ESERCIZI PROPOSTI

I) *Calcolare gli integrali* **1)-148)**, *utilizzando le formule di* ***integrazione immediata*** (cfr. ¶ **9. a)**), *dopo aver analizzato gli esempi* ***a)-l)***, *di seguito riportati*:

a) $\int 5x^3 dx = 5\int x^3 dx = 5\cdot\frac{x^{3+1}}{3+1}+c = \frac{5}{4}x^4+c$

b) $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1}+c = \frac{x^{5/3}}{5/3}+c = \frac{3}{5}\sqrt[3]{x^5}+c$

c) $\frac{3}{5}\int \frac{1}{\sqrt[5]{x^2}}dx = \frac{3}{5}\int \frac{1}{x^{2/5}}dx = \frac{3}{5}\int x^{-2/5}dx = \frac{3}{5}\cdot\frac{x^{-\frac{2}{5}+1}}{-\frac{2}{5}+1}+c = \frac{3}{5}\cdot\frac{x^{3/5}}{3/5}+c = \frac{3}{5}\cdot\frac{5}{3}\sqrt[5]{x^3}+c =$

$= \sqrt[5]{x^3}+c$

d) $\int\left(\frac{1}{\sqrt{x}}-\sqrt{x}\right)dx = \int\frac{1}{\sqrt{x}}dx - \int\sqrt{x}dx = \int x^{-1/2}dx - \int x^{1/2}dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c =$

$= \frac{x^{1/2}}{1/2} - \frac{x^{3/2}}{3/2}+c = 2\sqrt{x} - \frac{2}{3}\sqrt{x^3}+c = 2\sqrt{x} - \frac{2}{3}x\sqrt{x}+c$

e) $\int (x-1)(x-2)\,dx = \int\left(x^2-2x-x+2\right)dx = \int\left(x^2-3x+2\right)dx =$

$= \int x^2dx - 3\int xdx + 2\int dx = \frac{x^{2+1}}{2+1} - 3\cdot\frac{x^{1+1}}{1+1} + 2x + c = \frac{x^3}{3} - \frac{3}{2}x^2 + 2x + c$

f) $\int\frac{x+x\sqrt[3]{x}+2}{\sqrt[3]{x}}dx = \int\left(\frac{x}{\sqrt[3]{x}} + \frac{x\sqrt[3]{x}}{\sqrt[3]{x}} + \frac{2}{\sqrt[3]{x}}\right)dx = \int\left(\sqrt[3]{\frac{x^3}{x}} + x + \frac{2}{\sqrt[3]{x}}\right)dx =$

$= \int\sqrt[3]{x^2}dx + \int xdx + 2\int x^{-1/3}dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{x^{1+1}}{1+1} + 2\cdot\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c = \frac{3}{5}x^{5/3} + \frac{1}{2}x^2 - 3x^{2/3} + c =$

$= \frac{3}{5}\sqrt[3]{x^5} + \frac{1}{2}x^2 - 3\sqrt[3]{x^2} + c = \frac{3}{5}x\sqrt[3]{x^2} + \frac{1}{2}x^2 - 3\sqrt[3]{x^2} + c$

g) $\int\frac{x^5-1}{x-1}dx = \int Q(x)dx + \int\frac{R(x)}{P_2(x)}dx = \int\left(x^4+x^3+x^2+x+1\right)dx =$

$$= \int x^4 dx + \int x^3 dx + \int x^2 dx + \int x dx + \int dx = \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + c$$

essendo $Q(x) = x^4 + x^3 + x^2 + x + 1$ ed $R(x) = 0$, dopo aver effettuato la divisione classica tra polinomi

h) $\int \frac{x^2+3}{1+x^2} dx = \int dx + \int \frac{2}{x^2+1} dx = x + 2\text{arctg}x + c$ $\qquad (Q(x) = 1 \text{ ed } R(x) = 2)$

1) $\int 7x^6 dx$ $\qquad \left[x^7 + c\right]$

2) $\int \frac{4}{21\sqrt[7]{x^5}} dx$ $\qquad \left[\frac{2}{3}\sqrt[7]{x^2} + c\right]$

3) $\int \frac{1}{6\sqrt[3]{x^2}} dx$ $\qquad \left[\frac{1}{2}\sqrt[3]{x} + c\right]$

4) $3\int \frac{dx}{\sqrt{x^5}}$ $\qquad \left[-2\frac{1}{\sqrt{x^3}} + c\right]$

5) $\int \left(\sqrt{x} + x + 2\right) dx$ $\qquad \left[\frac{2}{3}\sqrt{x^3} + \frac{1}{2}x^2 + 2x + c\right]$

6) $\int \left(\frac{1}{x^3} + \frac{1}{x^2} - 3x + 4\right) dx$ $\qquad \left[-\frac{1}{2x^2} - \frac{1}{x} - \frac{3x^2}{2} + 4x + c\right]$

7) $\int \frac{1}{12} x^{-(7/6)} dx$ $\qquad \left[-\frac{1}{2}x^{-1/6} + c\right]$

8) $\int \left(4x^3 - 3x^2 + \frac{1}{x^2} - \frac{3}{x^4} - 1\right) dx$ $\qquad \left[x^4 - x^3 - \frac{1}{x} + \frac{1}{x^3} - x + c\right]$

9) $\int (x+2)(x-3) dx$ $\qquad \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + c\right]$

10) $\int \left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} - \frac{2}{\sqrt{x^3}}\right) dx$ $\qquad \left[\frac{3}{5}\sqrt[3]{x^5} + 3\sqrt[3]{x} + \frac{4}{\sqrt{x}} + c\right]$

11) $\int \left(x^2 + x^{-2} - 2x^{-3}\right) dx$ $\qquad \left[\frac{1}{3}x^3 - x^{-1} + x^{-2} + c\right]$

12) $\int \left(x^{3/4} - x^{4/5}\right) dx$ $\left[\frac{4}{7}\sqrt[4]{x^7} - \frac{5}{9}\sqrt[5]{x^9} + c\right]$

13) $\int (2-5x)^3 \, dx$ $\left[-\frac{1}{20}(2-5x)^4 + c\right]$

14) $\int \left(x^{1/3} + x^{3/5}\right) dx$ $\left[\frac{3}{4}x\sqrt[3]{x} + \frac{5}{8}x\sqrt[5]{x^3} + c\right]$

15) $\int 2x\left(1-2x^2\right)^3 dx$ $\left[-\frac{1}{8}\left(1-2x^2\right)^4 + c\right]$

16) $\int \left(2\sqrt[3]{x^2} - \frac{1}{3}\sqrt{x} + \frac{2}{3}\sqrt[5]{x^4} + \frac{1}{\sqrt{x}}\right) dx$ $\left[\frac{6}{5}x\sqrt[3]{x^2} - \frac{2}{9}x\sqrt{x} + \frac{10}{27}x\sqrt[5]{x^4} + 2\sqrt{x} + c\right]$

