LE DERIVATE

1. GENERALITÀ

Definizione 1.1)

La *derivata* è un operatore che ad una funzione *f* associa un'altra funzione e che obbedisce alle seguenti regole:

(1) $D\left(a_0x^n\right) = \left(a_0x^n\right)' = na_0x^{n-1}$ *derivata di un monomio*

ESEMPI

$D\left(3x^2\right) = 2\cdot 3\cdot x^{2-1} = 6x$

$D\left(5x^4\right) = 4\cdot 5\cdot x^{4-1} = 20x^3$

$D\left(-4x^3\right) = 3\cdot(-4)\cdot x^{3-1} = -12x^2$

(2) $D\left(a_0x\right) = a_0$ *derivata di un monomio con* $n = 1$

ESEMPI

$D(7x) = 1\cdot 7\cdot x^{1-1} = 7x^0 = 7\cdot 1 = 7$, $D(10x) = 10$, $D(3x) = 3$

(3) $D(x) = 1$ *derivata di un monomio con* $a_0 = 1 = n$

(4) $D(c) = 0$ *derivata di una costante*

ESEMPI

$D(4) = 0$, $D(10) = 0$, $D(25) = 0$

(5) $D\left(x^n\right) = nx^{n-1}$ *derivata di un monomio con* $a_0 = 1$

Più in generale risulta:

(5.1) $D\left(x^\alpha\right) = \alpha x^{\alpha-1}$ (α *reale qualsiasi*)

Ricordando le regole delle potenze:

$a)\ a^{\frac{1}{n}} = \sqrt[n]{a}$

$b)\ a^{-n} = \dfrac{1}{a^n}$

$c)\ a^{-\frac{1}{n}} = \dfrac{1}{\sqrt[n]{a}}$

seguono varie proprietà applicate nei seguenti

ESEMPI

$$D\left(x^3\right) = 3x^{3-1} = 3x^2,\ D\left(x^5\right) = 5x^{5-1} = 5x^4,\ D\left(x^8\right) = 8x^{8-1} = 8x^7,\ D\left(x^{\sqrt{2}}\right) = \sqrt{2}\, x^{\sqrt{2}-1},$$

$$D\left(x^{1+\sqrt[3]{2}}\right) = \left(1+\sqrt[3]{2}\right) x^{\sqrt[3]{2}}$$

Se $\alpha = \dfrac{1}{n}$ allora si ha:

$$D\left(x^{\frac{1}{n}}\right) = \frac{1}{n}x^{\frac{1}{n}-1} = \frac{1}{n}x^{\frac{1-n}{n}} = \frac{1}{n}\cdot\frac{1}{x^{\frac{n-1}{n}}}$$

che si può scrivere, in modo più semplice, come segue:

(6) $D\left(\sqrt[n]{x}\right) = \dfrac{1}{n\sqrt[n]{x^{n-1}}}$ *derivata della radice n-esima*

ESEMPI

$$D\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x^{2-1}}} = \frac{1}{2\sqrt{x}},\ D\left(\sqrt[3]{x}\right) = \frac{1}{3\sqrt[3]{x^{3-1}}} = \frac{1}{3\sqrt[3]{x^2}},\ D\left(\sqrt[5]{x}\right) = \frac{1}{5\sqrt[5]{x^{5-1}}} = \frac{1}{5\sqrt[5]{x^4}}$$

Più in generale si ottiene:

(7) $D\left[\sqrt[n]{f(x)}\right] = \dfrac{f'(x)}{n\sqrt[n]{\left[f(x)\right]^{n-1}}}$ *derivata della radice n-esima di una funzione*

