Choice and Efficiency

1. The decision maker

We focus on how individuals make decisions. In doing so, we will set a definition of rationality. Our main character is the individual decision maker. The decision maker does nothing but choosing an element from a given set.

Our assumptions:

1. the decision maker has well defined preferences over a choice set;
2. the decision maker is rational.

Thanks to these assumptions, we will prove that the decision maker's preferences can be represented by a utility function and that the decision maker chooses her preferred element in the choice set as if she were maximizing her utility function in the choice set.

2. The choice set

The choice set is represented by X: an arbitrary set of objects such as a list of consumption bundles. Thus we could have:

$$X = \{nuts, berries, shoes, shirt\}$$

or, more abstractly:

$$X = \{a, x, y, w, z\}$$

3. Preferences

Preferences are represented by a preference relation, denoted by $\succeq$ (please, do note that $\succeq$ is not $\geq$). This preference relation is what the decision maker uses to compare any pair of alternatives $x, y \in X$.

The expression

$$x \succeq y$$

is read as "x is at least as good as y" or "x is weakly preferred to y" for our decision maker. "Weakly" means that either the decision maker likes x better than y or else she is indifferent between the two.

Two further relations are then derived: the strict preference relation and the indifference relation.

The strict preference relation $\succ$ is so defined:

$$x \succ y \quad \text{iff} \quad x \succeq y \quad \text{but not} \quad y \succeq x.$$

On the other hand, the indifference relation is defined as:

$$x \sim y \quad \text{iff} \quad x \succeq y \quad \text{and} \quad y \succeq x.$$

4. Rationality

We identify "rationality" with two assumptions on the preference relation: completeness and transitivity.

Definition 1 *We will say that $\succeq$ is complete iff $\forall x, \forall y \in X$ you either have $x \succeq y$ or $y \succeq x$ or both.*

So, completeness means that any and every two elements of the choice set can be compared.

Definition 2 *We will say that $\succeq$ is transitive iff $\forall x, \forall y, \forall z \in X$, if $x \succeq y$ and $y \succeq z$ then $x \succeq z$.*

5. Representing utility

The very fact that $\succeq$ is complete and transitive allows us to represent $\succeq$ by a utility function.

Definition 3 *A function $u : X \to \mathbb{R}$ is a utility function representing $\succeq$ if $\forall x, \forall y \in X$ we have:*

$$x \succeq y \Leftrightarrow u(x) \geq u(y).$$

The following representation theorem is a cornerstone for economic analysis. We will limit ourselves to the statement (proving it is not that hard, though! Give it a try!)

Theorem 1 *A preference relation $\succeq$ can be represented by a utility function iff it is complete and transitive.*

6. Choice

A rational agent chooses then the element x of the choice set X to which is associated the highest utility $u(x)$.

Formally, a rational agent choice problem is represented in these terms:

$$max_{x \in X} \;\; u(x).$$

The very core of the matter is the decision maker being forced to face some constraints (we are talking "economics" after all!!). Under this perspective, the consumer problem is the classic example.

Le m be a fixed amount of money available for the consumer. We then let $p = (p_1, \ldots, p_k)$ be a vector of prices for $1, \ldots, k$ goods. Let then the set of affordable objects for the consumer be:

$$B = \{x \in X : p_1 x_1 + \ldots + p_k x_k \leq m\}$$

B is called the *budget set* and $p_1 x_1 + \ldots + p_k x_k \leq m$ is called the *budget constraint.*

In vector notation, we can write $px \leq m$ and have the consumer problem written as:

$$\textit{choose } x \textit{ to maximize } u(x) \textit{ subject to } x \in B$$

or equivalently:

$$\textit{choose } x \textit{ to maximize } u(x) \textit{ subject to } x \in X \textit{ and } px \leq m$$

or as we usually do as:

$$max_{x \in B} \ u(x)$$

7. The budget constraint

Let us consider an economy with two goods: $X = \mathbb{R}^2_+$. We shall call (x_1, x_2) a consumption bundle of x_1 units of good 1 and x_2 units of good 2. Any such bundle is thus a point on a two-dimensional graph.

