

CAPITOLO VII

DERIVATE

1. GENERALITÀ

Definizione 1.1)

La *derivata* è un operatore che ad una funzione f associa un'altra funzione e che obbedisce alle seguenti regole:

$$(1) D(a_0 x^n) = (a_0 x^n)' = n a_0 x^{n-1} \quad \text{derivata di un monomio}$$

ESEMPI

$$D(3x^2) = 2 \cdot 3 \cdot x^{2-1} = 6x$$

$$D(5x^4) = 4 \cdot 5 \cdot x^{4-1} = 20x^3$$

$$D(-4x^3) = 3 \cdot (-4) \cdot x^{3-1} = -12x^2$$

$$(2) D(a_0 x) = a_0 \quad \text{derivata di un monomio con } n = 1$$

ESEMPI

$$D(7x) = 1 \cdot 7 \cdot x^{1-1} = 7x^0 = 7 \cdot 1 = 7, D(10x) = 10, D(3x) = 3$$

$$(3) D(x) = 1 \quad \text{derivata di un monomio con } a_0 = 1 = n$$

$$(4) D(c) = 0 \quad \text{derivata di una costante}$$

ESEMPI

$$D(4) = 0, D(10) = 0, D(25) = 0$$

$$(5) D(x^n) = n x^{n-1} \quad \text{derivata di un monomio con } a_0 = 1$$

Più in generale risulta:

$$(5.1) D(x^a) = a x^{a-1} \quad (a \text{ reale qualsiasi})$$

Ricordando le regole delle potenze:

$$a) a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$b) a^{-n} = \frac{1}{a^n}$$

$$c) a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$$

seguono varie proprietà applicate nei seguenti

ESEMPI

$$D(x^3) = 3x^{3-1} = 3x^2, D(x^5) = 5x^{5-1} = 5x^4, D(x^8) = 8x^{8-1} = 8x^7, D(x^{\sqrt{2}}) = \sqrt{2} x^{\sqrt{2}-1},$$

$$D(x^{1+\sqrt[3]{2}}) = (1+\sqrt[3]{2}) x^{\sqrt[3]{2}}$$

Se $a = \frac{1}{n}$ allora si ha:

$$D\left(x^{\frac{1}{n}}\right) = \frac{1}{n} x^{\frac{1}{n}-1} = \frac{1}{n} x^{\frac{1-n}{n}} = \frac{1}{n} \cdot \frac{1}{x^{\frac{n-1}{n}}}$$

che si può scrivere, in modo più semplice, come segue:

$$(6) D(\sqrt[n]{x}) = \frac{1}{n \sqrt[n]{x^{n-1}}} \quad \text{derivata della radice } n\text{-esima}$$

ESEMPI

$$D(\sqrt{x}) = \frac{1}{2 \sqrt{x^{2-1}}} = \frac{1}{2 \sqrt{x}}, D(\sqrt[3]{x}) = \frac{1}{3 \sqrt[3]{x^{3-1}}} = \frac{1}{3 \sqrt[3]{x^2}}, D(\sqrt[5]{x}) = \frac{1}{5 \sqrt[5]{x^{5-1}}} = \frac{1}{5 \sqrt[5]{x^4}}$$

Più in generale si ottiene:

$$(7) D\left[\sqrt[n]{f(x)}\right] = \frac{f'(x)}{n \sqrt[n]{[f(x)]^{n-1}}} \quad \text{derivata della radice } n\text{-esima di una funzione}$$

ESEMPI

$$D(\sqrt{x^3 - 4x + 2}) = \frac{(x^3 - 4x + 2)'}{2 \sqrt{(x^3 - 4x + 2)^{2-1}}} = \frac{3x^2 - 4}{2 \sqrt{x^3 - 4x + 2}}$$

$$D\left(\sqrt[3]{4x^3+6x^2-5}\right) = \frac{(4x^3+6x^2-5)^{\prime}}{3\sqrt[3]{(4x^3+6x^2-5)^{3-1}}} = \frac{12x^2+12x}{3\sqrt[3]{(4x^3+6x^2-5)^2}} = \frac{4x^2+4x}{\sqrt[3]{(4x^3+6x^2-5)^2}}$$

$$D\left(\sqrt[5]{5x+3}\right) = \frac{(5x+3)^{\prime}}{5\sqrt[5]{(5x+3)^{5-1}}} = \frac{5}{5\sqrt[5]{(5x+3)^4}} = \frac{1}{\sqrt[5]{(5x+3)^4}}$$

(8) $D[a f(x) \pm b g(x)] = a D f(x) \pm b D g(x)$ *derivata della somma (o differenza) e linearità*

ESEMPI

$$D(5x^4 + 3x^2 + 7x + 6) = 5D(x^4) + 3D(x^2) + 7D(x) + 6D(1) = 20x^3 + 6x + 7$$

$$D(6x^3 - 2x^2 + 4x - 6) = 6D(x^3) - 2D(x^2) + 4D(x) - 6D(1) = 18x^2 - 4x + 4$$

$$D(-4x^3 + 7x) = -4D(x^3) + 7D(x) = -12x^2 + 7$$

(9) $D[f(x)g(x)] = [f(x)g(x)]^{\prime} = f'(x)g(x) + f(x)g'(x)$ *derivata del prodotto*