ESEMPI

$$D\left(\sqrt{x^3-4x+2}\right) = \frac{\left(x^3-4x+2\right)'}{2\sqrt{\left(x^3-4x+2\right)^{2-1}}} = \frac{3x^2-4}{2\sqrt{x^3-4x+2}}$$

$$D\left(\sqrt[3]{4x^3+6x^2-5}\right) = \frac{\left(4x^3+6x^2-5\right)'}{3\sqrt[3]{\left(4x^3+6x^2-5\right)^{3-1}}} = \frac{12x^2+12x}{3\sqrt[3]{\left(4x^3+6x^2-5\right)^2}} = \frac{4x^2+4x}{\sqrt[3]{\left(4x^3+6x^2-5\right)^2}}$$

$$D\left(\sqrt[5]{5x+3}\right) = \frac{(5x+3)'}{5\sqrt[5]{(5x+3)^{5-1}}} = \frac{5}{5\sqrt[5]{(5x+3)^4}} = \frac{1}{\sqrt[5]{(5x+3)^4}}$$

(8) $D\left[a\,f(x) \pm b\,g(x)\right] = a\,D\,f(x) \pm b\,D\,g(x)$ *derivata della somma (o differenza) e linearità*

ESEMPI

$$D\left(5x^4+3x^2+7x+6\right) = 5D\left(x^4\right)+3D\left(x^2\right)+7D\left(x\right)+6D\left(1\right) = 20x^3+6x+7$$

$$D\left(6x^3-2x^2+4x-6\right) = 6D\left(x^3\right)-2D\left(x^2\right)+4D\left(x\right)-6D\left(1\right) = 18x^2-4x+4$$

$$D\left(-4x^3+7x\right) = -4D\left(x^3\right)+7D\left(x\right) = -12x^2+7$$

(9) $D\left[f(x)\,g(x)\right] = \left[f(x)\,g(x)\right]' = f'(x)\,g(x) + f(x)\,g'(x)$ *derivata del prodotto*

ESEMPI

$$D\left[\left(x^2-1\right)(5x+2)\right] = \left(x^2-1\right)'(5x+2)+\left(x^2-1\right)(5x+2)' = 2x\,(5x+2)+\left(x^2-1\right)5 =$$

$$= 10x^2+4x+5x^2-5 = 15x^2+4x-5$$

$$D\left[(2x+3)\left(x^2+2x\right)\right] = (2x+3)'\left(x^2+2x\right)+(2x+3)\left(x^2+2x\right)' = 2\left(x^2+2x\right)+(2x+3)(2x+2) =$$

$$= 2x^2+4x+4x^2+4x+6x+6 = 6x^2+14x+6$$

$$D\left[\left(3x^3+4x+1\right)\left(3x^2+4\right)\right] = \left(3x^3+4x+1\right)'\left(3x^2+4\right)+\left(3x^3+4x+1\right)\left(3x^2+4\right)' =$$

$$= \left(9x^2+4\right)\left(3x^2+4\right)+\left(3x^3+4x+1\right)6x = 27x^4+36x^2+12x^2+16+18x^4+24x^2+6x =$$

$$= 45x^4+72x^2+6x+16$$

(10) $D\left[\dfrac{f(x)}{g(x)}\right] = \left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{f'(x)\,g(x)-f(x)\,g'(x)}{\left[g(x)\right]^2}, \quad g(x) \neq 0$ *derivata del quoziente*

ESEMPI

$$D\left(\frac{x}{x^2+1}\right) = \frac{(x)^{|}(x^2+1)-x\,(x^2+1)^{|}}{(x^2+1)^2} = \frac{x^2+1-x\cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$$D\left(\frac{x^3}{4-x}\right) = \frac{(x^3)^{|}(4-x)-x^3(4-x)^{|}}{(4-x)^2} = \frac{3x^2(4-x)-x^3(-1)}{(4-x)^2} = \frac{-2x^3+12x^2}{(4-x)^2}$$

$$D\left(\frac{1+x^2}{4+x^2}\right) = \frac{(1+x^2)^{|}(4+x^2)-(1+x^2)\,(4+x^2)^{|}}{(4+x^2)^2} = \frac{2x\,(4+x^2)-(1+x^2)\,2x}{(4+x^2)^2} =$$

$$= \frac{6x}{(4+x^2)^2}$$

(11) $D f\,\left[g(x)\right] = f^{|}[.....]\,g^{|}[.....] = f^{|}\left[g(x)\right]\,g^{|}(x)$ *derivata di funzioni composte*