Prices of the goods p_1 and $_2$ are known and given, the consumer is a price taker. Let m be the consumer's income. The consumer's budget constraint is thus: $p_1x_1 + p_2x_2 \leq m$.

The *budget set* is the set of all bundles that are affordable at a given price and income:

$$B = \{x \in X : p_1x_1 + p_2x_2 \leq m\}.$$

The budget line is the set of bundles such that:

$$p_1x_1 + p_2x_2 = m$$

This can also be written as:

$$x_2 = \frac{p_1}{p_2}x_1 + \frac{m}{p_2}.$$

This is a linear function with vertical intercept m/p_2, horizontal intercept m/p_1 and a slope of $-p_1/p_2$.

The slope of the budget line measures the rate at which the market is willing to exchange good 1 for good 2. So, the slope measures the opportunity cost of consuming good 1: the consumer has an alternative opportunity to consuming 1 unit of good 1, consisting in selling the unit of good 1, for an amount equal to p_1, and then using the proceeds to buy good 2 – that is $\frac{p_1}{p_2}$ units of good 2.

We also know that when prices or income change, then the budget line moves. We won't expand on this.

8. Indifference curves

A useful graphical description of preferences exploits a construction known as indifference curves.

Indifference curves are level curves, that is, they represent the set of $(x_1; x_2)$ for which the utility is constant at some level, say U as in the following figure:

We make two general assumptions on preference relations. The first is *monotonicity* (i.e. "more is better").

Definition 4 *We typically assume that more is better when consider goods, not bads. More precisely, consider (x_1, x_2) as a bundle of goods and let (x'_1, x'_2) be any other bundle with at least as much of both goods and more of one. That is, $x'_1 \geq x_1$ and $x'_2 \geq x_2$ with at least one strict inequality. (Strict) monotonicity of preferences requires that:*

$$\text{if } (x'_1, x'_2) > (x_1, x_2) \text{ then } (x'_1, x'_2) \succ (x_1, x_2)$$

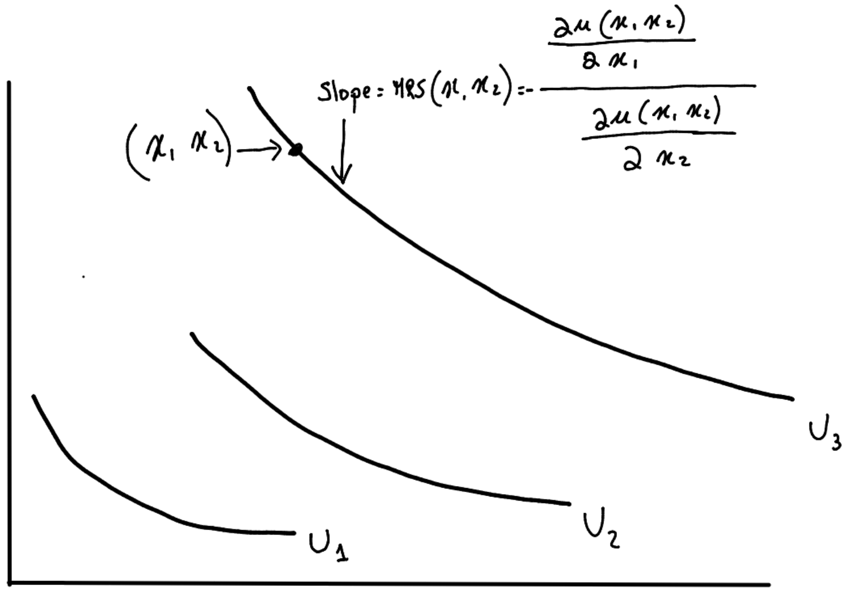

Figure 1: Indifference curves and the MRS

Thus, (strict) monotonic preferences imply that more of (resp. less) of both goods is a better (resp. worse) bundle. Monotonicity of preferences implies that the utility function which represents preferences is monotonic increasing in both its arguments:

$$\frac{\partial u(x_1,x_2)}{\partial x_1}, \frac{\partial u(x_1,x_2)}{\partial x_2} > 0; \text{for any } (x_1,x_2) \in X$$

Strict monotonicity also implies that indifference curves have a negative slope:

$$-MRS(x_1,x_2) < 0$$

Of course, it goes without saying that higher level indifference curves correspond to higher utility levels.