ESEMPI

$$\begin{aligned} D[(x^2-1)(5x+2)] &= (x^2-1)^{\prime}(5x+2) + (x^2-1)(5x+2)^{\prime} = 2x(5x+2) + (x^2-1)5 = \\ &= 10x^2 + 4x + 5x^2 - 5 = 15x^2 + 4x - 5 \end{aligned}$$

$$\begin{aligned} D[(2x+3)(x^2+2x)] &= (2x+3)^{\prime}(x^2+2x) + (2x+3)(x^2+2x)^{\prime} = 2(x^2+2x) + (2x+3)(2x+2) = \\ &= 2x^2 + 4x + 4x^2 + 4x + 6x + 6 = 6x^2 + 14x + 6 \end{aligned}$$

$$\begin{aligned} D[(3x^3+4x+1)(3x^2+4)] &= (3x^3+4x+1)^{\prime}(3x^2+4) + (3x^3+4x+1)(3x^2+4)^{\prime} = \\ &= (9x^2+4)(3x^2+4) + (3x^3+4x+1)6x = 27x^4 + 36x^2 + 12x^2 + 16 + 18x^4 + 24x^2 + 6x = \\ &= 45x^4 + 72x^2 + 6x + 16 \end{aligned}$$

(10) $D\left[\frac{f(x)}{g(x)}\right] = \left[\frac{f(x)}{g(x)}\right]^{\prime} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}, \quad g(x) \neq 0$ *derivata del quoziente*

ESEMPI

$$D\left(\frac{x}{x^2+1}\right) = \frac{(x)(x^2+1) - x(x^2+1)'}{(x^2+1)^2} = \frac{x^2+1-x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$$D\left(\frac{x^3}{4-x}\right) = \frac{(x^3)(4-x) - x^3(4-x)'}{(4-x)^2} = \frac{3x^2(4-x) - x^3(-1)}{(4-x)^2} = \frac{-2x^3+12x^2}{(4-x)^2}$$

$$D\left(\frac{1+x^2}{4+x^2}\right) = \frac{(1+x^2)(4+x^2) - (1+x^2)'(4+x^2)'}{(4+x^2)^2} = \frac{2x(4+x^2) - (1+x^2)' 2x}{(4+x^2)^2} =$$

$$= \frac{6x}{(4+x^2)^2}$$

(11) $Df[g(x)] = f'[\dots]g'[\dots] = f'[g(x)]g'(x)$

derivata di funzioni composte

ESEMPI

$$D\left(\sqrt{4x^2-3}\right) = \left[(4x^2-3)^{\frac{1}{2}}\right]' = \frac{1}{2\sqrt{4x^2-3}}(4x^2-3)' = \frac{8x}{2\sqrt{4x^2-3}} = \frac{4x}{\sqrt{4x^2-3}}$$

$$D\left[\sqrt[5]{(2x^3+1)^2}\right] = \left[\sqrt[5]{(2x^3+1)^2}\right]' = \left[(2x^3+1)^{\frac{2}{5}}\right]' = \frac{2}{5}(2x^3+1)^{-\frac{3}{5}}[(2x^3+1)'] =$$

$$= \frac{2}{5\sqrt[5]{(2x^3+1)^3}} 6x^2 = \frac{12x^2}{5\sqrt[5]{(2x^3+1)^3}}$$

$$D\left(\sqrt[3]{x^2-7x}\right) = \left[(x^2-7x)^{\frac{1}{3}}\right]' = \frac{1}{3}(x^2-7x)^{\frac{1}{3}-1}(x^2-7x)' = \frac{2x-7}{3\sqrt[3]{(x^2-7x)^2}}$$

$$D\left[(3x^2-x)^5\right] = 5(3x^2-x)^{5-1}(3x^2-x)' = 5(3x^2-x)^4(6x-1)$$

(12) $D\left[e^{f(x)}\right] = e^{f(x)}f'(x)$

derivata di funzioni esponenziali

ESEMPI

$$D\left(e^{x^2+5x+2}\right) = e^{x^2+5x+2}(x^2+5x+2)' = e^{x^2+5x+2}(2x+5)$$

$$D\left(e^{3x^3+7x}\right) = e^{3x^3+7x}(3x^3+7x)' = e^{3x^3+7x}(9x^2+7)$$

$$D \left(e^{\frac{x^3+3}{x^2+1}} \right) = e^{\frac{x^3+3}{x^2+1}} \left(\frac{x^3+3}{x^2+1} \right)' = e^{\frac{x^3+3}{x^2+1}} \left[\frac{3x^2(x^2+1) - (x^3+3)2x}{(x^2+1)^2} \right] = e^{\frac{x^3+3}{x^2+1}} \frac{x^4 + 3x^2 - 6x}{(x^2+1)^2}$$

$$D \left(e^{\frac{2x^2+4x}{3x^2+5}} \right) = e^{\frac{2x^2+4x}{3x^2+5}} \left(\frac{2x^2+4x}{3x^2+5} \right)' = e^{\frac{2x^2+4x}{3x^2+5}} \left[\frac{4x(3x^2+5) - (2x^2+4x)6x}{(3x^2+5)^2} \right] =$$