ESEMPI

$$D\left(\sqrt{4x^2-3}\right) = \left[(4x^2-3)^{\frac{1}{2}}\right]^{|} = \frac{1}{2\,\sqrt{4x^2-3}}\,(4x^2-3)^{|} = \frac{8x}{2\,\sqrt{4x^2-3}} = \frac{4x}{\sqrt{4x^2-3}}$$

$$D\left[\sqrt[5]{(2x^3+1)^2}\right] = \left[\sqrt[5]{(2x^3+1)^2}\right]^{|} = \left[(2x^3+1)^{\frac{2}{5}}\right]^{|} = \frac{2}{5}\,(2x^3+1)^{-\frac{3}{5}}\left[(2x^3+1)\right]^{|} =$$

$$= \frac{2}{5\,\sqrt[5]{(2x^3+1)^3}}\,6x^2 = \frac{12x^2}{5\,\sqrt[5]{(2x^3+1)^3}}$$

$$D\left(\sqrt[3]{x^2-7x}\right) = \left[(x^2-7x)^{\frac{1}{3}}\right]^{|} = \frac{1}{3}\,(x^2-7x)^{\frac{1}{3}-1}\,(x^2-7x)^{|} = \frac{2x-7}{3\,\sqrt[3]{(x^2-7x)^2}}$$

$$D\left[(3x^2-x)^5\right] = 5\,(3x^2-x)^{5-1}(3x^2-x)^{|} = 5\,(3x^2-x)^4(6x-1)$$

(12) $D\left[e^{f(x)}\right] = e^{f(x)}\,f^{|}(x)$ *derivata di funzioni esponenziali*

ESEMPI

$$D\left(e^{x^2+5x+2}\right) = e^{x^2+5x+2}\,(x^2+5x+2)^{|} = e^{x^2+5x+2}\,(2x+5)$$

$$D\left(e^{3x^3+7x}\right) = e^{3x^3+7x}\,(3x^3+7x)^{|} = e^{3x^3+7x}\,(9x^2+7)$$

$$D\left(e^{\frac{x^3+3}{x^2+1}}\right) = e^{\frac{x^3+3}{x^2+1}}\left(\frac{x^3+3}{x^2+1}\right)' = e^{\frac{x^3+3}{x^2+1}}\left[\frac{3x^2\left(x^2+1\right)-\left(x^3+3\right)\,2x}{\left(x^2+1\right)^2}\right] = e^{\frac{x^3+3}{x^2+1}}\ \frac{x^4+3x^2-6x}{\left(x^2+1\right)^2}$$

$$D\left(e^{\frac{2x^2+4x}{3x^2+5}}\right) = e^{\frac{2x^2+4x}{3x^2+5}}\left(\frac{2x^2+4x}{3x^2+5}\right)' = e^{\frac{2x^2+4x}{3x^2+5}}\left[\frac{\left(4x+4\right)\ \left(3x^2+5\right)-\left(2x^2+4x\right)\ 6x}{\left(3x^2+5\right)^2}\right] =$$

$$= e^{\frac{2x^2+4x}{3x^2+5}}\ \frac{12x^3+20x+12x^2+20-12x^2-24x}{\left(3x^2+5\right)^2} = e^{\frac{2x^2+4x}{3x^2+5}}\ \frac{12x^3-4x+20}{\left(3x^2+5\right)^2}$$

(13) $D\left[\ln f\left(x\right)\right] = \dfrac{f'\left(x\right)}{f\left(x\right)}$ *derivata di funzioni logaritmiche*

<u>ESEMPI</u>

$$D\left[\ln\left(3x^2+2\right)\right] = \frac{1}{3x^2+2}\left(3x^2+2\right)' = \frac{6x}{3x^2+2}$$

$$D\left[\ln\left(x^3-2x-1\right)\right] = \frac{1}{x^3-2x-1}\left(x^3-2x-1\right)' = \frac{3x^2-2}{x^3-2x-1}$$

$$D\left[\ln\left(\frac{2x^2+4x}{3x^2+5}\right)\right] = \frac{3x^2+5}{2x^2+4x}\left(\frac{2x^2+4x}{3x^2+5}\right)' = \frac{3x^2+5}{2x^2+4x}\ \frac{-24x^2+20x}{\left(3x^2+5\right)^2} = \frac{-24x^2+20x}{\left(2x^2+4x\right)\left(3x^2+5\right)}$$

