Lezione 6 25/11/2021

giovedì 25 novembre 2021 14:18

Lezione #6
25/11/2021

SISTEMI RIGIDI

$$\vec{F}^{(RIS)} = M\vec{a}_{CDM}$$

Un sistema rigido è in equilibrio quando

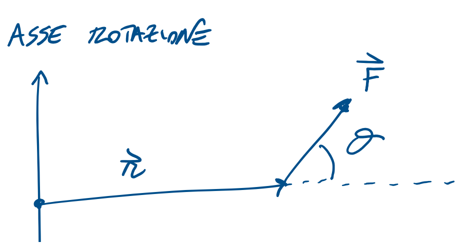
$$\begin{cases} \vec{F}^{(RIS)} = \vec{0} & \text{moto traslazionale} \\ \vec{M} = \vec{0} & \text{" rotazionale} \end{cases}$$

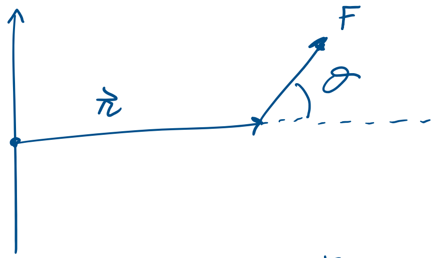
$$\vec{M} = \vec{r} \times \vec{F}$$

"vettori" ⇒ prodotto vettoriale

momento di una forza

$$\vec{M} \text{ è un vettore} \begin{cases} \text{modulo} \\ \text{direzione} \\ \text{verso} \end{cases}$$

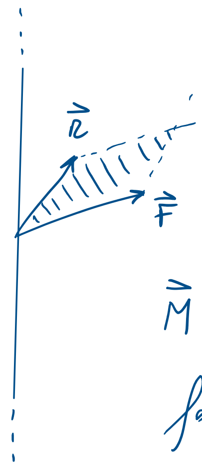




$$M = rF\sin\theta$$

•) Modulo $\quad M = rF\sin\theta$

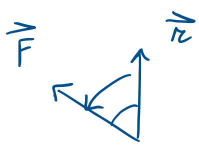
••) Direzione



$\vec{M}$ è $\perp$ al piano formato da $\vec{r}$ e $\vec{F}$

•••) Verso

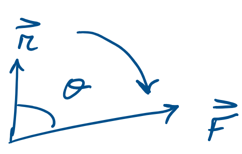
a) $M > 0$ se $\vec{r} \curvearrowleft \vec{F}$ in senso antiorario



antiorario

$\vec{M} > 0 \quad \odot$ uscente

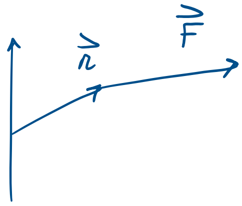
b) $M < 0$ se $\vec{r} \curvearrowright \vec{F}$ in senso orario



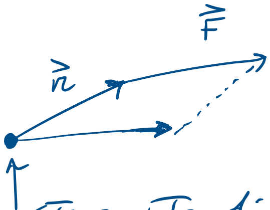
$\vec{M} < 0$

$\otimes$ entrante

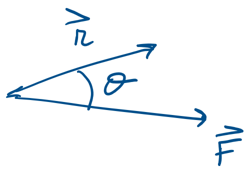
Come calcolare il momento:

1) 

spostare $\vec{F}$ in modo tale che abbia lo stesso origine $\vec{r}$



stesso pto di applicazione!