$$= e^{\frac{2x^2+4x}{3x^2+5}} \frac{-24x^2 + 20x}{(3x^2+5)^2}$$

$$(13) D [\ln f(x)] = \frac{f'(x)}{f(x)}$$

derivata di funzioni logaritmiche

ESEMPI

$$D [\ln(3x^2 + 2)] = \frac{1}{3x^2 + 2} (3x^2 + 2)' = \frac{6x}{3x^2 + 2}$$

$$D [\ln(x^3 - 2x - 1)] = \frac{1}{x^3 - 2x - 1} (x^3 - 2x - 1)' = \frac{3x^2 - 2}{x^3 - 2x - 1}$$

$$D \left[\ln \left(\frac{2x^2 + 4x}{3x^2 + 5} \right) \right] = \frac{3x^2 + 5}{2x^2 + 4x} \left(\frac{2x^2 + 4x}{3x^2 + 5} \right)' = \frac{3x^2 + 5}{2x^2 + 4x} \frac{-24x^2 + 20x}{(3x^2 + 5)^2} = \frac{-24x^2 + 20x}{(2x^2 + 4x)(3x^2 + 5)}$$

Osservazione: riteniamo opportuno richiamare l'attenzione dello studente su alcune proprietà dei logaritmi che si riveleranno particolarmente utili soprattutto per lo studio di funzioni:

- $\log_a(b \cdot c) = \log_a b + \log_a c$ con $a, b > 0$ e $a \neq 1$
- $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$ con $a, b, c > 0$ e $a \neq 1$
- $\log_a(b^n) = n \log_a b$ con $a, b > 0$ e $a \neq 1$ e n intero positivo
- $\log_a(\sqrt[n]{b}) = \frac{1}{n} \log_a b$ con $a, b > 0$ e $a \neq 1$ e n intero positivo
- $\log_a a = 1$ con $a > 0$ e $a \neq 1$
- $\log_a 1 = 0$ con $a > 0$ e $a \neq 1$
- $\log_a 0 = -\infty$ con $a > 0$ e $a \neq 1$
- $\log_b N = \frac{\log_a N}{\log_a b}$ *formula del cambio di base con N intero positivo*

Inoltre, sfruttando la definizione classica di logaritmo, è facile verificare l'equivalenza delle seguenti espressioni:

$$z = \log_a b; \quad a^z = b; \quad a^{\log_a b} = b;$$

In generale si è soliti indicare con \ln o anche con \log il logaritmo naturale o *Neperiano*, cioè in base e .

2. TABELLA DELLE DERIVATE PIÙ COMUNI

Riportiamo qui di seguito una tabella riassuntiva delle derivate di alcune funzioni elementari, scrivendo a sinistra la funzione e, nella stessa linea, a destra, la sua derivata:

$y = c$	$y' = 0$
$y = x$	$y' = 1$
$y = x^n, n \in \mathfrak{R}$	$y' = nx^{n-1}$
$y = x^a, a \in \mathfrak{R} \text{ e } x > 0$	$y' = a x^{a-1}$
$y = \sqrt{x}$	$y' = \frac{1}{2\sqrt{x}}$
$y = \sqrt[n]{x^m}, n > m$	$y' = \frac{m}{n \sqrt[n]{x^{n-m}}}$
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \operatorname{tg} x$	$y' = \frac{1}{\cos^2 x} = 1 + \operatorname{tg}^2 x$
$y = \operatorname{ctg} x$	$y' = -\frac{1}{\sin^2 x} = -1 - \operatorname{cotg}^2 x$
$y = e^x$	$y' = e^x$
$y = a^x, a > 0$	$y' = a^x \log a$
$y = x^x$	$y' = x^x (1 + \ln x)$
$y = \ln x, x > 0$	$y' = \frac{1}{x}$
$y = \log_a x, x > 0, a > 0, a \neq 1$	$y' = \frac{1}{x} \log_a e$

$$y = \arcsin x, \quad -\frac{\mathbf{p}}{2} < y < \frac{\mathbf{p}}{2}$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

$$y = \arccos x, \quad 0 < y < \mathbf{p}$$

$$y' = -\frac{1}{\sqrt{1-x^2}}$$

$$y = \operatorname{arctg} x$$

$$y' = \frac{1}{1+x^2}$$

$$y = \operatorname{arcctg} x$$

$$y' = -\frac{1}{1+x^2}$$

Riportiamo adesso un elenco di derivate di funzioni elementari ottenuto dalla tabella precedente sostituendo alla variabile indipendente x una certa funzione $f(x)$ di cui si conosca la derivata ed applicando poi la regola di derivazione delle funzioni composte:

$$y = [f(x)]^n$$

$$y' = n [f(x)]^{n-1} f'(x)$$

$$y = \sqrt{f(x)}$$

$$y' = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$y = \sqrt[n]{[f(x)]^m}$$