17) $\int \left(\frac{3}{\sqrt[3]{x}} - \frac{2}{\sqrt{x}} + \frac{3}{\sqrt[4]{x^3}} - 1\right) dx$ $\left[\frac{9}{2}\sqrt[3]{x^2} - 4\sqrt{x} + 12\sqrt[4]{x} - x + c\right]$

18) $\int (7+2x)^2 \, dx$ $\left[\frac{1}{6}(7+2x)^3 + c\right]$

19) $\int \frac{dx}{(3-x)^3}$ $\left[\frac{1}{2(3-x)^2} + c\right]$

20) $\int \frac{dx}{(3x-1)^4}$ $\left[-\frac{1}{9(3x-1)^3} + c\right]$

21) $\int \frac{x\sqrt[3]{x} - 1}{\sqrt[3]{x^2}} dx$ $\left[\frac{3}{5}x\sqrt[3]{x^2} - 3\sqrt[3]{x} + c\right]$

22) $\int x\left(2x^2-1\right)^3 dx$ $\left[\frac{1}{16}\left(2x^2-1\right)^4 + c\right]$

23) $\int x^{1/2}(x+2)^2 \, dx$ $\left[\frac{2}{7}x^3\sqrt{x} + \frac{8}{5}x^2\sqrt{x} + \frac{8}{3}x\sqrt{x} + c\right]$

24) $\int \left(\frac{1}{x^2} + \sqrt[4]{x} - \sqrt{x}\right) dx$ $\left[-\frac{1}{x} + \frac{4}{5}x\sqrt[4]{x} - \frac{2}{3}x\sqrt{x} + c\right]$

25) $\int \frac{x^2 + \sqrt{x^3} + 3}{\sqrt{x}} dx$ $\left[\frac{2}{5}x^2\sqrt{x} + \frac{1}{2}x^2 + 6\sqrt{x} + c\right]$

26) $\int \left(\frac{3x^3 + 1 + x}{x}\right)^2 dx$ $\left[\frac{9}{5}x^5 - \frac{1}{x} + x + 3x^2 + 2x^3 + 2\log x + c\right]$

27) $\int\left(3x^2-\sqrt{x}+\frac{1}{x^2}-\frac{1}{x^3}\right)dx$ $\left[x^3-\frac{2}{3}x\sqrt{x}-\frac{1}{x}+\frac{1}{2x^2}+c\right]$

28) $\int x^2\left(2x+1\right)^2dx$ $\left[\frac{4}{5}x^5+x^4+\frac{1}{3}x^3+c\right]$

29) $\int\left(1-x\right)^2\left(1+2x\right)dx$ $\left[\frac{1}{2}x^4-x^3+x+c\right]$

30) $\int\left(\sqrt[5]{x}+\frac{2}{\sqrt[4]{x^3}}-\frac{1}{\sqrt[3]{x}}\right)dx$ $\left[\frac{5}{6}x\sqrt[5]{x}+8\sqrt[4]{x}-\frac{3}{2}\sqrt[3]{x^2}+c\right]$

31) $\int\left(2+\sqrt{x}+\sqrt[3]{x^2}\right)dx$ $\left[\frac{3}{5}x\sqrt[3]{x^2}+\frac{2}{3}x\sqrt{x}+2x+c\right]$

32) $\int\left(\frac{1}{2}-8\sqrt[5]{x^3}+4\sqrt[3]{x}-\frac{5}{\sqrt[3]{x^2}}+\frac{3}{\sqrt[4]{x}}\right)dx$ $\left[\frac{x}{2}-5x\sqrt[5]{x^3}+3x\sqrt[3]{x}-15\sqrt[3]{x}+4\sqrt[4]{x^3}+c\right]$

33) $\int x\left(3x-2\right)^3dx$ $\left[\frac{27}{5}x^5-\frac{27}{2}x^4+12x^3-4x^2+c\right]$

34) $\int\left(\frac{1}{\sqrt[3]{x^2}}+\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$ $\left[3\sqrt[3]{x}+\frac{2}{3}x\sqrt{x}+2\sqrt{x}+c\right]$

35) $\int\left(\frac{3}{2}-3x^2+\frac{3}{2\sqrt{x}}-\frac{1}{2\sqrt{x}}\right)dx$ $\left[\frac{3}{2}x-x^3+2\sqrt{x}+c\right]$

36) $\int\left(x-1\right)^3\left(5x-4\right)dx$ $\left[x^5-\frac{19}{4}x^4+9x^3-\frac{17}{2}x^2+4x+c\right]$

37) $\int\left(3\frac{\sqrt{x}}{\sqrt[3]{x^2}}+\frac{1}{\sqrt[3]{x^2}}\right)dx$ $\left[18\sqrt[3]{x^5}+3\sqrt[3]{x}+c\right]$

38) $\int\frac{1-3x+3x^3-x^4}{x^6}dx$ $\left[-\frac{1}{5x^5}-\frac{3}{4x^4}-\frac{3}{2x^2}+\frac{1}{x}+c\right]$

39) $\int\frac{dx}{2\sqrt{x+3}}$ $\left[\sqrt{x+3}+c\right]$

40) $\int\frac{x^{3/4}+x^{1/2}+1}{x^{2/3}}dx$ $\left[\frac{12}{13}x\sqrt[12]{x}+\frac{6}{5}\sqrt[6]{x^5}+3\sqrt[3]{x}+c\right]$

41) $\int\left(\frac{2x-1}{x^3}-x^2-\frac{1}{x}\right)dx$ $\left[-\frac{2}{x}+\frac{1}{2x^2}-\frac{x^3}{3}-\log\left|x\right|+c\right]$

42) $\int \frac{dx}{6\left(\sqrt[3]{2x+3}\right)}$ $\left[\frac{1}{8}\sqrt[3]{(2x+3)^2}+c\right]$

43) $\int \left(2+x^{1/2}\right)(x-1)^2\,dx$ $\left[\frac{2}{7}x^3\sqrt{x}-\frac{4}{5}x^2\sqrt{x}+\frac{2}{3}x\sqrt{x}+\frac{2}{3}x^3-2x^2+2x+c\right]$

44) $\int \frac{\left(1-2\sqrt[3]{x}\right)^2}{\sqrt[3]{x^2}}dx$ $\left[4x-6\sqrt[3]{x^2}+3\sqrt[3]{x}+c\right]$

45) $\int \frac{x^3-4x^2+3}{x^{1/2}}dx$ $\left[\frac{2}{7}x^3\sqrt{x}-\frac{8}{5}x^2\sqrt{x}+6\sqrt{x}+c\right]$

46) $\int \frac{x^6-1}{x^2+x+1}\,dx$ $\left[\frac{1}{5}x^5-\frac{1}{4}x^4+\frac{1}{2}x^2-x+c\right]$

47) $\int \frac{(x-1)\left(x^{1/2}-1\right)^2}{\sqrt{x^3}}dx$ $\left[\frac{2}{3}x^{3/2}-2x+2\log|x|+\frac{2}{\sqrt{x}}+c\right]$