2) 

$$M = rF\sin\theta$$

3) $\vec{r} \curvearrowright \vec{F}$ $\begin{cases} \text{antiorario} \Rightarrow M = rF\sin\theta \quad \odot \\ \text{orario} \Rightarrow M = -rF\sin\theta \quad \otimes \end{cases}$

$$[M] = [r][F] = Nm$$

Esempi:

LEVE BIOMECCANICHE

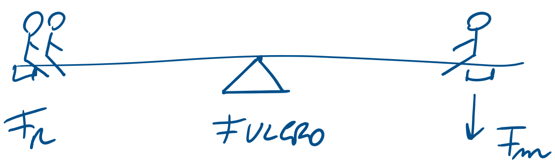
ARTICOLAZIONI

LEVA: una macchina meccanica vincolata in grado di trasferire energia

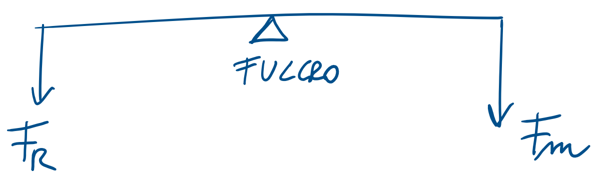
$\vec{F}_m$ = Forza motrice

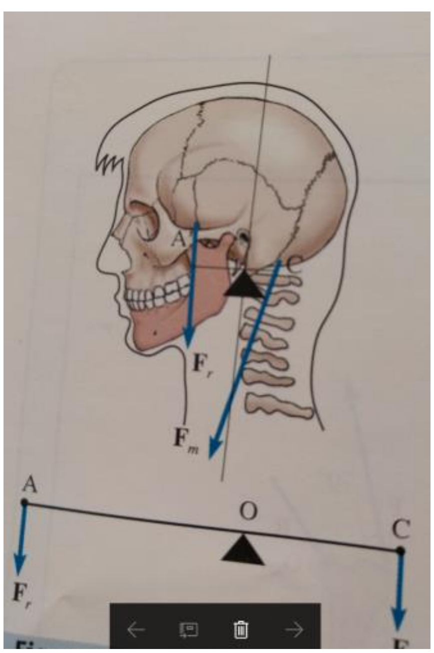
$\vec{F}_R$ = " resistiva

Fulcro = vincolo, asse di rotazione, fisso



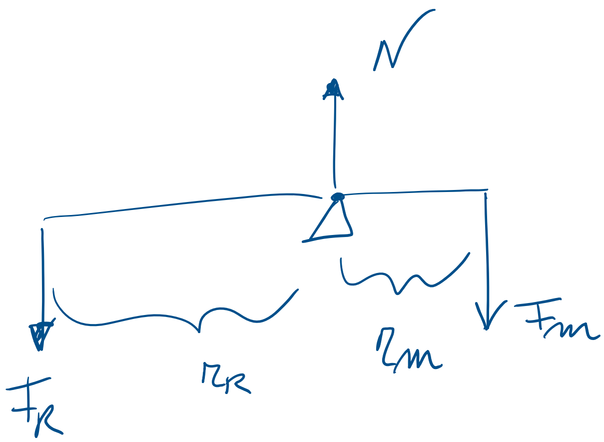
LEVA I TIPO:





EQUILIBRIO:

$$\begin{cases} \vec{F}^{(RIS)} = \vec{0} \\ \vec{M}^{(RIS)} = \vec{0} \end{cases}$$



$F_R = 80\,N$ = forza peso cranio

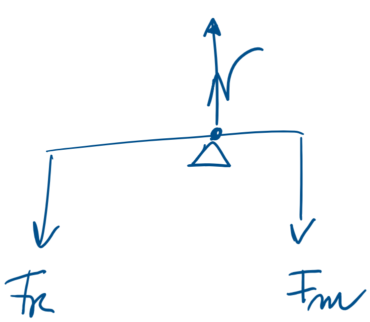
$r_m = 2$ cm

$r_R = 8$ cm

F_m = forza motrice dei muscoli splenici = ?

N = ?

