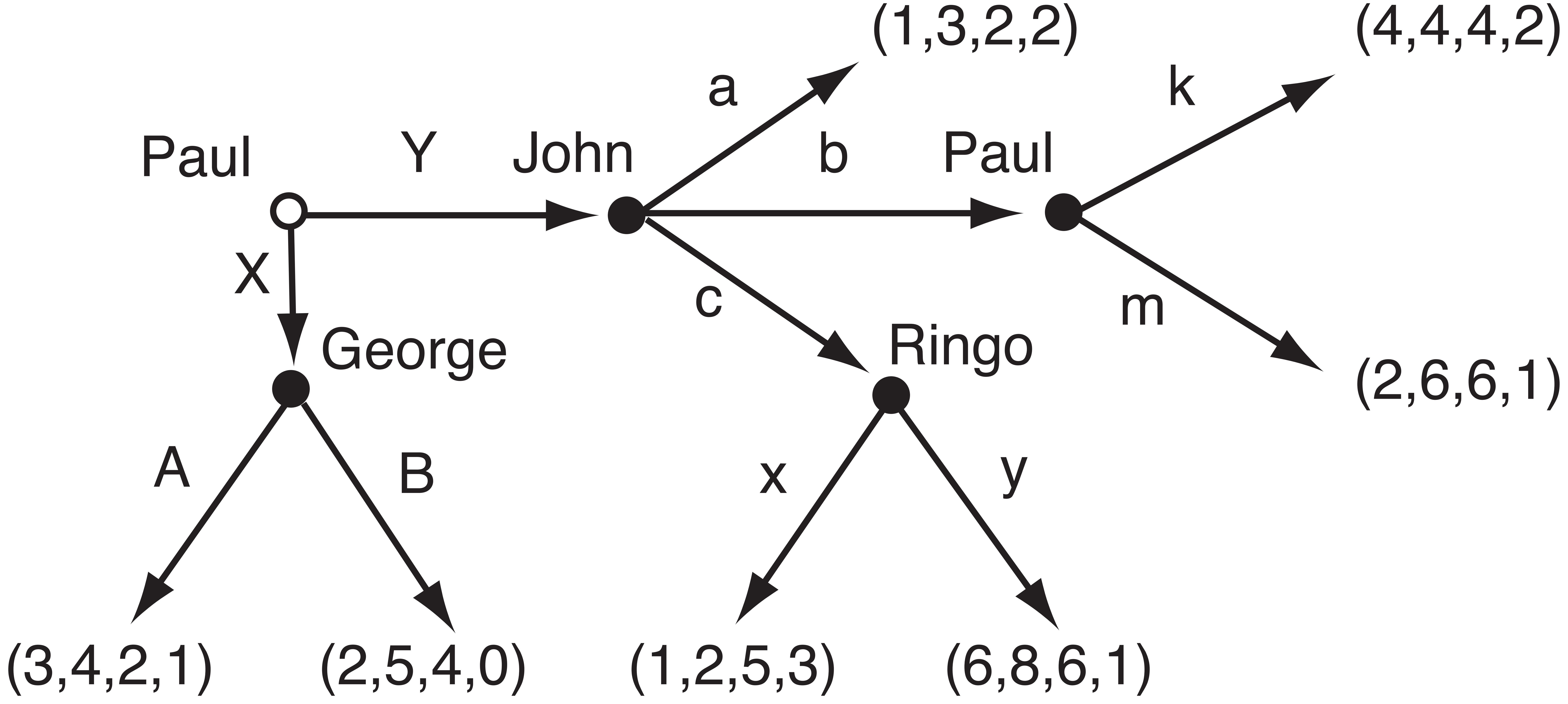
The Economics of strategic relationships

Part three

2022 — 2023

Backward Induction in Extensive Form Games



(1,3,2,2)
(4,4,4,2)
a
k
Paul
Y
John
b
Paul
X
c
m
George
Ringo
(2,6,6,1)
A
B
x
y
(3,4,2,1)
(2,5,4,0)
(1,2,5,3)
(6,8,6,1)