48) $\int \frac{x^2-x+1}{x-1}\,dx$ $\left[\frac{1}{2}x^2+\log|x-1|+c\right]$

49) $\int \frac{1+x^2}{x}\,dx$ $\left[\log|x|+\frac{1}{2}x^2+c\right]$

50) $\int \frac{x^2+2x}{(x+1)^2}\,dx$ $\left[x+\frac{1}{x+1}+c\right]$

51) $\int \frac{x^2-2x}{(x-1)^2}\,dx$ $\left[x+\frac{1}{x-1}+c\right]$

52) $\int \frac{x^3-3x^2+3x}{(x-1)^3}\,dx$ $\left[x-\frac{1}{2(x-1)^2}+c\right]$

53) $\int \frac{x^3+3x^2+3x}{(x-1)^3}dx$ $\left[x+\frac{1}{2(x+1)^2}+c\right]$

54) $\int \frac{xdx}{(x+1)^3}$ $\left[-\frac{1}{1+x}+\frac{1}{2(x+1)^2}+c\right]$

55) $\int \frac{x^3-8}{x-2}dx$ $\left[\frac{1}{3}x^3+x^2+4x+c\right]$

56) $\int \frac{1+x^2}{(1-x)^2}dx$ $\left[x+\log(x-1)^2-\frac{2}{x-1}+c\right]$

57) $\int \frac{x^3+2x^2-5}{x^2+1}dx$ $\left[\frac{1}{2}x^2+2x-\frac{1}{2}\log\left|x^2+1\right|-7\operatorname{arctg}x+c\right]$

58) $\int \frac{x^4-2x^2+2x+1}{x^2-1}dx$ $\left[\frac{1}{3}x^3-x+\log\left|x^2-1\right|+c\right]$

59) $\int \frac{2\sqrt{1-x^2}+1}{\sqrt{1-x^2}}dx$ $\left[2x+\arcsin x+c\right]$

60) $\int \frac{1-x+3x^2-x^3+x^4}{x^3-1}dx$ $\left[\frac{1}{2}x^2-x+\log\left|x^3-1\right|+c\right]$

61) $\int \frac{\left(2-3\sqrt[3]{x}\right)^2}{\sqrt[3]{x}}dx$ $\left[\frac{27}{4}x\sqrt[3]{x}+6\sqrt[3]{x^2}-12x+c\right]$

62) $\int \frac{2-12x+9x^2+3x^3+x^4}{13+4x+x^2}dx$ $\left[\frac{1}{3}x^3-\frac{1}{2}x^2+\frac{1}{2}\log\left|x^2+4x+13\right|+c\right]$

63) $\int \left(x-x^{-1}\right)^3 dx$ $\left[\frac{1}{4}x^4-\frac{3}{2}x^2+3\log|x|+\frac{1}{2x^2}+c\right]$

64) $\int \frac{x^4+x^3+x^2+x+1}{1+x^2}dx$ $\left[\frac{1}{3}x^3+\frac{1}{2}x^2+\operatorname{arctg}x+c\right]$

65) $\int \left(e^x+\frac{1}{e^x}\right)dx$ $\left[e^x-e^{-x}+c\right]$

66) $\int \frac{1+x+4x^3}{1+4x^2}dx$ $\left[\frac{1}{2}x^2+\frac{1}{2}\operatorname{arctg}2x+c\right]$

67) $\int \frac{6x^2}{2x^3-1}dx$ $\left[\log\left|2x^3-1\right|+c\right]$

68) $\int \frac{3x^2-6x}{x^3-3x^2-1}dx$ $\left[\log\left|x^3-3x^2-1\right|+c\right]$

69) $\int \frac{x^4-4x}{x^5-10x^2+1}dx$ $\left[\frac{1}{5}\log\left|x^5-10x^2+1\right|+c\right]$

70) $\int \frac{x-1}{\frac{1}{2}x^2-x+\frac{3}{4}}dx$ $\left[\log\left|\frac{1}{2}x^2-x+\frac{3}{4}\right|+c\right]$

71) $\int \frac{x^3 - x}{x^4 - 2x^2 - \frac{1}{3}}\,dx$ $\left[\frac{1}{4}\log\left|x^4 - 2x^2 - \frac{1}{3}\right| + c\right]$

72) $\int \frac{dx}{\sqrt{x+1}}$ $\left[2\sqrt{x+1} + c\right]$

73) $\int\left(\frac{1}{\sqrt{3+x}} + \frac{1}{\sqrt{x-2}} + \frac{1}{\sqrt{x}}\right)dx$ $\left[2\sqrt{3+x} + 2\sqrt{x-2} + 2\sqrt{x} + c\right]$

74) $\int\left(\frac{1}{\sqrt{x+1}} + \frac{1}{3\sqrt{x-1}}\right)dx$ $\left[2\sqrt{x+1} + \frac{2}{3}\sqrt{x-1} + c\right]$

75) $\int \frac{8x-3}{\sqrt{4x^2 - 3x}}\,dx$ $\left[2\sqrt{4x^2 - 3x} + c\right]$

76) $\int\left(\frac{x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1-x^2}}\right)dx$ $\left[\sqrt{1+x^2} - \sqrt{1-x^2} + c\right]$

77) $\int \frac{4x^3 - 9x^2 + 2}{\sqrt{x^4 - 3x^3 + 2x}}\,dx$ $\left[2\sqrt{x^4 - 3x^3 + 2x} + c\right]$

78) $\int \frac{x-1}{\sqrt{2x^2 - 4x + \frac{1}{2}}}\,dx$ $\left[\frac{1}{2}\sqrt{2x^2 - 4x + \frac{1}{2}} + c\right]$

79) $\int \frac{2x-3}{\sqrt{3x^2 - 9x + 1/2}}\,dx$ $\left[\frac{2}{3}\sqrt{3x^2 - 9x + \frac{1}{2}} + c\right]$

80) $\int \frac{x\,dx}{\sqrt[3]{\left(x^2 - 2\right)^2}}$ $\left[\frac{3}{2}\sqrt[3]{x^2 - 2} + c\right]$

81) $\int \frac{6x-5}{\sqrt[3]{\left(3x^2 - 5x + 1/2\right)}}dx$ $\left[3\sqrt[3]{3x^2 - 5x + \frac{1}{2}} + c\right]$

82) $\int \frac{dx}{\sqrt[5]{\left(4x - 1/2\right)}}$ $\left[\frac{5}{4}\sqrt[5]{4x - \frac{1}{2}} + c\right]$

83) $\int \sin x \cos x\,dx$ $\left[\frac{1}{2}\sin^2 x + c\right]$

84) $\int \sin 3x\,dx$ $\left[-\frac{1}{3}\cos 3x + c\right]$

85) $\int \sin^3 x \cos x dx$ $\left[\frac{1}{4}\sin^4 x + c\right]$

86) $\int \left(x^2 - \cos x + \sin 4x\right) dx$ $\left[\frac{1}{3}x^3 - \sin x - \frac{1}{4}\cos 4x + c\right]$