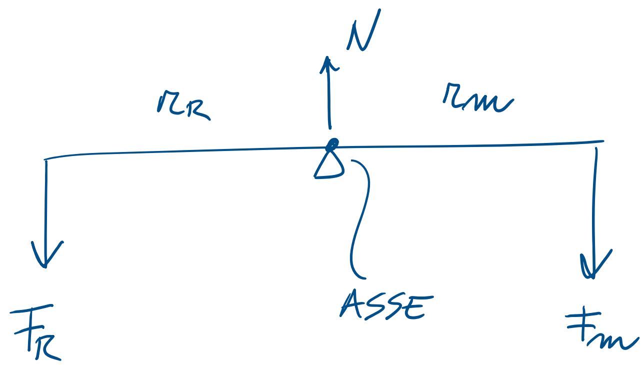
Equilibrio
$$\begin{cases} \vec{F}^{(RIS)} = \vec{0} \\ \vec{M} = \vec{0} \end{cases}$$



$$F^{(RIS)} \begin{cases} F_x^{RIS} = 0 = 0 \\ F_y^{RIS} = -F_R + N - F_m = 0 \end{cases}$$

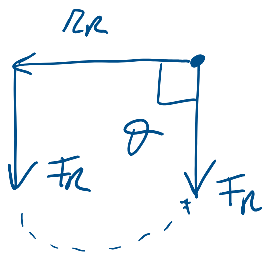
$$\boxed{N = F_m + F_R}$$

$$\vec{M}^{RIS} = \vec{0}$$

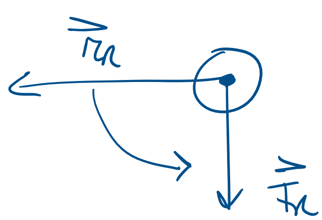


$$\vec{M}^{RIS} = \vec{M}_R + \vec{M}_m = \vec{0}$$

$\vec{M}_R$:



$\theta = 90°$

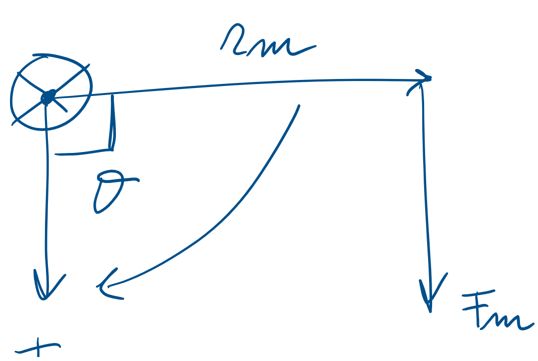


senso antiorario

$M_R > 0$

$$M_R = + r_R F_R \underbrace{\sin\theta}_{1}$$

$\vec{M}_m$:



$\theta = 90°$

$\downarrow \swarrow$

F_m

$\downarrow F_m$

Senso orario

$M<0$

$$M_m = -r_m F_m$$

$$M^{ris} = \boxed{r_R F_R - r_m F_m = 0}$$

$$F_m = \left(\frac{r_R}{r_m}\right) F_R$$

$$F_m = \left(\frac{8\,\cancel{10^{-2}}}{2\,\cancel{10^{-2}}}\right) F_R =$$

$$\boxed{F_m = 4 F_R}$$

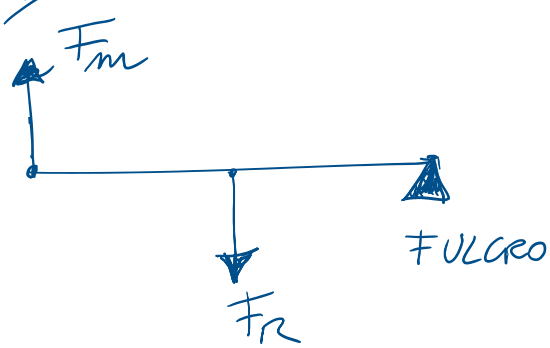
La F. nei muscoli splenici è 4 volte il peso della testa