Our second assumption will be convexity. We know that a set is convex if with $x, y \in X$ and $\alpha \in [0,1]$ then $\alpha x + (1-\alpha)y \in X$.

That is a set is convex if it contains the entire segment joining any two of its elements. Convexity of preferences requires that the set of weakly preferred allocations to any given

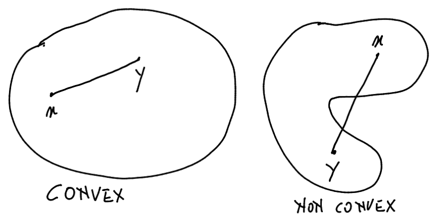

Figure 2: Convex and non convex sets

allocation is convex. Formally, for any $(x_1,x_2) \in X$; the set $\{(y_1,y_2) \in X : (y_1,y_2) \succeq (x_1,x_2)\}$ is convex. At the very same time, taking two arbitrary bundles on the same indifference curve, (x_1,x_2) and (x_1',x_2') by convexity the bundle $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + (1-\alpha)\begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$ is preferred to both (x_1,x_2) and (x_1',x_2'). Convexity of preferences implies that the utility function representing preferences is concave and that indifference curves are convex.

9. The consumer problem

We can now restrict ourselves to Cobb-Douglas preferences and solve the consumer problem:

$$\max_{x \in X} u((x_1, x_2)) \text{ s.t. } p_1 x_1 + p_2 x_2 = m$$

under the assumptions that the preference relation represented by $u(x_1, x_2)$ are monotonic and convex.

Painted with an extremely broad brush: a bundle (x_1^*, x_2^*) is an optimal choice for the consumer if the set of bundles that the consumer prefers to (x_1^*, x_2^*) (i.e. the set of bundles above the indifference curve through (x_1^*, x_2^*)) has an empty intersection with the bundles she can afford (i.e. the bundles beneath her budget line). It then follows that at (x_1^*, x_2^*) the indifference curve is tangent to the budget line:

$$MRS(x_1^*, x_2^*) = \frac{\alpha x_2^*}{\beta x_1^*} = -\frac{p_1}{p_2}$$

and at (x_1^*, x_2^*) the budget constraint must be satisfied:

$$p_1 x_1^* + p_2 x_2^* = m$$

Now that we have these two equations in two unknowns that can be solved for the optimal bundle, we can derive the demand function for good 1:

$$x_1^* = \frac{\alpha}{\alpha + \beta} \frac{m}{p_1}$$

and for good 2:

$$x_2^* = \frac{\alpha}{\alpha + \beta} \frac{m}{p_2}$$

10. Equilibrium and efficiency

We investigate the fundamental economic problem of allocation and price determination in a very simple economy. Our aim is to describe what outcomes might arise by giving individuals the opportunity to voluntarily exchange goods. Thus, we will follow two simple principles: (i) Rationality - individuals choose the best patterns of consumption that are affordable for them, and (ii) Equilibrium - prices adjust such that the amount that people demand of some good is equal to the amount that is supplied. We will determine equilibrium prices, equating demand and supply. We will show that these prices solve the allocation problem efficiently.

We shall consider a pure exchange economy: that is, an economy with no production, in which consumers have fixed endowments of goods. In our economy, markets are competitive: that is, consumers take prices as given, independently of their trading choices. We say therefore that consumers are price takers. Furthermore, we assume consumers are rational: that is, they choose the consumption bundle which maximizes their utility (and, as you know, they have a utility which represents their preferences to maximize if and only if they are rational). These assumptions are fundamental in our analysis. We assume that the economy is a 2x2 economy: that is, it is composed of two consumers who consume two types of goods.