87) $\int \left(2x + \cos 2x - \sin\frac{1}{2}x\right) dx$ $\left[x^2 + \frac{1}{2}\sin 2x + 2\cos\frac{1}{2}x + c\right]$

88) $\int \left(\cos\frac{x}{3} + x\cos x^2\right) dx$ $\left[3\sin\frac{x}{3} + \frac{1}{2}\sin x^2 + c\right]$

89) $\int \frac{\cos 2x}{\sin x + \cos x} dx$ $[\cos x + \sin x + c]$

90) $\int \frac{\cos 2x}{\cos x - \sin x} dx$ $[\sin x - \cos x + c]$

91) $\int \sec^2 x dx$ $[\text{tg}x + c]$

92) $\int \text{cosec}^2 x dx$ $[-\text{ctg}x + c]$

93) $\int \text{tg}^2 x dx$ $[\text{tg}x - x + c]$

94) $\int \text{ctg}^2 x dx$ $[-\text{ctg}x - x + c]$

95) $\int (3\cos 2x + 2\sin 2x) dx$ $\left[\frac{3}{2}\sin 2x - \cos 2x + c\right]$

96) $\int \frac{dx}{\sin^2 x \cos^2 x}$ $[\text{tg}x - \text{ctg}x + c]$

97) $\int \left(3 + \frac{4}{\sin^2 x}\right) dx$ $[3x - 4\text{ctg}x + c]$

98) $\int \left(\frac{2}{\sin^2 x} + \frac{2}{\cos^2 x}\right) dx$ $[-4\text{ctg}2x + c]$

99) $\int \left(\text{tg}^2 x - \text{ctg}^2 x\right) dx$ $[\text{tg}x + \text{ctg}x + c]$

100) $\int \sin^2 x dx$ $\left[\frac{x}{2}-\frac{1}{4}\sin 2x+c\right]$

101) $\int \cos^2 x dx$ $\left[\frac{x}{2}+\frac{1}{4}\sin 2x+c\right]$

102) $\int \left(3\sin^2 x+2\cos^2 x\right)dx$ $\left[\frac{5}{2}x-\frac{1}{4}\sin 2x+c\right]$

103) $\int\left(\sin 5x+\cos 4x+\frac{1}{\cos^2 3x}\right)dx$ $\left[-\frac{1}{5}\cos 5x+\frac{1}{4}\sin 4x+\frac{1}{3}\operatorname{tg}3x+c\right]$

104) $\int\left(\sin\frac{x}{2}-\cos\frac{x}{3}+\frac{1}{\sin^2 2x}\right)dx$ $\left[-\left(2\cos\frac{x}{2}+3\sin\frac{x}{3}+\frac{1}{2}\operatorname{ctg}2x\right)+c\right]$

105) $\int \operatorname{tg}x dx$ $\left[-\log|\cos x|+c\right]$

106) $\int 4\operatorname{tg}\frac{x}{3}dx$ $\left[-12\log\left|\cos\frac{x}{3}\right|+c\right]$

107) $\int\left(3\sin\frac{x}{3}-\cos\frac{x}{2}\right)dx$ $\left[-9\cos\frac{x}{3}+2\sin\frac{x}{2}+c\right]$

108) $\int(1-\cos x)^2 dx$ $\left[\frac{3}{2}x-2\sin x+\frac{1}{4}\sin 2x+c\right]$

109) $\int(1-\sin x)^2 dx$ $\left[\frac{3}{2}x+2\cos x-\frac{1}{4}\sin 2x+c\right]$

110) $\int\frac{1-\cos x}{1+\cos x}dx$ $\left[2\left(\operatorname{tg}\frac{x}{2}-\frac{x}{2}\right)+c\right]$

111) $\int\frac{1+\cos x}{1-\cos x}dx$ $\left[-2\left(\operatorname{ctg}\frac{x}{2}+\frac{x}{2}\right)+c\right]$

112) $\int\frac{1-\cos 4x}{1+\cos 4x}dx$ $\left[\frac{1}{2}\left(\operatorname{tg}2x-2x\right)+c\right]$

113) $\int\frac{dx}{\sqrt{4-x^2}}$ $\left[\arcsin\frac{x}{2}+c\right]$

114) $\int\frac{dx}{\sqrt{25-x^2}}$ $\left[-\arccos\frac{x}{5}+c\right]$

115) $\int \frac{dx}{\sqrt{a^2 - x^2}}$ $\left[\begin{cases} \arcsin\frac{x}{a} + c \\ -\arccos\frac{x}{a} + c \end{cases}\right]$

116) $\int \frac{2x dx}{1 + \left(x^2 + 1\right)^2}$ $\left[\begin{cases} \operatorname{arctg}\left(x^2 + 1\right) + c \\ -\operatorname{arcctg}\left(x^2 + 1\right) + c \end{cases}\right]$

117) $\int \frac{\sin x}{1 + \cos x} dx$ $\left[-\log\left|1 + \cos x\right| + c\right]$

118) $\int \frac{e^x}{2 + e^x} dx$ $\left[\log\left|2 + e^x\right| + c\right]$

119) $\int \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} dx$ $\left[\log\left|e^x + e^{-x}\right| + c\right]$

Si calcolino ora i seguenti integrali utilizzando le seguenti formule di Werner:

$$\sin a \cos b = \frac{1}{2}\left[\sin(a + b) + \sin(a - b)\right]$$

$$\cos a \sin b = \frac{1}{2}\left[\sin(a + b) - \sin(a - b)\right]$$

$$\cos a \cos b = \frac{1}{2}\left[\cos(a + b) + \cos(a - b)\right]$$

$$\sin a \sin b = \frac{1}{2}\left[\cos(a - b) - \cos(a + b)\right]$$

120) $\int \cos 6x \cos 2x dx$ $\left[\frac{1}{8}\left(\frac{1}{2}\sin 8x + \sin 4x\right) + c\right]$

121) $\int \sin 5x \sin x dx$ $\left[\frac{1}{4}\left(\frac{1}{2}\sin 4x - \frac{1}{3}\sin 6x\right) + c\right]$

122) $\int \cos 4x \sin x dx$ $\left[\frac{1}{2}\left(\frac{1}{3}\cos 3x - \frac{1}{5}\cos 5x\right) + c\right]$

123) $\int \sin 4x \cos 2x dx$ $\left[-\frac{1}{4}\left(\frac{1}{3}\cos 6x + \cos 2x\right) + c\right]$

124) $\int \cos 3x \sin 2x dx$ $\left[\frac{1}{2}\left(\cos x - \frac{1}{5}\cos 5x\right) + c\right]$

125) $\int \cos 2x \sin 4x dx$ $\left[-\frac{1}{4}\left(\cos 2x+\frac{1}{3}\cos 6x\right)+c\right]$

126) $\int \sin 4x \sin 3x dx$ $\left[\frac{1}{2}\left(\sin x-\frac{1}{7}\sin 7x\right)+c\right]$

127) $\int \cos 3x \cos 2x dx$ $\left[\frac{1}{2}\sin x+\frac{1}{10}\sin 5x+c\right]$