Backward induction

- This is a four-player extensive-form game in which there are no information sets and no moves by nature.
- The lack of information sets is particularly relevant: This means that whenever a player is called upon to move, he or she knows precisely what happened in earlier moves and (so) precisely where in the game tree things stand.
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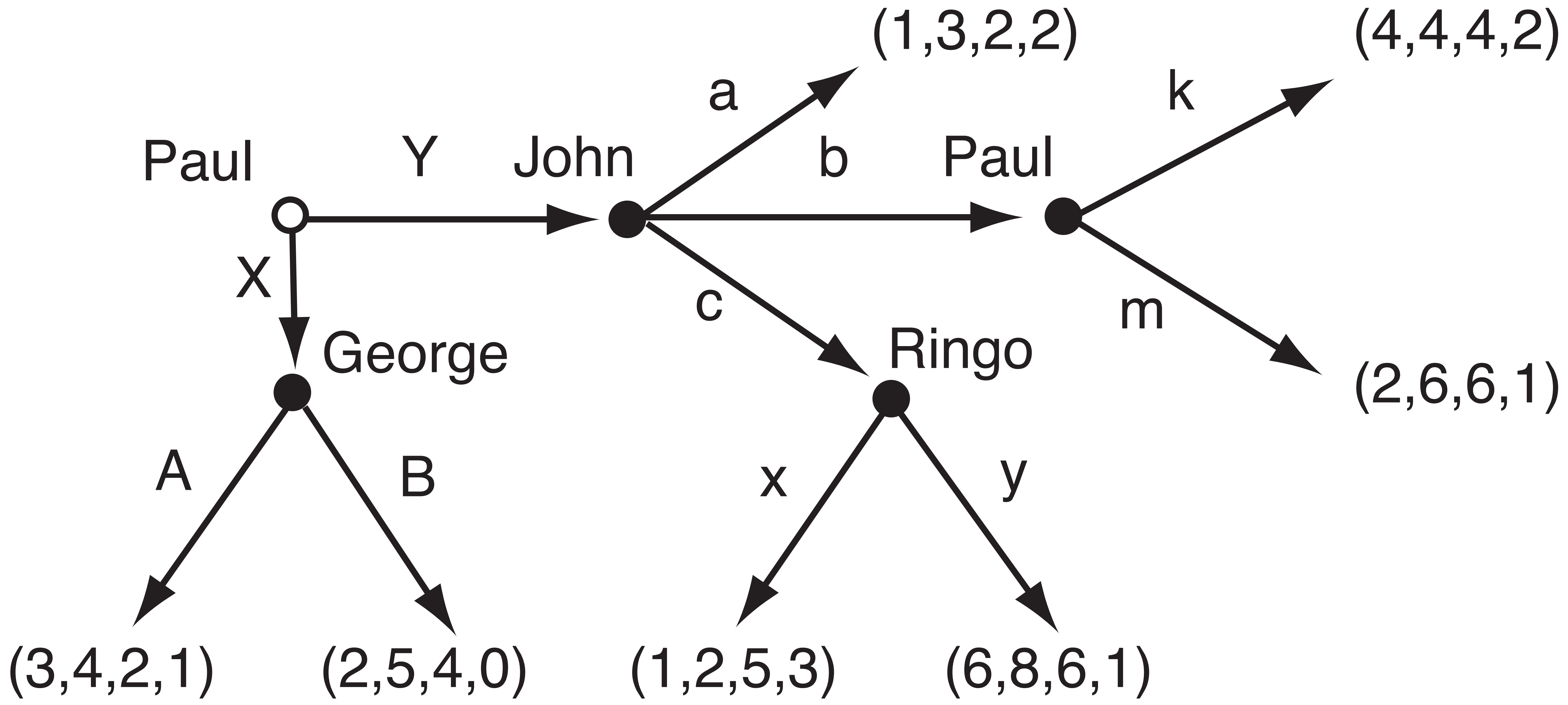
Backward Induction

Suppose that Paul begins by choosing *Y* and John follows this with a choice of *b*.

It is Paul's turn to move again:

- If Paul chooses *k*, Paul will get a payoff of 4,
- While choosing *m* will give Paul a payoff of 2.