Osservazione: riteniamo opportuno richiamare l'attenzione dello studente su alcune proprietà dei logaritmi che si riveleranno particolarmente utili soprattutto per lo studio di funzioni:

- $\log_a\left(b\cdot c\right) = \log_a b+\log_a c$ *con* $a, b>0$ *e* $a\neq 1$
- $\log_a\left(\frac{b}{c}\right) = \log_a b-\log_a c$ *con* $a, b, c>0$ *e* $a\neq 1$
- $\log_a\left(b^n\right) = n\log_a b$ *con* $a, b>0$ *e* $a\neq 1$ *e* *n intero positivo*
- $\log_a\left(\sqrt[n]{b}\right) = \frac{1}{n}\log_a b$ *con* $a, b>0$ *e* $a\neq 1$ *e* *n intero positivo*
- $\log_a a = 1$ *con* $a>0$ *e* $a\neq 1$
- $\log_a 1 = 0$ *con* $a>0$ *e* $a\neq 1$
- $\log_a 0 = -\infty$ *con* $a>0$ *e* $a\neq 1$
- $\log_b N = \dfrac{\log_a N}{\log_a b}$ *formula del cambio di base con N intero positivo*

Inoltre, sfruttando la definizione classica di logaritmo, è facile verificare l'equivalenza delle seguenti espressioni:

$$z = \log_a b; \qquad a^z = b; \qquad a^{\log_a b} = b;$$

In generale si è soliti indicare con *ln* o anche con *log* il logaritmo naturale o *Neperiano*, cioè in base *e*.

2. TABELLA DELLE DERIVATE PIÙ COMUNI

Riportiamo qui di seguito una tabella riassuntiva delle derivate di alcune funzioni elementari, scrivendo a sinistra la funzione e, nella stessa linea, a destra, la sua derivata:

$y = c$	$y' = 0$
$y = x$	$y' = 1$
$y = x^n,\ n \in \aleph$	$y' = nx^{n-1}$
$y = x^\alpha,\ \alpha \in \Re\ e\ x > 0$	$y' = \alpha x^{\alpha-1}$
$y = \sqrt{x}$	$y' = \frac{1}{2\sqrt{x}}$
$y = \sqrt[n]{x^m},\ n > m$	$y' = \frac{m}{n\sqrt[n]{x^{n-m}}}$
$y = sinx$	$y' = cosx$
$y = cosx$	$y' = -sinx$
$y = tgx$	$y' = \frac{1}{cos^2 x} = 1 + tg^2 x$
$y = ctgx$	$y' = -\frac{1}{sin^2 x} = -1 - cotg^2 x$
$y = e^x$	$y' = e^x$
$y = a^x,\ a > 0$	$y' = a^x\ loga$
$y = x^x$	$y' = x^x\ (1 + lnx)$
$y = lnx,\ x > 0$	$y' = \frac{1}{x}$
$y = log_a x,\ x > 0,\ a > 0,\ a \neq 1$	$y' = \frac{1}{x}\ log_a e$

$$y = arcsinx, \ -\frac{\pi}{2} < y < \frac{\pi}{2} \qquad y' = \frac{1}{\sqrt{1-x^2}}$$

$$y = arccosx, \ 0 < y < \pi \qquad y' = -\frac{1}{\sqrt{1-x^2}}$$

$$y = arctgx \qquad y' = \frac{1}{1+x^2}$$

$$y = arcctgx \qquad y' = -\frac{1}{1+x^2}$$

Riportiamo adesso un elenco di derivate di funzioni elementari ottenuto dalla tabella precedente sostituendo alla variabile indipendente x una certa funzione $f(x)$ di cui si conosca la derivata ed applicando poi la regola di derivazione delle funzioni composte:

$$y = \left[f(x)^n \right] \qquad y' = n\left[f(x) \right]^{n-1} f'(x)$$

$$y = \sqrt{f(x)} \qquad y' = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$y = \sqrt[n]{\left[f(x) \right]^m} \qquad y' = \frac{m}{n\sqrt[n]{\left[f(x) \right]^{n-m}}}$$

$$y = sin\, f(x) \qquad y' = cos\, f(x) f'(x)$$

$$y = cos\, f(x) \qquad y' = -sin\, f(x) f'(x)$$

$$y = tg\, f(x) \qquad y' = \frac{1}{cos^2 f(x)} f'(x)$$

$$y = ctg\, f(x) \qquad y' = -\frac{1}{sin^2 f(x)} f'(x)$$

$$y = arcsin\, f(x) \qquad y' = \frac{1}{\sqrt{1-\left[f(x) \right]^2}} f'(x)$$

$$y = arccos\, f(x) \qquad y' = -\frac{1}{\sqrt{1-\left[f(x) \right]^2}} f'(x)$$

$$y = arctg\, f(x) \qquad y' = \frac{1}{1+\left[f(x) \right]^2} f'(x)$$

$$y = arcctg\, f(x) \qquad y' = -\frac{1}{1+\left[f(x) \right]^2} f'(x)$$

$$y = e^{f(x)} \qquad y' = e^{f(x)} f'(x)$$

$$y = a^{f(x)} \qquad y' = a^{f(x)} \, ln \, a \, f'(x)$$

$$y = ln \, f(x) \qquad y' = \frac{1}{f(x)} f'(x)$$

$$y = log_a f(x) \qquad y' = \frac{1}{f(x)} log_a e \, f'(x)$$

$$y = \left[f(x) \right]^{g(x)} \qquad y' = \left[f(x) \right]^{g(x)} \left[g'(x) \, log \, f(x) + \frac{g(x)}{f(x)} f'(x) \right]$$

ESERCIZI PROPOSTI

Calcolare le derivate delle seguenti funzioni polinomiali:

$y = 3x^2 + 1$ [$6x$]

$y = 5x + 7$ [5]

$y = 2x – 5$ [2]

$y = 3x^2 – 6x + 4$ [$6x – 6$]

$y = 4x^3 – 2x^2 + 5x – 3$ [$12x^2 – 4x + 5$]

$y = 4x^2 – 1$ [$8x$]

$y = 1 + x + x^2$ [$1 + 2x$]

$y = x^3 – 2x$ [$3x^2 – 2$]

$y = 3x – 1$ [3]

$y = 4x^2$ [$8x$]

$y = 4x^2 + 5$ [$8x$]

$y = x^5 + 4x^2$ [$5x^4 + 8x$]

$y = x^3 + 2x^2 + 1$ [$3x^2 + 4x$]

$y = 3x^4 – 5x^3 + 4x – 7$ [$12x^3 – 15x^2 + 4$]

$y = 8x^5 – 24x^3 + 7$ [$40x^4 – 72x^2$]

$y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + 5x + 9$ [$x + x^2 + 5$]

$y = (2x + 3)(x^2 + 3x – 1)$ [$6x^2 + 18x + 7$]

$y = (x^2 – 1)(5x + 2)$ [$15x^2 + 4x – 5$]

$y = (x^2 + 1)^5$ [$10x(x^2 + 1)^4$]

$y = \frac{5x^3}{4} - \frac{7x^2}{2} - \frac{3x}{5} + 9$ $\left[\frac{15}{4}x^2 - 7x - \frac{3}{5}\right]$

$y = x(x – 1)^3$ [$–(2x + 1)(x – 1)^2$]

$y = (1 + x^2)(2x – 5)$ [$6x^2 – 10x + 2$]

$y = (2x – 1)^2(3 – 7x)^5$ [$(2x – 1)(3 – 7x)^4(– 98x + 47)$]

$y = (2x + 3)(x^2 + 3x – 1)$ [$6x^2 + 18x + 7$]