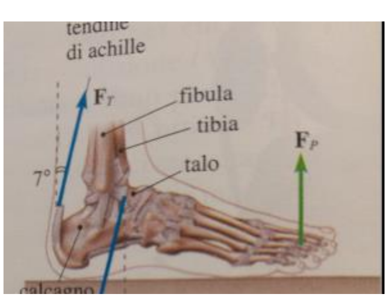
$$\boxed{F_m = 4 \cdot 80 = 320\,N}$$

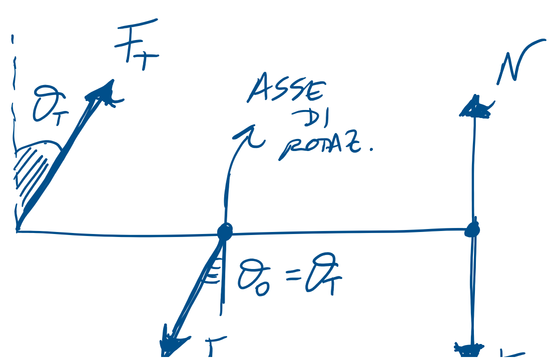
$$F^{(RIS)} \Rightarrow \quad N = F_m + F_R = 4F_R + F_R$$

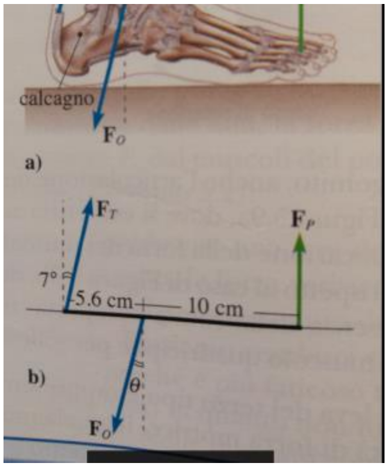
$$\boxed{N = 5\,F_R}$$

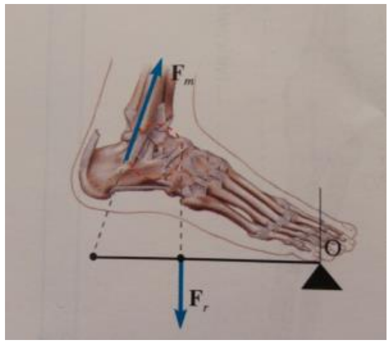
- LEVA II TIPO -

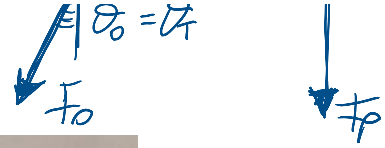












F_T : " tendine Achille

F_O : " osso del piede

$\boxed{F_T = ?}$

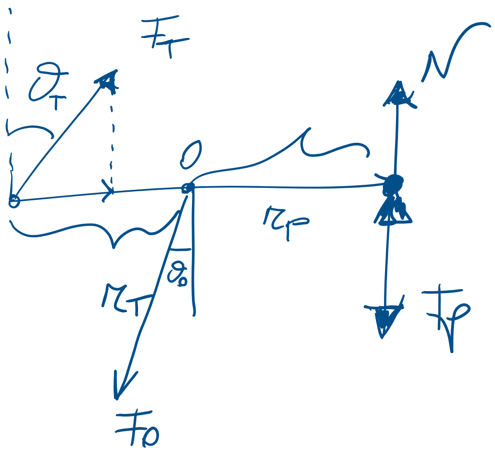
$\theta_T = \theta_O = 7^\circ$

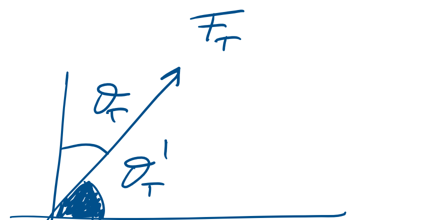
$\boxed{F_O = ?}$

$F_P = 900\,N$

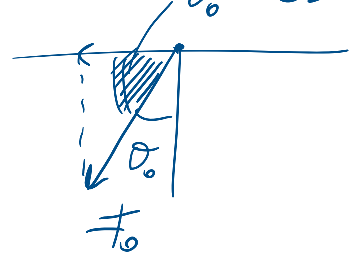
$r_P = 10\,cm$

$r_T = 5{,}6\,cm$



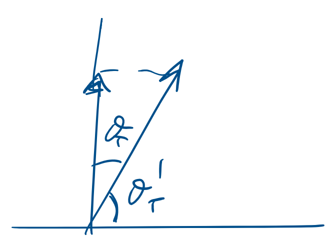


$\boxed{\theta'_T = 90 - 7 = 83^\circ}$



$\vec{F}^{ris} = \vec{0}$

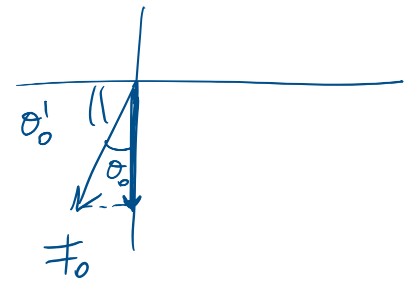