$$y' = \frac{m}{n \sqrt[n]{[f(x)]^{n-m}}}$$

$$y = \sin f(x)$$

$$y' = \cos f(x) f'(x)$$

$$y = \cos f(x)$$

$$y' = -\sin f(x) f'(x)$$

$$y = \operatorname{tg} f(x)$$

$$y' = \frac{1}{\cos^2 f(x)} f'(x)$$

$$y = \operatorname{ctg} f(x)$$

$$y' = -\frac{1}{\sin^2 f(x)} f'(x)$$

$$y = \arcsin f(x)$$

$$y' = \frac{1}{\sqrt{1-[f(x)]^2}} f'(x)$$

$$y = \arccos f(x)$$

$$y' = -\frac{1}{\sqrt{1-[f(x)]^2}} f'(x)$$

$$y = \operatorname{arctg} f(x)$$

$$y' = \frac{1}{1+[f(x)]^2} f'(x)$$

$$y = \operatorname{arcctg} f(x)$$

$$y' = -\frac{1}{1+[f(x)]^2} f'(x)$$

$$y = e^{f(x)}$$

$$y' = e^{f(x)} f'(x)$$

$$y = a^{f(x)}$$

$$y' = a^{f(x)} \ln a f'(x)$$

$$y = \ln f(x)$$

$$y' = \frac{1}{f(x)} f'(x)$$

$$y = \log_a f(x)$$

$$y' = \frac{1}{f(x)} \log_a e f'(x)$$

$$y = [f(x)]^{g(x)}$$

$$y' = [f(x)]^{g(x)} \left[g'(x) \log f(x) + \frac{g(x)}{f(x)} f'(x) \right]$$

ESERCIZI PROPOSTI

Calcolare le derivate delle seguenti funzioni polinomiali:

$y = 3x^2 + 1$	[6x]
$y = 5x + 7$	[5]
$y = 2x - 5$	[2]
$y = 3x^2 - 6x + 4$	[6x - 6]
$y = 4x^3 - 2x^2 + 5x - 3$	[12x ² - 4x + 5]
$y = 4x^2 - 1$	[8x]
$y = 1 + x + x^2$	[1 + 2x]
$y = x^3 - 2x$	[3x ² - 2]
$y = 3x - 1$	[3]
$y = 4x^2$	[8x]
$y = 4x^2 + 5$	[8x]
$y = x^5 + 4x^2$	[5x ⁴ + 8x]
$y = x^3 + 2x^2 + 1$	[3x ² + 4x]
$y = 3x^4 - 5x^3 + 4x - 7$	[12x ³ - 15x ² + 4]
$y = 8x^5 - 24x^3 + 7$	[40x ⁴ - 72x ²]
$y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + 5x + 9$	[x + x ² + 5]
$y = (2x + 3)(x^2 + 3x - 1)$	[6x ² + 18x + 7]
$y = (x^2 - 1)(5x + 2)$	[15x ² + 4x - 5]
$y = (x^2 + 1)^5$	[10x(x ² + 1) ⁴]
$y = \frac{5x^3}{4} - \frac{7x^2}{2} - \frac{3x}{5} + 9$	$\left[\frac{15}{4}x^2 - 7x - \frac{3}{5}\right]$
$y = x(x - 1)^3$	[(4x-1)(x - 1) ²]
$y = (1 + x^2)(2x - 5)$	[6x ² - 10x + 2]
$y = (2x - 1)^2(3 - 7x)^5$	[(2x - 1)(3 - 7x) ⁴ (-98x + 47)]
$y = (2x + 3)(x^2 + 3x - 1)$	[6x ² + 18x + 7]
$y = (1 - 2x^2)(3x + 1)$	[-18x ² - 4x + 3]

$y = (3 - 2x - x^2)(x^4 - 2x^2)$	$[2x(-3x^4 - 5x^3 + 10x^2 + 6x - 6)]$
$y = x^2(4 + x)(5x + 1)$	$[x(20x^2 + 63x + 8)]$
$y = (8x - 1)^{10}$	$[80(8x - 1)^9]$
$y = (x - 1)^2(x - 2)$	$[(x - 1)(3x - 5)]$
$y = (5 + x^3)(1 - 2x - 4x^3)^2$	$[(1 - 2x - 4x^3)(-36x^5 - 10x^3 - 117x^2 - 20)]$
$y = (1 - 3x)^4(1 + x)$	$[(11 + 15x)(3x - 1)^3]$
$y = (2 - x)^2(x^3 + 2x)$	$[(2 - x)(-5x^3 + 6x^2 - 6x + 4)]$
$y = (x - 2)^3(x + 1)^2$	$[(x + 1)(x - 2)^2(5x - 1)]$
$y = (x^2 + x + 1)^3(x - 1)^4$	$[(x^2 + x + 1)^2(x - 1)^3(10x^2 + x + 1)]$
$y = (x^6 + 1)(3x + 1)^8$	$[6(3x + 1)^7(7x^6 + x^5 + 4)]$
$y = (x^2 + 2x - 3)^3(4 - x^2)^7$	$[2(12 + 33x - 17x^2 - 10x^3)(x^2 + 2x - 3)^2(4 - x^2)^6]$
$y = 2(x + 2)^2(x^2 + 4x - 3)$	$[4(x + 2)(2x^2 + 8x + 1)]$
$y = x^2(x^4 + 1)^3 + 3x(x^2 + 1)$	$[2x(x^4 + 1)^2(7x^4 + 1) + 3(3x^2 + 1)]$

Calcolare le derivate delle seguenti funzioni razionali fratte:

$y = \frac{5}{x+1}$	$\left[-\frac{5}{(x+1)^2} \right]$
$y = \frac{x+1}{2x}$	$\left[-\frac{1}{2x^2} \right]$
$y = \frac{x-3}{x-4}$	$\left[-\frac{1}{(x-4)^2} \right]$
$y = \frac{2x-3}{3x-4}$	$\left[\frac{1}{(3x-4)^2} \right]$
$y = \frac{x+1}{x-3}$	$\left[-\frac{4}{(x-3)^2} \right]$
$y = \frac{3x-4}{x^2-1}$	$\left[\frac{x(5-3x)}{(x^2-1)^2} \right]$