$y = (1 – 2x^2)(3x + 1)$ [$– 18x^2 – 4x + 3$]

$y = (3 - 2x - x^2)(x^4 - 2x^2)$ $[2x(-3x^4 - 5x^3 + 10x^2 + 6x - 6)]$

$y = x^2(4 + x)(5x + 1)$ $[x(20x^2 + 63x + 8)]$

$y = (8x - 1)^{10}$ $[80(8x - 1)^9]$

$y = (x - 1)^2(x - 2)$ $[(x - 1)(3x - 5)]$

$y = (5 + x^3)(1 - 2x - 4x^3)^2$ $[(1 - 2x - 4x^3)(-36x^5 - 10x^3 - 117x^2 - 20)]$

$y = (1 - 3x)^4(1 + x)$ $[(11 + 15x)(3x - 1)^3]$

$y = (2 - x)^2(x^3 + 2x)$ $[(2 - x)(-5x^3 + 6x^2 - 6x + 4)]$

$y = (x - 2)^3(x + 1)^2$ $[(x + 1)(x - 2)^2(5x - 1)]$

$y = (x^2 + x + 1)^3(x - 1)^4$ $[(x^2 + x + 1)^2(x - 1)^3(10x^2 + x + 1)]$

$y = (x^6 + 1)(3x + 1)^8$ $[6(3x + 1)^7(7x^6 + x^5 + 4)]$

$y = (x^2 + 2x - 3)^3(4 - x^2)^7$ $[2(12 + 33x - 17x^2 - 10x^3)(x^2 + 2x - 3)^2(4 - x^2)^6]$

$y = 2(x + 2)^2(x^2 + 4x - 3)$ $[4(x + 2)(2x^2 + 8x + 1)]$

$y = x^2(x^4 + 1)^3 + 3x(x^2 + 1)]$ $[2x(x^4 + 1)^2(7x^4 + 1) + 3(3x^2 + 1)]$

Calcolare le derivate delle seguenti funzioni razionali fratte:

$y = \frac{5}{x+1}$ $\left[-\frac{5}{(x+1)^2}\right]$

$y = \frac{x+1}{2x}$ $\left[-\frac{1}{2x^2}\right]$

$y = \frac{x-3}{x-4}$ $\left[-\frac{1}{(x-4)^2}\right]$

$y = \frac{2x-3}{3x-4}$ $\left[\frac{1}{(3x-4)^2}\right]$

$y = \frac{x+1}{x-3}$ $\left[-\frac{4}{(x-3)^2}\right]$

$y = \frac{3x-4}{x^2-1}$ $\left[\frac{x(5-3x)}{(x^2-1)^2}\right]$

$y=\frac{1}{x^2-1}$ $\left[-\frac{2x}{\left(x^2-1\right)^2}\right]$

$y=\frac{4x^2+1}{x}$ $\left[\frac{(2x+1)(2x-1)}{x^2}\right]$

$y=\frac{x^2-3x-1}{x+1}$ $\left[\frac{x^2+2x-2}{(x+1)^2}\right]$

$y=\frac{(1-x)^2}{x}$ $\left[\frac{(x-1)(x+1)}{x^2}\right]$

$y=\frac{3x^2-5}{x^2-1}$ $\left[\frac{4x}{\left(x^2-1\right)^2}\right]$

$y=\frac{4x^2-5x+3}{x^2-6x+5}$ $\left[-\frac{19x^2-34x+7}{\left(x^2-6x+5\right)^2}\right]$

$y=\frac{3x^2-2x+3}{x^2-2x-1}$ $\left[-\frac{4\left(x^2+3x-2\right)}{\left(x^2-2x-1\right)^2}\right]$

$y=\frac{1}{x}$ $\left[-\frac{1}{x^2}\right]$

$y=\frac{1}{2x^2}-\frac{3}{x}+x^3$ $\left[\frac{3x^5+3x-1}{x^3}\right]$

$y=3x^2+9x+\frac{1}{x}$ $\left[\frac{6x^3+9x^2-1}{x^2}\right]$

$y=\frac{x^2+x^6-3x^3}{x^4}$ $\left[-\frac{2}{x^3}+2x+\frac{3}{x^2}\right]$

$y=x^3-2x-\frac{5}{x}+\frac{2}{x^3}$ $\left[3x^2-2+\frac{5}{x^2}-\frac{6}{x^4}\right]$