128) $\int \cos 5x \cos 3x dx$ $\left[\frac{1}{4}\left(\sin 2x+\frac{1}{4}\sin 8x\right)+c\right]$

129) $\int \sin 5x \sin 2x dx$ $\left[\frac{1}{2}\left(\frac{1}{3}\sin 3x-\frac{1}{7}\sin 7x\right)+c\right]$

130) $\int \sin 7x \cos 3x dx$ $\left[-\frac{1}{4}\left(\frac{1}{5}\cos 10x+\frac{1}{2}\cos 4x\right)+c\right]$

131) $\int \cos 6x \sin 2x dx$ $\left[\frac{1}{8}\left(\cos 4x-\frac{1}{2}\cos 8x\right)+c\right]$

132) $\int \cos 4x \cos x dx$ $\left[\frac{1}{2}\left(\frac{1}{5}\sin 5x+\frac{1}{3}\sin 3x\right)+c\right]$

133) $\int \left(2+\cos x-2\cos^2 x\right)dx$ $\left[x+\sin x-\frac{1}{2}\sin 2x+c\right]$

134) $\int \frac{\cos x dx}{1+\cos x}$ $\left[x-\operatorname{tg}\frac{x}{2}+c\right]$

135) $\int \frac{dx}{5+9x^2}$ $\left[\frac{1}{3\sqrt{5}}\operatorname{arctg}\frac{3x}{\sqrt{5}}+c\right]$

136) $\int \frac{dx}{3+16x^2}$ $\left[\frac{1}{4\sqrt{3}}\operatorname{arctg}\frac{4x}{\sqrt{3}}+c\right]$

137) $\int \frac{dx}{2+25x^2}$ $\left[\frac{1}{5\sqrt{2}}\operatorname{arctg}\frac{5x}{\sqrt{2}}+c\right]$

138) $\int\left(\sqrt{\frac{2-x}{2+x}}-\sqrt{\frac{2+x}{2-x}}\right)dx$ $\left[2\sqrt{4-x^2}+c\right]$

139) $\int\left(\sqrt{\frac{b-x}{b+x}}-\sqrt{\frac{b+x}{b-x}}\right)dx$ $\left[2\sqrt{b^2-x^2}+c\right]$

140) $\int \sqrt{1-\cos x}\,dx$ $\left[-2\sqrt{2}\cos\frac{x}{2}+c\right]$

141) $\int \sqrt{1+\cos x}\,dx$ $\left[2\sqrt{2}\sin\frac{x}{2}+c\right]$

142) $\int 2xe^{x^2}\,dx$ $\left[e^{x^2}+c\right]$

143) $\int e^{\sin x}\cos x\,dx$ $\left[e^{\sin x}+c\right]$

144) $\int e^{\cos x}\sin x\,dx$ $\left[-e^{\cos x}+c\right]$

145) $\int \frac{5e^x\,dx}{3+3e^{2x}}$ $\left[\frac{5}{3}\operatorname{arctg}e^x+c\right]$

146) $\int e^{(1/3)+x}\,dx$ $\left[e^{1/3}e^x+c\right]$

147) $\int \frac{\log x}{x}\,dx$ $\left[\frac{1}{2}\log^2 x+c\right]$

148) $\int e^x\left(\log x+\frac{1}{x}\right)dx$ $\left[e^x\log x+c\right]$

II) *Calcolare i seguenti integrali utilizzando il* ***metodo di sostituzione*** *e tenendo in considerazione gli esempi riportati nella teoria* (cfr. **¶ 9. *c)***):

1) $\int \sin^2 x\cos x\,dx$ $\left[\frac{1}{3}\sin^3 x+c\right]$

2) $\int \cos^2 x\sin x\,dx$ $\left[-\frac{1}{3}\cos^3 x+c\right]$

3) $\int \sin^n x\cos x\,dx$ $\left[\frac{1}{n+1}\sin^{n+1} x+c\right]$

4) $\int \cos^n x\sin x\,dx$ $\left[-\frac{1}{n+1}\cos^{n+1} x+c\right]$

5) $\int \frac{1+\sin x}{(x-\cos)^2}\,dx$ $\left[\frac{1}{\cos x-x}+c\right]$

6) $\int \sin^4 x \cos^3 x dx$ $\left[\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c\right]$

7) $\int \frac{\sin x}{1+\cos^2 x} dx$ $\left[\begin{cases} -\operatorname{arctg}(\cos x) + c \\ \operatorname{arctg}(\cos x) + c \end{cases}\right]$

8) $\int \cos^4 x \sin^3 x dx$ $\left[\frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + c\right]$

9) $\int \frac{dx}{1+\cos x}$ $\left[\operatorname{tg}\frac{x}{2} + c\right]$

10) $\int \sin^2 x \cos^3 x dx$ $\left[\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c\right]$

11) $\int \frac{dx}{(x-2)\log^3(x-2)}$ $\left[-\frac{1}{2}\frac{1}{\log^2(x-2)} + c\right]$

12) $\int \sin^3 x \sqrt{\cos x} dx$ $\left[2\sqrt{\cos^3 x}\left(\frac{1}{7}\cos^2 x - \frac{1}{3}\right) + c\right]$

13) $\int \frac{\cos x dx}{2+3\sin x}$ $\left[\frac{1}{3}\log|2+3\sin x| + c\right]$

14) $\int \frac{\sin 2x}{1+\cos^2 2x} dx$ $\left[-\frac{1}{2}\operatorname{arctg}(\cos 2x) + c\right]$

15) $\int \frac{1+\sin x}{(x-\cos x)^3} dx$ $\left[-\frac{1}{2}\frac{1}{(x-\cos x)^2} + c\right]$

16) $\int \frac{\cos 3x}{1+\sin^2 3x} dx$ $\left[\frac{1}{3}\operatorname{arctg}(\sin 3x) + c\right]$

17) $\int \frac{\sin x + \cos x}{(\cos x - \sin x)^4} dx$ $\left[\frac{1}{3}\frac{1}{(\cos x - \sin x)^3} + c\right]$

18) $\int \frac{\sin 2x}{\cos^2 x} dx$ $\left[\log(\sec^2 x) + c\right]$

19) $\int \sqrt{\cos^2 x + 1} \sin 2x dx$ $\left[-\frac{2}{3}\sqrt{(\cos^2 x + 1)^3} + c\right]$

20) $\int \frac{dx}{\operatorname{tg} x \sin^2 x}$ $\left[-\frac{1}{2}\operatorname{ctg}^2 x + c\right]$

21) $\int \frac{\sqrt[3]{1+\mathrm{tg}x}}{\cos^2 x}\,dx$ $\left[\frac{3}{4}\sqrt[3]{(1+\mathrm{tg}x)^4}+c\right]$

22) $\int x^2 e^{x^3-3}dx$ $\left[\frac{1}{3}e^{x^3-3}+c\right]$

23) $\int \frac{\sin\left(1-\sqrt{x}\right)}{\sqrt{x}}dx$ $\left[2\cos\left(1-\sqrt{x}\right)+c\right]$