$y = \frac{1}{x^2 - 1}$	$\left[-\frac{2x}{(x^2 - 1)^2} \right]$
$y = \frac{4x^2 + 1}{x}$	$\left[\frac{(2x + 1)(2x - 1)}{x^2} \right]$
$y = \frac{x^2 - 3x - 1}{x + 1}$	$\left[\frac{x^2 + 2x - 2}{(x + 1)^2} \right]$
$y = \frac{(1 - x)^2}{x}$	$\left[\frac{(x - 1)(x + 1)}{x^2} \right]$
$y = \frac{3x^2 - 5}{x^2 - 1}$	$\left[\frac{4x}{(x^2 - 1)^2} \right]$
$y = \frac{4x^2 - 5x + 3}{x^2 - 6x + 5}$	$\left[-\frac{19x^2 - 34x + 7}{(x^2 - 6x + 5)^2} \right]$
$y = \frac{3x^2 - 2x + 3}{x^2 - 2x - 1}$	$\left[-\frac{4(x^2 + 3x - 2)}{(x^2 - 2x - 1)^2} \right]$
$y = \frac{1}{x}$	$\left[-\frac{1}{x^2} \right]$
$y = \frac{1}{2x^2} - \frac{3}{x} + x^3$	$\left[\frac{3x^5 + 3x - 1}{x^3} \right]$
$y = 3x^2 + 9x + \frac{1}{x}$	$\left[\frac{6x^3 + 9x^2 - 1}{x^2} \right]$
$y = \frac{x^2 + x^6 - 3x^3}{x^4}$	$\left[-\frac{2}{x^3} + 2x + \frac{3}{x^2} \right]$
$y = x^3 - 2x - \frac{5}{x} + \frac{2}{x^3}$	$\left[3x^2 - 2 + \frac{5}{x^2} - \frac{6}{x^4} \right]$
$y = \frac{2x}{3 - x}$	$\left[\frac{6}{(3 - x)^2} \right]$
$y = \frac{1 + x^2}{4 + x^2}$	$\left[\frac{6x}{(4 + x^2)^2} \right]$

$y = \frac{x^2 - 4}{x^2 + 4}$	$\left[\frac{16x}{(x^2 + 4)^2} \right]$
$y = \frac{x^3}{4 - x}$	$\left[\frac{2x^2(6 - x)}{(4 - x)^2} \right]$
$y = \frac{x^2 + 1}{5x - 7}$	$\left[\frac{5x^2 - 14x - 5}{(5x - 7)^2} \right]$
$y = \frac{8x + x^5}{x + 1}$	$\left[\frac{4x^5 + 5x^4 + 8}{(x + 1)^2} \right]$
$y = \frac{4x^2 - 5}{x + 1}$	$\left[\frac{4x^2 + 8x + 5}{(x + 1)^2} \right]$
$y = 2x - \frac{x}{x^2 + 1}$	$\left[\frac{2x^4 + 5x^2 + 1}{(x^2 + 1)^2} \right]$
$y = \left(2x + \frac{5}{x} \right)^3$	$\left[3 \left(2x + \frac{5}{x} \right)^2 \left(2 - \frac{5}{x^2} \right) \right]$
$y = \left(x - 1 - \frac{3}{x} \right)^4$	$\left[4 \left(x - 1 - \frac{3}{x} \right)^3 \left(1 + \frac{3}{x^2} \right) \right]$
$y = \frac{x^2 - 4}{x^2 + 4}$	$\left[\frac{16x}{(x^2 + 4)^2} \right]$
$y = \frac{x^2 + x + 2}{x^2 - 1}$	$\left[\frac{-x^2 - 6x - 1}{(x^2 - 1)^2} \right]$
$y = \frac{7}{(x^3 + 8)^2}$	$\left[-\frac{42x^2}{(x^3 + 8)^3} \right]$
$y = \frac{x}{x^3 + x^2 + 2}$	$\left[\frac{-2x^3 - x^2 + 2}{(x^3 + x^2 + 2)^2} \right]$

$y = \frac{x^3}{2x^2 - 3x + 1}$	$\left[\frac{x^2(2x^2 - 6x + 3)}{(2x^2 - 3x + 1)^2} \right]$
$y = x^2 + 2 + \frac{x^3}{4 - x}$	$\left[\frac{4x(8 - x)}{(4 - x)^2} \right]$
$y = \frac{x^3 + x + 1}{x - 1}$	$\left[\frac{2x^3 - 3x^2 - 2}{(x - 1)^2} \right]$
$y = \frac{3x - 1}{(4 - 5x)^2}$	$\left[\frac{15x + 2}{(4 - 5x)^3} \right]$
$y = \frac{4x^2 - 5}{x + 1}$	$\left[\frac{4x^2 + 8x + 5}{(x + 1)^2} \right]$
$y = 5(x^2 + 9) - \frac{7}{x}$	$\left[\frac{10x^3 + 7}{x^2} \right]$
$y = \left(2x + \frac{1}{x}\right)^3 (x^2 - 1)$	$\left[\left(\frac{2x^2 + 1}{x}\right)^2 \left(\frac{6x^4 - x^2 + 1}{x^2}\right) \right]$