$y=\frac{2x}{3-x}$ $\left[\frac{6}{(3-x)^2}\right]$

$y=\frac{1+x^2}{4+x^2}$ $\left[\frac{6x}{\left(4+x^2\right)^2}\right]$

$$y=\frac{x^2-4}{x^2+4} \qquad \left[\frac{16x}{\left(x^2+4\right)^2}\right]$$

$$y=\frac{x^3}{4-x} \qquad \left[\frac{2x^2(6-x)}{(4-x)^2}\right]$$

$$y=\frac{x^2+1}{5x-7} \qquad \left[\frac{5x^2-14x-5}{(5x-7)^2}\right]$$

$$y=\frac{8x+x^5}{x+1} \qquad \left[\frac{4x^5+5x^4+8}{(x+1)^2}\right]$$

$$y=\frac{4x^2-5}{x+1} \qquad \left[\frac{4x^2+8x+5}{(x+1)^2}\right]$$

$$y=2x-\frac{x}{x^2+1} \qquad \left[\frac{2x^4+5x^2+1}{\left(x^2+1\right)^2}\right]$$

$$y=\left(2x+\frac{5}{x}\right)^3 \qquad \left[3\left(2x+\frac{5}{x}\right)^2\left(2-\frac{5}{x^2}\right)\right]$$

$$y=\left(x-1-\frac{3}{x}\right)^4 \qquad \left[4\left(x-1-\frac{3}{x}\right)^3\left(1+\frac{3}{x^2}\right)\right]$$

$$y=\frac{x^2-4}{x^2+4} \qquad \left[\frac{16x}{\left(x^2+4\right)^2}\right]$$

$$y=\frac{x^2+x+2}{x^2-1} \qquad \left[\frac{-x^2-6x-1}{\left(x^2-1\right)^2}\right]$$

$$y=\frac{7}{\left(x^3+8\right)^2} \qquad \left[-\frac{42x^2}{\left(x^3+8\right)^3}\right]$$

$$y=\frac{x}{x^3+x^2+2} \qquad \left[\frac{-2x^3-x^2+2}{\left(x^3+x^2+2\right)^2}\right]$$

$y=\frac{x^3}{2x^2-3x+1}$ $\left[\frac{x^2\left(2x^2-6x+3\right)}{\left(2x^2-3x+1\right)^2}\right]$

$y=x^2+2+\frac{x^3}{4-x}$ $\left[\frac{4x\left(8-x\right)}{\left(4-x\right)^2}\right]$

$y=\frac{x^3+x+1}{x-1}$ $\left[\frac{2x^3-3x^2-2}{\left(x-1\right)^2}\right]$

$y=\frac{3x-1}{\left(4-5x\right)^2}$ $\left[\frac{15x+2}{\left(4-5x\right)^3}\right]$

$y=\frac{4x^2-5}{x+1}$ $\left[\frac{4x^2+8x+5}{\left(x+1\right)^2}\right]$

$y=5\left(x^2+9\right)-\frac{7}{x}$ $\left[\frac{10x^3+7}{x^2}\right]$

$y=\left(2x+\frac{1}{x}\right)^3\left(x^2-1\right)$ $\left[\left(\frac{2x^2+1}{x}\right)^2\left(\frac{6x^4-x^2+1}{x^2}\right)\right]$

Calcolare le derivate delle seguenti funzioni esponenziali e logaritmiche:

$y=ln\left(2x-1\right)$ $\left[\frac{2}{2x-1}\right]$

$y=ln\left(x+3\right)$ $\left[\frac{1}{x+3}\right]$

$y = e^{x+1}$ $[e^{x+1}]$

$y = xe^x$ $[(1+x)e^x]$

$y=e^{5-x^2}$ $\left[-2xe^{5-x^2}\right]$

$y=\frac{x^4+1}{e^4+1}$ $\left[\frac{4x^3}{e^4+1}\right]$

$y = xlnx$ $[lnx + 1]$

$y = x^2lnx + 3x$ $[2xlnx + x + 3]$

$y=e^{-\frac{3}{x^2}}$ $\left[\frac{2}{x^3}e^{-\frac{3}{x^2}}\right]$

$y = \frac{lnx - 1}{lnx + 1}$ $\left[\frac{2}{x(lnx+1)^2}\right]$

$y = x^3e^x + e^x - 1$ $[e^x(x^3 + 3x^2 + 1)]$

$y = e^x(2 - e^x)$ $[2e^x(1 - e^x)]$

$y = e^x(x^3 - x + 7)$ $[e^x(x^3 + 3x^2 - x + 6)]$

$y = xln^3x - 3xln^2x + 6xlnx - 6x$ $[ln^3x]$

$y = x^2(lnx)^3$ $[x(lnx)^2(2lnx + 3)]$

$y = 5xln^2x - 6x^3ln^5x$ $[-18x^2ln^5x - 30x^2ln^4x + 5ln^2x + 10lnx]$

$y = \frac{1}{x} \cdot lnx$ $\left[\frac{1}{x^2}(1 - lnx)\right]$

$y = (xlnx - 1)^2$ $[2(xlnx - 1)(lnx + 1)]$

$y = 3xlnx$ $[3(lnx + 1)]$

$y = x^2lnx$ $[x(2lnx + 1)]$

$y = xln^3x$ $[ln^2x(lnx + 3)]$

$y = ln\left(\frac{x}{x-1}\right) - \frac{2}{x} - \frac{1}{x^2}$ $\left[\frac{x^2 - 2}{x^3(x-1)}\right]$

$y = ln^2x + 3x + 5$ $\left[\frac{2lnx + 3x}{x}\right]$

$y = 4xln^4x + 7x^5ln^5x$ $\left[ln^3x\left(35x^4ln^2x + 35x^4lnx + 4lnx + 16\right)\right]$

$y = \frac{lnx}{x^2}$ $\left[\frac{1}{x^3}(1 - 2lnx)\right]$

$y = ln(x^2 - 7x - 8)$ $\left[\frac{2x - 7}{x^2 - 7x - 8}\right]$

$y = (x-1)ln^3x$ $\left[\frac{xln^3x + 3(x-1)ln^2x}{x}\right]$

$y = e^{x^2 - 2x}$ $\left[2(x-1)e^{x^2-2x}\right]$

$y = xe^{\frac{x-1}{x}}$ $\left[e^{\frac{x-1}{x}} \cdot \frac{x+1}{x}\right]$

$y = ln\left(\frac{x}{x+1}\right) + \frac{1}{x} - \frac{1}{2x^2}$ $\left[\frac{1}{x^3(x+1)}\right]$

$y = ln\left(\frac{x}{x-1}\right)$ $\left[-\frac{1}{x(x-1)}\right]$

$y = ln\left(x^3 + x^2 + 8\right)$ $\left[\frac{3x^2 + 2x}{x^3 + x^2 + 8}\right]$

$y = \frac{lnx - 1}{lnx + 1}$ $\left[\frac{2}{x(lnx+1)^2}\right]$

$y = \frac{e^x + 1}{2 - e^x}$ $\left[\frac{3e^x}{(2-e^x)^2}\right]$

$y = \frac{e^x}{x^3 + x^2}$ $\left[\frac{e^x\left(x^3 - 2x^2 - 2x\right)}{\left(x^3 + x^2\right)^2}\right]$

$y = \frac{lnx}{1 + lnx}$ $\left[\frac{1}{x(1+lnx)}\right]$

$y = ln(lnx)$ $\left[\frac{1}{xlnx}\right]$

$y = ln\left(e^x - 2\right)$ $\left[\frac{e^x}{e^x - 2}\right]$

$y = \frac{1}{2}ln\left(x^2 - 1\right) + x$ $\left[\frac{x^2 + x - 1}{x^2 - 1}\right]$