Let $\{1, 2\}$ denote the set of consumption goods. Let instead $\{A, B\}$ denote the set of consumers.

Consumer A and consumer B initial endowments are denoted, respectively, $w^A = (w_1^A, w_2^A)$ and $w^B = (w_1^B, w_2^B)$.

Similarly, $x^A = (x_1^A, x_2^A)$ and $x^B = (x_1^B, x_2^B)$ will denote consumer A's and consumer B's consumption bundles.

Consumer As and consumer B's preferences are represented, respectively, by complete and transitive preference orderings $\succ_A$ and $\succ_B$ defined on $\mathbb{R}_+^2$ or, which is equivalent, by utility functions $u^A(x_1, x_2)$ and $u^B(x_1, x_2)$.

Definition 5 *An allocation is a pair of bundles x^A and x^B. An allocation is feasible if:*

$$x_1^A + x_1^B = w_1^A + w_1^B$$

and

$$x_2^A + x_2^B = w_2^A + w_2^B$$

We now need a notion of efficiency in order to pairwise compare different allocations.

Definition 6 *A feasible allocation $x^A + x^B$ is Pareto efficient if there is no other feasible allocation $y^A + y^B$ such that $y^A \succeq x^A$ and $y^B \succeq x^B$ with at least one $\succ$.*

That is, an allocation is Pareto efficient if it is feasible and there is no other feasible allocation for which one consumer is at least as well off and the other consumer is strictly better off. This implies that at a Pareto efficient allocation (i) there is no way to make both consumers strictly better or, (ii) all of the gains from trade have been exhausted, that is, there are no mutually advantageous trades to be made.

This can geometrically be easily seen in a graphical representation of the Pareto Frontier.

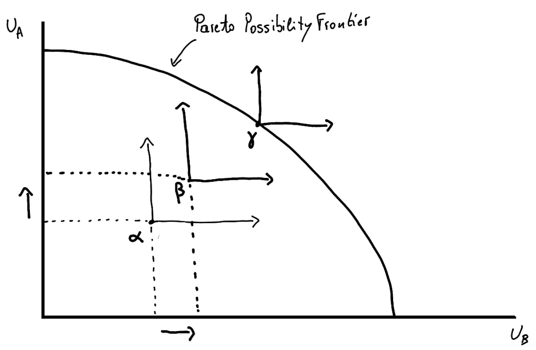

Figure 3: The Pareto Possibility Frontier

11. The Edgeworth Box

The Edgeworth Box is a simple way to illustrate some key aspects of a pure exchange economy.

Let us consider a $2x2$ economy, let the decision makers be $\{Adamo, Eva\}$ and finally let $\{Beer, Wine\}$ be the set of goods.

We first represent a cartesian space for Adam (henceforth A), let its origin be O_A.

We then represent Eve's space in the very same way.

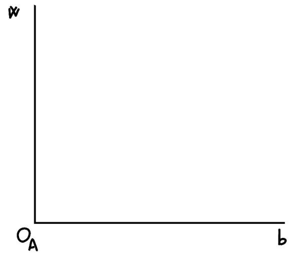

Figure 4: Adam's coordinates

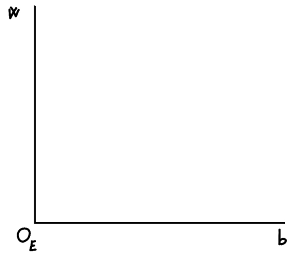

Figure 5: Eve's coordinates

Finally, we flip Eve's coordinates so that their origin lies orthogonally w.r.t. Adam's and we get:

Note, that every point in the box so obtained – e.g. point x – corresponds to an allocation of both goods to both agents. Also try and move the point within the box: notice that each movement corresponds to some trade between the two agents (Fig.6).