Calcolare le derivate delle seguenti funzioni irrazionali:

$y = \sqrt{x}$	$\left[\frac{1}{2\sqrt{x}} \right]$
$y = \sqrt[3]{x^2 - 7x}$	$\left[\frac{2x - 7}{\sqrt[3]{(x^2 - 7x)^2}} \right]$
$y = \sqrt{x^2 + 2}$	$\left[\frac{x}{\sqrt{x^2 + 2}} \right]$
$y = \sqrt{4x^2 - 3}$	$\left[\frac{4x}{\sqrt{4x^2 - 3}} \right]$
$y = \sqrt{x^3 - 4x + 2}$	$\left[\frac{3x^2 - 4}{2\sqrt{x^3 - 4x + 2}} \right]$
$y = \sqrt{x^4 + x^2 - 2x}$	$\left[\frac{2x^3 + x - 1}{\sqrt{x^4 + x^2 - 2x}} \right]$

$$y = (x-1)\sqrt{x^2+1} \quad \left[\frac{2x^2-x+1}{\sqrt{x^2+1}} \right]$$

$$y = \frac{1}{\sqrt{1-x^2}} \quad \left[\frac{x}{\sqrt{(1-x^2)^3}} \right]$$

$$y = \sqrt[5]{5x+3} \quad \left[\frac{1}{\sqrt[5]{(5x+3)^4}} \right]$$

$$y = \sqrt[3]{4x^3+6x^2-5} \quad \left[\frac{4x(x+1)}{\sqrt[3]{(4x^3+6x^2-5)^2}} \right]$$

$$y = \frac{\sqrt{x}}{x+2} \quad \left[\frac{2-x}{2\sqrt{x}(x+2)^2} \right]$$

$$y = \sqrt[5]{(2x^3+1)^2} \quad \left[\frac{12x^2}{5\sqrt[5]{(2x^3+1)^3}} \right]$$

$$y = 8x + \sqrt{x} \quad \left[8 + \frac{1}{2\sqrt{x}} \right]$$

$$y = \sqrt{x^2+1} - \sqrt{x-1} \quad \left[\frac{x}{\sqrt{x^2+1}} - \frac{1}{2\sqrt{x-1}} \right]$$

$$y = 2\sqrt{x} + 2x + 1 \quad \left[\frac{1}{\sqrt{x}} + 2 \right]$$

$$y = (x^2 + 2\sqrt{x})^5 \quad \left[5(x^2 + 2\sqrt{x})^4 \left(2x + \frac{1}{\sqrt{x}} \right) \right]$$

$$y = 2\sqrt{x^3} - \frac{7}{\sqrt[2]{x}} + 4\sqrt[4]{x^3} \quad \left[3\sqrt{x} + \frac{1}{\sqrt[2]{x^8}} + \frac{3}{\sqrt[4]{x}} \right]$$

$$y = \sqrt[4]{x} - \frac{1}{\sqrt[4]{x}} \quad \left[\frac{\sqrt{x}+1}{4x\sqrt[4]{x}} \right]$$

$$y = x^7 - 3\sqrt[3]{x} + \frac{1}{\sqrt[4]{x^3}} \quad \left[7x^6 - \frac{1}{\sqrt[3]{x^2}} - \frac{3}{4x\sqrt[4]{x^3}} \right]$$

$$y = \frac{2x-1}{\sqrt{x^2+1}} \quad \left[\frac{x+2}{\sqrt{(x^2+1)^3}} \right]$$

$$y = \frac{1}{x+\sqrt{x^2-1}} \quad \left[1 - \frac{x}{\sqrt{x^2-1}} \right]$$

$$y = \frac{x-1}{\sqrt{x^2-1}} \quad \left[\frac{1}{(x+1)\sqrt{x^2-1}} \right]$$

$$y = \frac{1}{x} - \frac{4}{x^2} + \sqrt{x} - 5\sqrt[3]{x^2} \quad \left[-\frac{1}{x^2} + \frac{8}{x^3} + \frac{1}{2\sqrt{x}} - \frac{10}{3\sqrt[3]{x}} \right]$$

$$y = 8\sqrt{x} - 7\sqrt[3]{x^2} \quad \left[\frac{4}{\sqrt{x}} - \frac{14}{3} \cdot \frac{1}{\sqrt[3]{x}} \right]$$

$$y = 4\sqrt{x} + 16\sqrt[4]{x} \quad \left[\frac{2\sqrt{x} + 4\sqrt[4]{x}}{x} \right]$$

$$y = 25x^2 + \frac{1}{\sqrt{x}} - 7 \quad \left[50x - \frac{1}{2x\sqrt{x}} \right]$$

$$y = \sqrt{2}x^3 - 4\sqrt[5]{x} + 25 \quad \left[3\sqrt{2}x^2 - \frac{4}{5\sqrt[5]{x^4}} \right]$$

$$y = x\sqrt[3]{x^2-1} \quad \left[\frac{5x^2-3}{3\sqrt[3]{(x^2-1)^2}} \right]$$

$$x + \frac{2}{\sqrt{x}} + \frac{5}{\sqrt[5]{x}} \quad \left[\frac{x^2 - \sqrt{x} - \sqrt[5]{x^4}}{x^2} \right]$$