$$\begin{cases} F_x^{ris} = +F_T \cos\theta'_T + \\ \qquad - F_O \cos\theta'_O \\ F_y^{ris} = +F_T \sin\theta'_T + \end{cases}$$



$$F_y^{ris} = + F_T \sin\theta'_T +$$

$$- F_0 \sin\theta'_0 + N_T$$

$$- F_p$$



$$\begin{cases} F_x^{ris} = F_T \cos\theta'_T - F_0 \cos\theta'_0 = 0 \\ F_y^{ris} = F_T \sin\theta'_T - F_0 \sin\theta'_0 + N - F_p = 0 \end{cases}$$

$$F_T \cancel{\cos\theta'_T} - F_0 \cancel{\cos\theta'_0} = 0 \qquad \boxed{F_T = F_0}$$

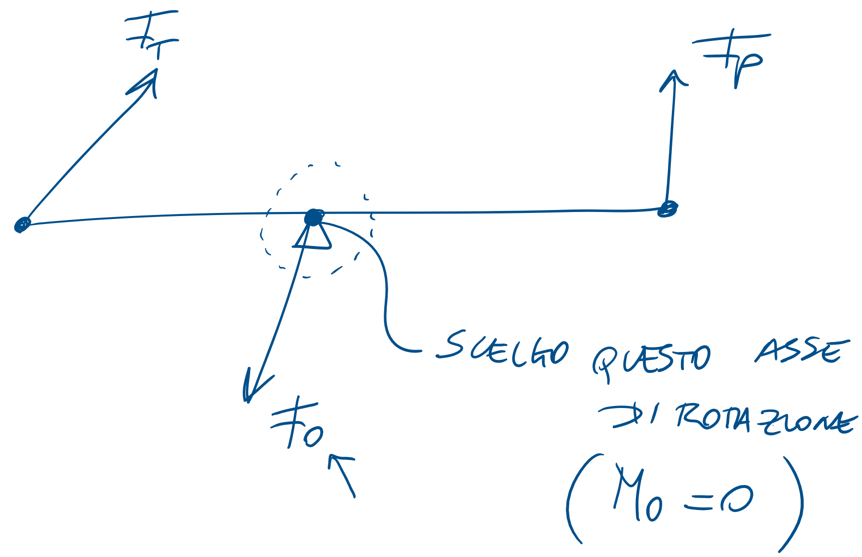
$$\cancel{F_T} \cancel{\sin\theta'_T} - F_0 \cancel{\sin\theta'_0} + N - F_p = 0$$

$\cancel{F_0}$

$$\boxed{N = F_p}$$

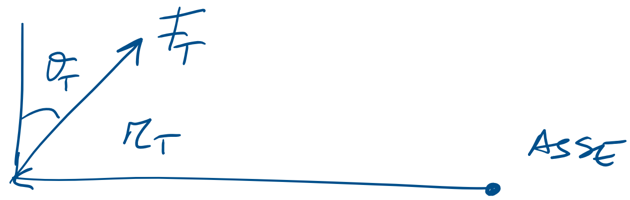
F_T $\uparrow$ F_p

$\vec{M}^{RIS} = \vec{0}$

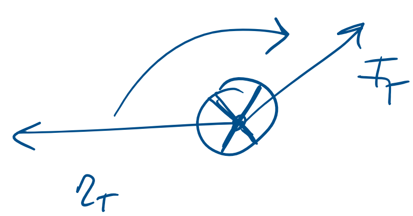


$\vec{M}^{RIS} = \vec{M}_T + \vec{M}_P = \vec{0}$

$\vec{M}_T$:



$\boxed{\theta = 90° + \theta_T = 97°}$

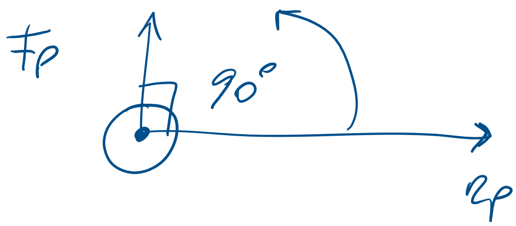


senso orario

$M_T < 0$

$$\boxed{M_T = - r_T F_T \sin(97^\circ)}$$

M_p :



senso antiorario

$M > 0$

$$M_p = r_p F_p$$

$$M^{ris} = 0 \Rightarrow - r_T F_T \sin(97^\circ) + r_p F_p = 0$$

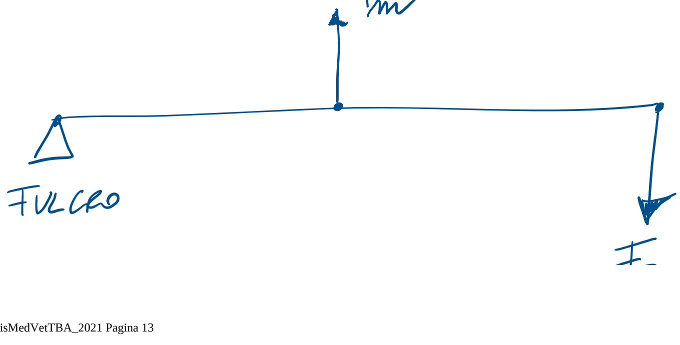
$$F_T = \left(\frac{r_p}{r_T \sin(97^\circ)} \right) F_p$$

$$\boxed{F_T \simeq 1{,}8\ F_P}$$

Sul Tendine di Achille si esercita per ogni passo una forza pari al doppio della nostra forza peso!!!