24) $\int x^3\left(e^{x^4}-e^{-x^4}\right)dx$ $\left[\frac{1}{4}\left(e^{x^4}+e^{-x^4}\right)+c\right]$

25) $\int \frac{e^{2x}}{3+e^{2x}}dx$ $\left[\log\sqrt{e^{2x}+3}+c\right]$

26) $\int \left(e^{2+\sqrt{x}}\right)x^{-1/2}dx$ $\left[2e^{2+\sqrt{x}}+c\right]$

27) $\int \frac{\sin\log x}{x}\,dx$ $\left[-\cos\left(\log x\right)+c\right]$

28) $\int \frac{\sin 2x+e^{2x}}{\cos 2x-e^{2x}}\,dx$ $\left[-\log\sqrt{e^{2x}-\cos 2x}+c\right]$

III) *Calcolare i seguenti integrali utilizzando il metodo di* ***integrazione per parti****, tenendo in considerazione gli esempi riportati in* **¶ 9. *b)*** *della teoria*:

1) $\int \log x dx$ $\left[x\log x-x+c\right]$

2) $\int x\sin 3x dx$ $\left[\frac{1}{9}\left(\sin 3x-3x\cos 3x+c\right)\right]$

3) $\int x^2\sin x dx$ $\left[-x^2\cos x+2x\sin x+2\cos x+c\right]$

4) $\int x\log x dx$ $\left[\frac{x^2}{4}\left(2\log x-1\right)+c\right]$

5) $\int x^2\cos 2x dx$ $\left[\frac{1}{4}\left[\left(2x^2-1\right)\sin 2x+2x\cos 2x\right]+c\right]$

6) $\int \frac{x dx}{\cos^2 x}$ $\left[x\mathrm{tg}x+\log|\cos x|+c\right]$

7) $\int xe^x dx$ $\left[(x-1)e^x + c\right]$

8) $\int \sin^2(3x+1)\,dx$ $\left[\frac{1}{2}x - \frac{1}{12}\sin(6x+2) + c\right]$

9) $\int e^x \cos x dx$ $\left[\frac{1}{2}e^x(\sin x + \cos x) + c\right]$

10) $\int xe^{3x} dx$ $\left[\frac{1}{9}e^{3x}(3x-1) + c\right]$

11) $\int \cos^4 \frac{x}{2}\,dx$ $\left[\frac{3}{8}x + \frac{1}{2}\sin x + \frac{1}{16}\sin 2x + c\right]$

12) $\int e^x \sin x dx$ $\left[\frac{1}{2}e^x(\sin x - \cos x) + c\right]$

13) $\int (x^2-1)e^x dx$ $\left[(x-1)^2 e^x + c\right]$

14) $\int x\operatorname{arctg} x dx$ $\left[\frac{1}{2}\left[-x + (x^2+1)\operatorname{arctg} x\right] + c\right]$

15) $\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$ $\left[x - \sqrt{1-x^2}\arcsin x + c\right]$

16) $\int x^{1/2} \log x dx$ $\left[\frac{2}{9}x\sqrt{x}(3\log x - 2) + c\right]$

17) $\int \arcsin x dx$ $\left[x\arcsin x + \sqrt{1-x^2} + c\right]$

18) $\int \operatorname{arctg} bx dx$ $\left[x\operatorname{arctg} bx - \frac{1}{2b}\log\left|1+b^2x^2\right| + c\right]$

19) $\int \sin^2 x dx$ $\left[\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + c\right]$

20) $\int \log^2 x dx$ $\left[x\log^2 x - 2x\log x + 2x + c\right]$

21) $\int \cos^2 x dx$ $\left[\frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + c\right]$

22) $\int e^{2x}\sin x\,dx$ $\qquad \left[\frac{1}{5}e^{2x}\left(2\sin x-\cos x\right)+c\right]$

23) $\int \cos^3 x\,dx$ $\qquad \left[\sin x\left(1-\frac{1}{3}\sin^2 x\right)+c\right]$

24) $\int e^x\cos 3x\,dx$ $\qquad \left[\frac{1}{10}e^x\left(3\sin 3x+\cos 3x\right)+c\right]$

25) $\int \sin^3 x\,dx$ $\qquad \left[\cos x\left(\frac{1}{3}\cos^2 x-1\right)+c\right]$

26) $\int \log(3x+2)\,dx$ $\qquad \left[\frac{1}{3}\left[(3x+2)\log(3x+2)-3x\right]+c\right]$

27) $\int \sin(\log x)\,dx$ $\qquad \left[\frac{x}{2}\left[\sin(\log x)-\cos(\log x)\right]+c\right]$

28) $\int (x+1)e^{x+4}\,dx$ $\qquad \left[xe^{x+4}+c\right]$

29) $\int \cos(\log x)\,dx$ $\qquad \left[\frac{x}{2}\left[\cos(\log x)+\sin(\log x)\right]+c\right]$

30) $\int \operatorname{arctg}5x\,dx$ $\qquad \left[x\operatorname{arctg}5x-\frac{1}{10}\log\left|1+25x^2\right|+c\right]$

31) $\int \log(x+1)\,dx$ $\qquad \left[(x+1)\log(x+1)-(x+1)+c\right]$

32) $\int \cos^4 x\,dx$ $\qquad \left[\frac{1}{4}\sin x\cos^3 x+\frac{3}{8}\left(\sin x\cos x+x\right)+c\right]$

IV) *Calcolare gli* ***integrali definiti*** **1)-71)** (cfr. **¶ 8.**), *dopo aver analizzato gli esempi* ***a)-c)*** *di seguito riportati*:

a) $\int_0^{p/2}\sin x\,dx=\left(-\cos x\right)\Big|_0^{p/2}=-\cos\frac{p}{2}-\left(-\cos 0\right)=-\cos\frac{p}{2}+\cos 0=1$

b) $\int_0^1 xe^{4x}\,dx=\left(\frac{1}{4}xe^{4x}\right)\Big|_0^1-\int_0^1\frac{1}{4}e^{4x}\,dx=\left(\frac{1}{4}xe^{4x}\right)\Big|_0^1-\left(\frac{1}{16}e^{4x}\right)\Big|_0^1=$

$$= \left(\frac{1}{4}e^4 - \frac{1}{4}\cdot 0\right) - \left(\frac{1}{16}e^4 - \frac{1}{16}\cdot e^0\right) = \frac{1}{4}e^4 - \frac{1}{16}e^4 + \frac{1}{16} = \frac{1}{16}\left(3e^4 + 1\right)$$

avendo applicato la formula di integrazione per parti con:

$$\begin{cases} f'(x) = 1 \\ g(x) = \frac{1}{4}e^{4x} \end{cases} \quad \Rightarrow \quad \begin{cases} f(x) = x \\ g'(x) = e^{4x} \end{cases}$$

c) $\displaystyle\int_0^1 \sqrt{1+x^2}\,dx$

Per calcolare il corrispondente integrale indefinito occorre applicare il metodo di integrazione per sostituzione, ponendo:

$$\sqrt{1+x^2} = x + t \quad (x + t > 0) \quad \Rightarrow \quad 1 + x^2 = (x+t)^2 = x^2 + 2xt + t^2 \quad \Rightarrow$$

$$\Rightarrow \quad x = \frac{1-t^2}{2t} \quad \Rightarrow \quad \sqrt{1+x^2} = x + t = \frac{1-t^2}{2t} + t = \frac{1-t^2+2t^2}{2t} = \frac{1+t^2}{2t} \quad \Rightarrow$$

$$\Rightarrow \quad dx = \frac{-2t(2t) - 2\left(1-t^2\right)}{4t^2}dt = \frac{-4t^2 - 2 + 2t^2}{4t^2}dt = \frac{-2t^2-2}{4t^2}dt = -\frac{t^2+1}{2t^2}dt$$