$$y = \frac{x^4+4x}{\sqrt{x}} \quad \left[\frac{7}{2}x^2\sqrt{x} + \frac{2}{\sqrt{x}} \right]$$

$$y = 2x\sqrt{x} - \frac{1}{4}\sqrt[3]{x\sqrt{x}} + 2x^6 - 3 \quad \left[3\sqrt{x} - \frac{1}{8\sqrt{x}} + 12x^5 \right]$$

$$y = \frac{\sqrt[3]{x} \cdot \sqrt[5]{x^2}}{x\sqrt{x}} \quad \left[-\frac{23}{30x\sqrt[30]{x^{23}}} \right]$$

$$y = \frac{1+\sqrt[4]{x^3}}{1-\sqrt[4]{x^3}} \quad \left[\frac{3}{2\sqrt[4]{x}(1-\sqrt[4]{x^3})^2} \right]$$

$$y = \frac{2 - \sqrt[3]{x^2}}{2 + \sqrt[3]{x^2}} \quad \left[-\frac{8}{3\sqrt[3]{x} \left(2 + \sqrt[3]{x^2}\right)^2} \right]$$

$$y = \frac{x^3 + \sqrt{x}}{x^2 + 3} \quad \left[x^4 + 9x^2 - \frac{3}{2}x\sqrt{x} + \frac{3}{2\sqrt{x}} \right]$$

$$y = (15x^2 - 12x + 8)\sqrt{(x+1)^3} \quad \left[\frac{105}{2}x^2\sqrt{x+1} \right]$$

$$y = \left(\sqrt[3]{x} - \frac{1}{2}x^2 + \sqrt[5]{x^3} \right)^2 \quad \left[2 \left(\sqrt[3]{x} - \frac{1}{2}x^2 + \sqrt[5]{x^3} \right) \left(\frac{1}{3\sqrt[3]{x^2}} - x + \frac{3}{5\sqrt[5]{x^2}} \right) \right]$$

$$y = \frac{x - \sqrt{x^2 + 4}}{7x + 1} \quad \left[\frac{\sqrt{x^2 + 4} - x + 28}{(7x + 1)^2 \sqrt{x^2 + 4}} \right]$$

$$y = \frac{\sqrt{2 - x^2} + x}{3 + x} \quad \left[\frac{-3x - 2 + 3\sqrt{2 - x^2}}{(3 + x)^2 \sqrt{2 - x^2}} \right]$$

$$y = \frac{3 - x^2}{\sqrt[3]{x + 2}} \quad \left[\frac{-5x^2 - 12x - 3}{3\sqrt[3]{(x + 2)^4}} \right]$$

$$y = \frac{x^5 - x^3}{\sqrt{x + 1}} \quad \left[\frac{x^2(9x^2 - x - 6)}{2\sqrt{x + 1}} \right]$$

$$y = \frac{x}{\sqrt{x + 1}} \quad \left[\frac{x + 2}{2(x + 1)\sqrt{x + 1}} \right]$$

$$y = x \cdot \sqrt{\frac{1 - x}{1 + x}} \quad \left[-\frac{2x}{(1 + x)^2 \sqrt{\frac{1 - x}{1 + x}}} \right]$$

$$y = \frac{\sqrt{x^2 - 1}}{x} \left(\frac{1}{x^2} + 2 \right) \quad \left[\frac{-2x^4 + x^2 + 4}{x^4 \sqrt{x^2 - 1}} \right]$$

Calcolare le derivate delle seguenti funzioni esponenziali e logaritmiche:

$$y = \ln(2x - 1) \quad \left[\frac{2}{2x - 1} \right]$$

$$y = \ln(x + 3) \quad \left[\frac{1}{x + 3} \right]$$

$y = e^{x+1}$	$[e^{x+1}]$
$y = xe^x$	$[(1+x)e^x]$
$y = e^{5-x^2}$	$[-2xe^{5-x^2}]$
$y = \frac{x^4+1}{e^4+1}$	$\left[\frac{4x^3}{e^4+1}\right]$
$y = x \ln x$	$[\ln x + 1]$
$y = x^2 \ln x + 3x$	$[2x \ln x + x + 3]$
$y = e^{-\frac{3}{x^2}}$	$\left[\frac{2}{x^3} e^{-\frac{3}{x^2}}\right]$
$y = e^{\sqrt{x}}$	$\left[\frac{1}{2\sqrt{x}} e^{\sqrt{x}}\right]$
$y = \frac{\ln x - 1}{\ln x + 1}$	$\left[\frac{2}{x(\ln x + 1)^2}\right]$
$y = x^3 e^x + e^x - 1$	$[e^x(x^3 + 3x^2 + 1)]$
$y = e^x(2 - e^x)$	$[2e^x(1 - e^x)]$
$y = e^x(x^3 - x + 7)$	$[e^x(x^3 + 3x^2 - x + 6)]$
$y = x \ln^3 x - 3x \ln^2 x + 6x \ln x - 6x$	$[\ln^3 x]$
$y = x^2(\ln x)^3$	$[x(\ln x)^2(2 \ln x + 3)]$
$y = 5x \ln^2 x - 6x^3 \ln^5 x$	$[-18x^2 \ln^5 x - 30x^2 \ln^4 x + 5 \ln^2 x + 10 \ln x]$
$y = \frac{1}{x} \cdot \ln x$	$\left[\frac{1}{x^2}(1 - \ln x)\right]$
$y = (x \ln x - 1)^2$	$[2(x \ln x - 1)(\ln x + 1)]$
$y = 3x \ln x$	$[3(\ln x + 1)]$
$y = x^2 \ln x$	$[x(2 \ln x + 1)]$
$y = x \ln^3 x$	$[\ln^2 x(\ln x + 3)]$
$y = \frac{1}{2} \ln^2 x - \ln x + \sqrt{x}$	$\left[\frac{2 \ln x - 2 + \sqrt{x}}{2x}\right]$
$y = \ln(x + \sqrt{4 + x^2})$	$\left[\frac{1}{\sqrt{4 + x^2}}\right]$