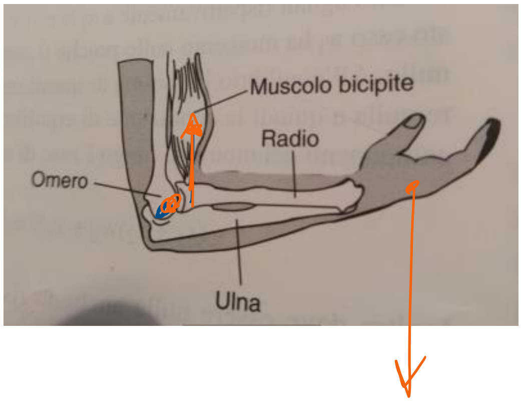
$$\boxed{F_O = F_T = 1{,}8 F_P}$$

LEVA III TIPO



FULCRO

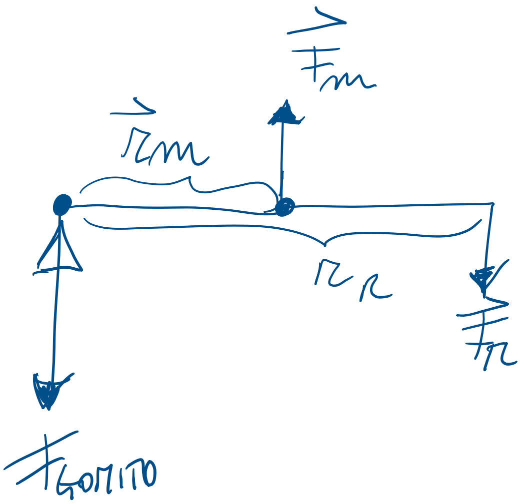
F_R



$F_{GOMITO} = ?$

$F_m = ?$

$$\begin{cases} r_m = 0{,}05\,m \\ r_R = 0{,}15\,m \\ F_R = 12\,N \end{cases}$$

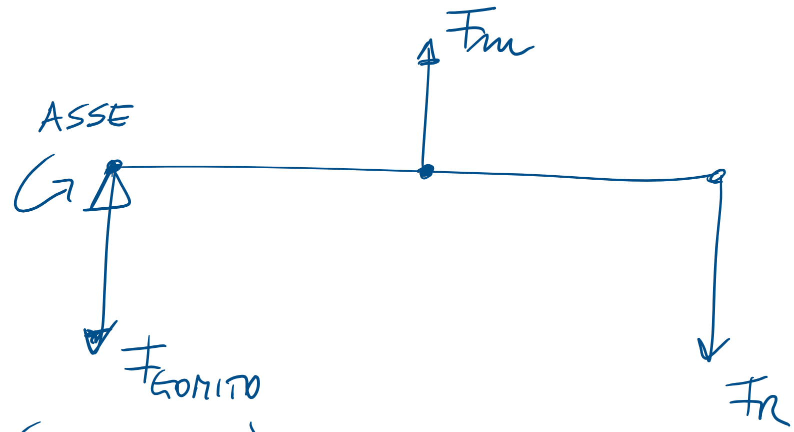


$$\vec{F}^{RIS} = \vec{0}$$

$$\begin{cases} F_x^{RIS} = 0 = 0 \\ F_y^{RIS} = -F_{GOMITO} + F_m - F_R = 0 \end{cases}$$

$$\boxed{F_{GOMITO} = F_m - F_R}$$

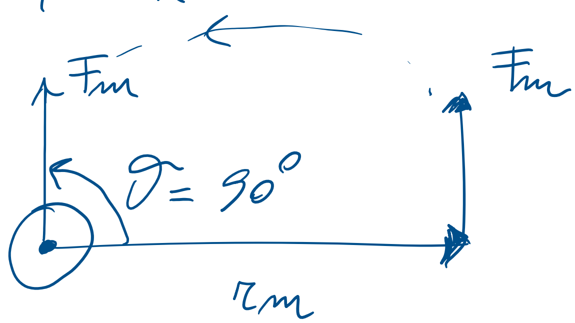
$\vec{M}^{(ris)} = \vec{0}$



$(M_{GOMITO} = 0)$

$\vec{M}^{ris} = \vec{M}_m + \vec{M}_r$

M_m



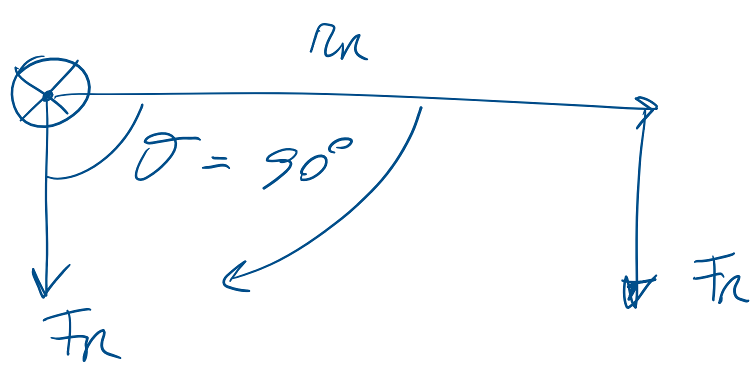
senso antiorario

$M > 0$

$\theta = 90°$

$M_m = r_m F_m$

M_r



senso orario

$M < 0$

$$M_R = - r_R F_R$$

$$r_m F_m - r_R F_R = 0$$

$$\boxed{F_m = \frac{r_R}{r_m} F_R}$$

$$F_m = \cancel{\frac{0{,}15}{0{,}05}} 3 F_R$$

$$\boxed{F_m = 3 F_R}$$

La forza esercitata dai bicipiti è il triplo del peso

$$F_{GOMITO} = 3F_R - F_R$$

$$\boxed{F_{GOMITO} = 2F_R}$$

Sull'articolazione del gomito si sente una forza doppia rispetto al peso applicato!!