Bisogna ora analizzare i valori assunti dalla nuova variabile *t*, quando *x* varia tra 0 ed 1:

$$\boxed{x = 0} \quad \Rightarrow \quad x = \frac{1-t^2}{2t} = 0 \quad \Rightarrow \quad 1 - t^2 = 0 \quad \Rightarrow \quad t^2 - 1 = 0 \quad \Rightarrow \quad t = \pm 1$$

ovvero:

$x = 0 \quad \Rightarrow \quad t = +1$ dovendo risultare $x + t > 0 \quad \Rightarrow \quad t > -x \quad \Rightarrow \quad t > 0$

$$\boxed{x = 1} \quad \Rightarrow \quad x = \frac{1-t^2}{2t} = 1 \quad \Rightarrow \quad 1 - t^2 = 2t \quad \Rightarrow \quad t^2 + 2t - 1 = 0 \quad \Rightarrow \quad t = -1 \pm \sqrt{2}$$

ovvero:

$$x = 1 \quad \Rightarrow \quad t = -1 + \sqrt{2}$$

Quindi si può scrivere:

$$\int_0^1 \sqrt{1+x^2}\,dx = \int_1^{\sqrt{2}-1} \left(\frac{1+t^2}{2t}\right)\cdot\left(-\frac{t^2+1}{2t^2}\right)dt = -\frac{1}{4}\int_1^{\sqrt{2}-1} \frac{\left(t^2+1\right)^2}{t^3}dt = -\frac{1}{4}\int_1^{\sqrt{2}-1} \frac{t^4 + 2t^2 + 1}{t^3}dt =$$

$$= -\frac{1}{4}\left[\int_1^{\sqrt{2}-1} \frac{t^4}{t^3}dt\right] - \frac{1}{4}\left[\int_1^{\sqrt{2}-1} \frac{2t^2}{t^3}dt\right] - \frac{1}{4}\left[\int_1^{\sqrt{2}-1} \frac{1}{t^3}dt\right] = -\frac{1}{4}\int_1^{\sqrt{2}-1} t\,dt - \frac{1}{2}\int_1^{\sqrt{2}-1} \frac{1}{t}dt - \frac{1}{4}\int_1^{\sqrt{2}-1} \frac{1}{t^3}dt =$$

$$= -\frac{1}{4}\cdot\frac{t^2}{2}\Bigg|_1^{\sqrt{2}-1} - \frac{1}{2}\cdot\log|t|\Bigg|_1^{\sqrt{2}-1} - \frac{1}{4}\cdot\frac{-1}{2t^2}\Bigg|_1^{\sqrt{2}-1} =$$

$$= \frac{1}{4}\left[-\frac{1}{2}\left(\sqrt{2}-1\right)^2 + \frac{1}{2} - 2\log\left|\sqrt{2}-1\right| + \frac{1}{2\left(\sqrt{2}-1\right)^2} - \frac{1}{2} \right] =$$

$$= \frac{1}{4}\left[-\frac{1}{2}\left(\sqrt{2}-1\right)^2 - 2\log\left|\sqrt{2}-1\right| + \frac{1}{2\left(\sqrt{2}-1\right)^2} \right] = \frac{1}{2}\left[\sqrt{2} - \log\left(\sqrt{2}-1\right) \right] =$$

$$= \frac{1}{2}\left[\sqrt{2} + \log\left(\sqrt{2}+1\right) \right]$$

1) $\int_0^2 (1+x)^2\,dx$ $\left[\frac{26}{3}\right]$

2) $\int_1^3 (2x-1)\,dx$ $[6]$

3) $\int_1^3 3\left(x^2+1\right)dx$ $[32]$

4) $\int_1^2 \left(3x^2 + x - 1\right)dx$ $\left[\frac{15}{2}\right]$

5) $\int_{-1}^3 \frac{1}{2}(3x-1)\,dx$ $[4]$

6) $\int_{-1}^{3/2} \left(x^2 - 5x + 1\right)dx$ $\left[\frac{5}{6}\right]$

7) $\int_{-1}^1 \left(3x^2 - \frac{1}{2}x + 1\right)dx$ $[4]$

8) $\int_{-2}^3 \left(5x^2 - 2\right)dx$ $\left[\frac{145}{3}\right]$

9) $\int_0^1 \left(x^3-3x^2+2x+3\right)dx$ $\left[\frac{13}{4}\right]$

10) $\int_0^2 x\left(3x+1\right)dx$ $[54]$

11) $\int_0^{1/2} \left(2x^2-3\right)dx$ $\left[-\frac{17}{12}\right]$

12) $\int_0^2 \left(\frac{1}{\sqrt{2+x}}+\frac{1}{\sqrt{2x}}\right)dx$ $\left[6-2\sqrt{2}\right]$

13) $\int_{-1}^1 \left(\frac{3}{4}x^2-\frac{1}{2}\sqrt[3]{x}-2\right)dx$ $\left[-\frac{7}{2}\right]$

14) $\int_0^2 \left(2x^2-3x-2\right)dx$ $\left[-\frac{14}{3}\right]$

15) $\int_1^4 \left(x^{-3}-x^{-2}+\sqrt{x}-3x^2\right)dx$ $\left[-\frac{5627}{96}\right]$

16) $\int_{-2}^1 \left(3x+x^{-2}-2x^{-3}\right)dx$ $\left[-\frac{21}{4}\right]$

17) $\int_{-2}^2 \left(\frac{1}{2}x^2+2x-\frac{1}{4}\right)dx$ $\left[\frac{5}{3}\right]$

18) $\int_{-1}^1 \frac{xdx}{\sqrt{1+x^2}}$ $[0]$

19) $\int_0^{p/2} \left(3\sin 2x\right)dx$ $[3]$

20) $\int_{-3}^{1} (x-2)(x-1)^2\,dx$ $\left[-\frac{256}{3}\right]$

21) $\int_{1}^{2} \frac{(x-1)\,dx}{\sqrt{x^2-2x+3}}$ $\left[\sqrt{3}-\sqrt{2}\right]$