$$y = \ln\left(\frac{x}{x-1}\right) - \frac{2}{x} - \frac{1}{x^2} \quad \left[\frac{x^2 - 2}{x^3(x-1)} \right]$$

$$y = \ln^2 x + 3x + 5 \quad \left[\frac{2\ln x + 3x}{x} \right]$$

$$y = 4x \ln^4 x + 7x^5 \ln^5 x \quad \left[\ln^3 x (35x^4 \ln^2 x + 35x^4 \ln x + 4\ln x + 16) \right]$$

$$y = \frac{\ln x}{x^2} \quad \left[\frac{1}{x^3} (1 - 2\ln x) \right]$$

$$y = x\sqrt{1+x^2} + \ln(x + \sqrt{1+x^2}) \quad \left[2\sqrt{1+x^2} \right]$$

$$y = \ln(x^2 - 7x - 8) \quad \left[\frac{2x-7}{x^2-7x-8} \right]$$

$$y = (x-1)\ln^3 x \quad \left[\frac{x \ln^3 x + 3(x-1)\ln^2 x}{x} \right]$$

$$y = e^{x^2-2x} \quad \left[2(x-1)e^{x^2-2x} \right]$$

$$y = x e^{\frac{x-1}{x}} \quad \left[e^{\frac{x-1}{x}} \cdot \frac{x+1}{x} \right]$$

$$y = \ln \sqrt[3]{x^2+4} \quad \left[\frac{2x}{5(x^2+4)} \right]$$

$$y = \ln\left(\frac{x}{x+1}\right) + \frac{1}{x} - \frac{1}{2x^2} \quad \left[\frac{1}{x^3(x+1)} \right]$$

$$y = \ln\left(\frac{x}{x-1}\right) \quad \left[-\frac{1}{x(x-1)} \right]$$

$$y = \ln(x^3 + x^2 + 8) \quad \left[\frac{3x^2 + 2x}{x^3 + x^2 + 8} \right]$$

$$y = \ln \sqrt[3]{4x^2 + 5x - 1} \quad \left[\frac{8x+5}{3(4x^2+5x-1)} \right]$$

$$y = \ln \sqrt{\frac{x^2+1}{2x+3}} \quad \left[\frac{x^2+3x-1}{(2x+3)(x^2+1)} \right]$$

$$y = \sqrt{\ln(x^2 + 4)} \quad \left[\frac{x}{(x^2 + 4)\sqrt{\ln(x^2 + 4)}} \right]$$

$$y = \sqrt[3]{\ln x + \sqrt{x^2 + 1}} \quad \left[\frac{1}{3\sqrt[3]{\ln x + \sqrt{x^2 + 1}}} \left(\frac{1}{x} + \frac{1}{\sqrt{x^2 + 1}} \right) \right]$$

$$y = \frac{1}{x^2} \ln^3(2x + \sqrt[5]{x}) \quad \left[-\frac{2}{x^3} \ln^3(2x + \sqrt[5]{x}) + \frac{6}{x^2} \frac{\ln^2(2x + \sqrt[5]{x})}{2x + \sqrt[5]{x}} + \frac{1}{5\sqrt[5]{x^4}} \right]$$

$$y = \ln \left(\frac{\sqrt{e^x}}{1 + \sqrt{1 + e^x}} \right) \quad \left[\frac{e^{\sqrt{x}} (2\sqrt{1 + e^x} + e^x + 2)}{4\sqrt{1 + e^x}} \right]$$

$$y = \frac{\ln x - 1}{\ln x + 1} \quad \left[\frac{2}{x(\ln x + 1)^2} \right]$$

$$y = \frac{e^x + 1}{2 - e^x} \quad \left[\frac{3e^x}{(2 - e^x)^2} \right]$$

$$y = \frac{e^x}{x^3 + x^2} \quad \left[\frac{e^x (x^3 - 2x^2 - 2x)}{(x^3 + x^2)^2} \right]$$

$$y = \frac{\ln x}{1 + \ln x} \quad \left[\frac{1}{x(1 + \ln x)} \right]$$

$$y = \ln(\ln x) \quad \left[\frac{1}{x \ln x} \right]$$

$$y = \ln(e^x - 2) \quad \left[\frac{e^x}{e^x - 2} \right]$$

$$y = \ln(5e^x \sqrt{x^2 - 1}) \quad \left[\frac{x^2 + x - 1}{x^2 - 1} \right]$$

$$y = \frac{1}{2} \ln(x^2 - 1) + x \quad \left[\frac{x^2 + x - 1}{x^2 - 1} \right]$$