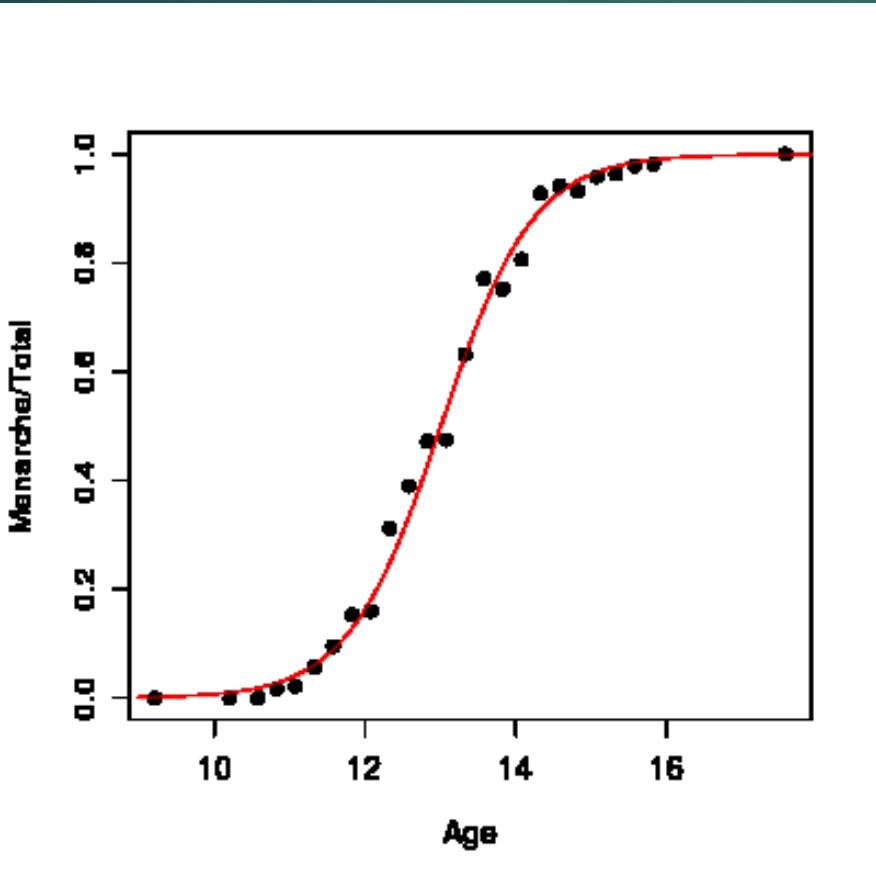
dove  $\ln$  è il logaritmo naturale e  $\frac{p}{1-p}$  è detto odds.

Ha come funzione inversa

$$p = \frac{e^{\text{logit}}}{1 + e^{\text{logit}}}$$

La funzione logit si applica ad esempio nella regressione logistica e nella variabile casuale logistica.

# Example...



important

► Correlation by itself does not imply a cause and effect relationship!!!!!!



Correlation matrix of gene expressions