We can then have some indifference curves drawn for the two (note: arrows indicate directions they have to follow in order to improve their welfare i.e. climbing to higher level indifference curves). (Fig.7)

Let us now consider the following graph (Fig.8)

A_G is Adam's indifference curve that intersects Eve's indifference curve E_G in point g. Question: can we imagine some trade corresponding to A improving his situation and to E not worsening hers? We can see from the figure that the answer is a bold yes e.g. in point h. As a matter of fact, i) A is better off as he moves from his curve A_G to his curve A_H which confers him a higher level of utility. Simultaneously, in h Eve is not worse off as she remains on the same indifference curve E_G.

- Question: can push this process any ffurther (i.e making A better off while not worsening E)?
- Answer: we can do that as long as A climbs to higher level indifference curves while, at the same time, intersecting E_G.
- Note: this will be possible until one reaches point p. From this point on making A better off amounts to worsening E's position. Point p is a Pareto efficient allocation. Confront it with our previous definition of Pareto efficiency: no one is worse off + at least one is better off. That is for every agent $\succeq$ holds and $\succ$ holds for at least one.

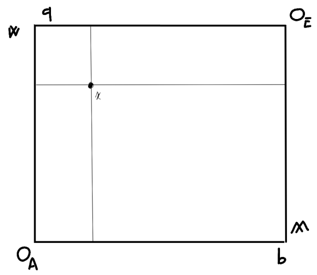

Figure 6: The Edgeworth box.

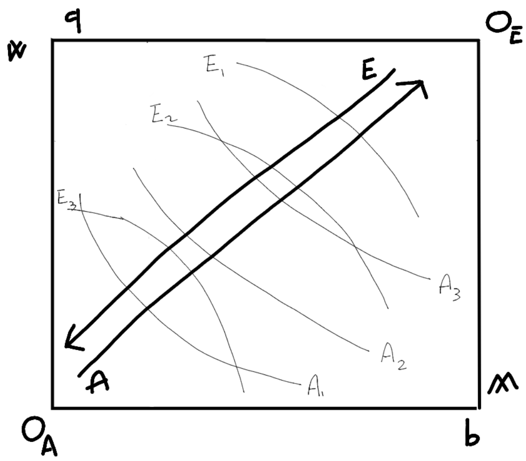

Figure 7: Indifference curves in the Edgeworth box.

Another question: can we imagine a simultaneous improvement for both agents at the same time? Have a look at Fig. 9.

- In p_2, A is at a higher level of utility than he is in g as he now is on curve A_{p_2} rather than in A_G
- same holds for E as E_{p_2} is higher than E_g.
- note that in this case, $\succ$ holds for A as it holds for E. That is: $p_2 \succ_A g$ and $p_2 \succ_E g$.
- note that in p_2 it holds true that A_{p_2} and E_{p_2} are tangent and thus have the same slope. This in turn means that A's marginal rate of substitution and E's marginal rate of substitution are equal. We can thus conclude that Pareto efficiency requires MRSs being equal and a that a tangency condition be met.

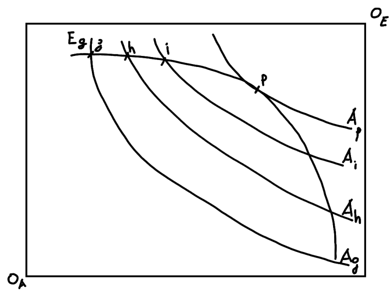

Figure 8: Improving Adam

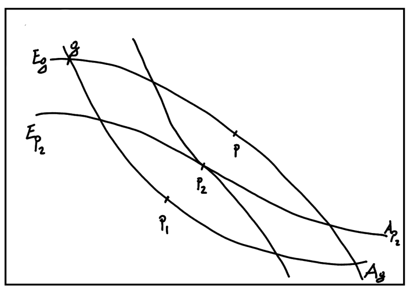

Figure 9: Improving both agents