22) $\int_{-p/2}^{p/2} (2\sin x-\cos x)\,dx$ $[-2]$

23) $\int_{1}^{3} \sqrt{2x+1}dx$ $\left[\frac{1}{3}\left(\sqrt{343}-3\sqrt{3}\right)\right]$

24) $\int_{0}^{p/2} \sin^2 x\cos x dx$ $\left[\frac{1}{3}\right]$

25) $\int_{p/2}^{p} \cos 3x dx$ $\left[\frac{1}{3}\right]$

26) $\int_{1}^{8} \sqrt{3x+1}dx$ $[26]$

27) $\int_{0}^{p/4} 2\sin^2 x dx$ $\left[\frac{p}{4}-\frac{1}{2}\right]$

28) $\int_{0}^{p} \frac{\cos 2x}{\cos x-\sin x}\,dx$ $[2]$

29) $\int_{a}^{2a} \frac{x dx}{\sqrt{x^2-a^2}}$ $\left[a\sqrt{3}\right]$

30) $\int_{0}^{p} (2\sin x-3\cos x)\,dx$ $[4]$

31) $\int_{2}^{3} \log x dx$ $\left[\log 27 + \operatorname{colog} 4 - 1\right]$

32) $\int_{0}^{p} \left(2\sin x + 5\cos x\right) dx$ $[4]$

33) $\int_{p/4}^{p/3} \left(1 - \frac{1}{\sin^2 x}\right) dx$ $\left[\frac{p}{12} - 1 + \frac{\sqrt{3}}{3}\right]$

34) $\int_{3}^{2} x e^x dx$ $\left[e^2\left(1 - 2e\right)\right]$

35) $\int_{-p/2}^{p/2} \sin^2 x \cos x dx$ $\left[\frac{2}{3}\right]$

36) $\int_{p/4}^{p/3} \left(\frac{3}{\cos^2 x} - \frac{2}{\sin^2 x}\right) dx$ $\left[\frac{1}{3}\left(11\sqrt{3} - 15\right)\right]$

37) $\int_{0}^{p/2} x \sin x dx$ $[1]$

38) $\int_{0}^{p} \frac{\cos 2x}{\cos x + \sin x} dx$ $[-2]$

39) $\int_{0}^{1} \arcsin x dx$ $\left[\frac{p}{2} - 1\right]$

40) $\int_{0}^{p/3} \sin x \cos x dx$ $\left[\frac{3}{8}\right]$

41) $\int_{-p/2}^{p/2} \frac{\cos x}{1+\cos x}\,dx$ $[p-2]$

42) $\int_{0}^{p/4} \mathrm{tg}x\,dx$ $\left[\log\sqrt{2}\right]$

43) $\int_{-a}^{+a} \frac{x^3\,dx}{\sqrt{a^4-x^4}}$ $[0]$

44) $\int_{p/4}^{p/2} \mathrm{ctg}x\,dx$ $\left[\log\sqrt{2}\right]$

45) $\int_{0}^{p/2} \sqrt{\cos^2 x+1}\sin 2x\,dx$ $\left[\frac{2}{3}\left(2\sqrt{2}-1\right)\right]$

46) $\int_{0}^{2\sqrt{2}} \frac{x\,dx}{\sqrt{1+x^2}}$ $[2]$

47) $\int_{-p/4}^{p/4} \frac{(1+\mathrm{tg}x)^2}{\cos^2 x}\,dx$ $\left[\frac{8}{3}\right]$

48) $\int_{1}^{2} \frac{\log x}{x}\,dx$ $\left[\frac{1}{2}\log^2 2\right]$

49) $\int_{p/4}^{p/3} e^x \sin x\,dx$ $\left[\frac{1}{4}\left(\sqrt{3}-1\right)e^{p/3}\right]$

50) $\int_{-p/2}^{p/2} \frac{tg\,\frac{x}{2}}{\cos^2\frac{x}{2}}\,dx$ $[0]$

51) $\int_{p/6}^{p/3} e^x \cos x\,dx$ $\left[\frac{1}{4}\left(\sqrt{3}+1\right)\left(e^{p/3}-e^{p/6}\right)\right]$

52) $\int_{1/2}^{3} \frac{1}{2x+3}\,dx$ $\left[\log\frac{3}{2}\right]$

53) $\int_{0}^{p/2} \left(5\sin x+4\cos x\right)dx$ $[9]$

54) $\int_{p/4}^{p/3} \frac{\sin x\left(\cos x+1\right)}{\cos x}\,dx$ $\left[\frac{1}{2}\left(\sqrt{2}-1+\log 2\right)\right]$

55) $\int_{0}^{p/3} \frac{\sin x\cos^2 x+1}{\cos^2 x}\,dx$ $\left[\frac{1}{2}+\sqrt{3}\right]$

56) $\int_{p/6}^{p/2} \left(\cos x-\cos^3 x\right)dx$ $\left[\frac{7}{24}\right]$

57) $\int_{0}^{p/4} \left(\sec^2 x+3\right)dx$ $\left[1+\frac{3}{4}p\right]$

58) $\int_{p/6}^{p/3} \left(\sec^2 x+\mathrm{cosec}^2 c\right)dx$ $\left[\frac{4}{3}\sqrt{3}\right]$

59) $\int_{0}^{p/4} \frac{\cos x-\sin x}{\cos x+\sin x}\,dx$ $\left[\log\sqrt{2}\right]$

60) $\int_{0}^{p} \frac{\cos^2 x}{1+\sin x}\,dx$ $[p-2]$

61) $\int_0^{p/4} \frac{\sin x\cos^2 x + 3\cos^2 x + 1}{\cos^2 x}\,dx$ $\left[2+\sqrt{2}\right]$

62) $\int_0^{p/2} \cos x\sqrt{1+\sin x}\,dx$ $\left[\frac{2}{3}\left(2\sqrt{2}-1\right)\right]$

63) $\int_0^1 x\,\mathrm{arctg}x\,dx$ $\left[\frac{1}{2}\left(\frac{p}{2}-1\right)\right]$

64) $\int_0^{p/2} x\cos x\,dx$ $\left[\frac{p-2}{2}\right]$

65) $\int_0^{p/2} \sin^3 x\,dx$ $\left[\frac{2}{3}\right]$

66) $\int_1^3 \sqrt{2x^2+x^4}\,dx$ $\left[\frac{11}{3}\sqrt{11}-\sqrt{3}\right]$

67) $\int_0^1 \frac{e^x}{e^x+2}\,dx$ $\left[\log\frac{2+e}{3}\right]$

68) $\int_1^3 \frac{x^2+x\sqrt{x}+3}{\sqrt{x}}dx$ $\left[\frac{1}{5}\left(48\sqrt{3}-12\right)\right]$

69) $\int_0^{\log 2} e^x\sqrt{e^x-1}dx$ $\left[\frac{2}{3}\right]$

70) $\int_1^2 \log x\,dx$ $[\log 4-1]$

71) $\int_1^2 \frac{e^x}{2+e^x}\,dx$ $\left[\log\frac{2+e^2}{2+e}\right]$