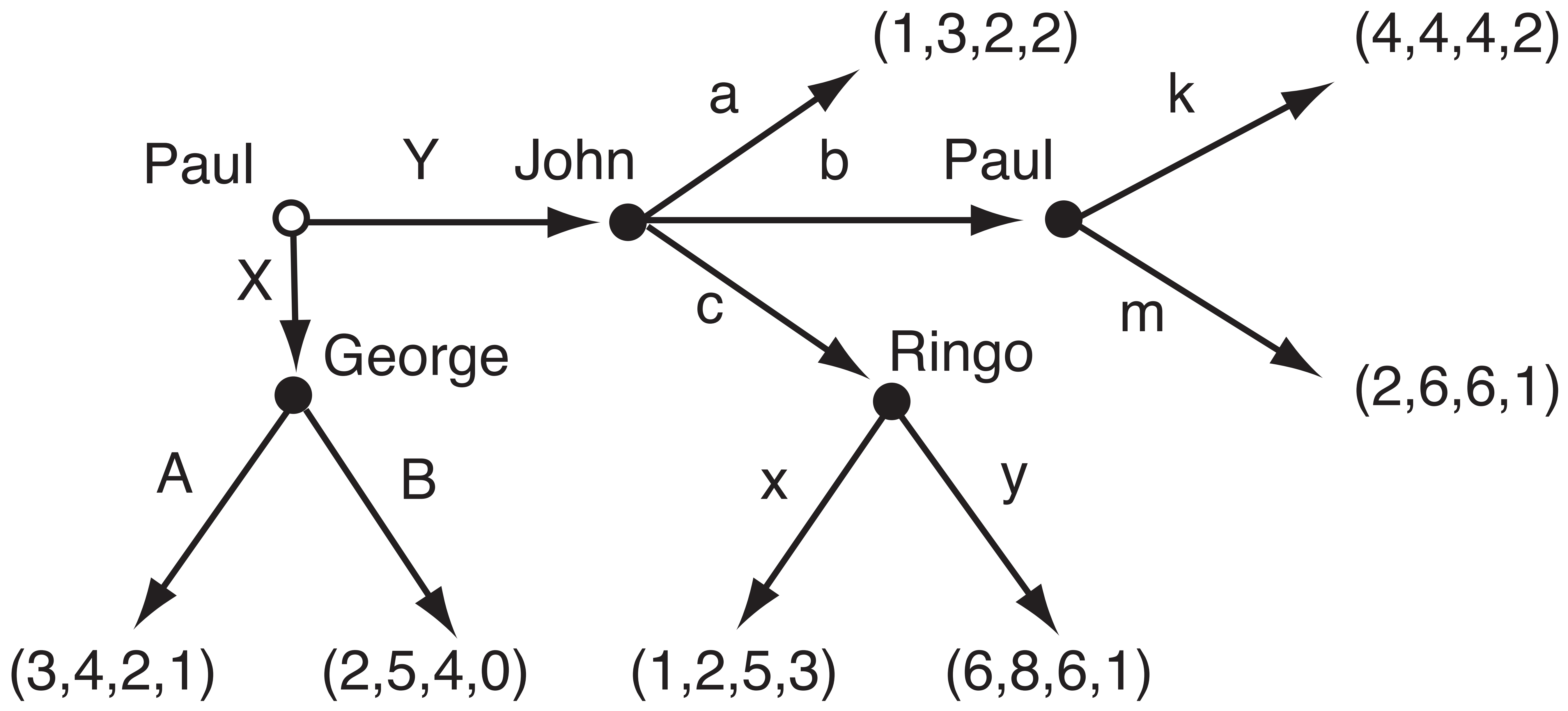


Backward Induction

It makes sense to suppose that Paul, put in this position, will choose k.

Similarly, if Paul chooses Y and John chooses c, Ringo has a choice of x, for a payoff to him (Ringo) of 3, or y, which brings him 1.

So, the analysis goes, if Paul chooses Y, John reasons: “If I choose a,I will get 3. If I choose b, Paul will choose k and I’ll get 4. If I choose c, Ringo will choose x as the better option for him, which gives me 2. So my best option is b.”



Backward induction

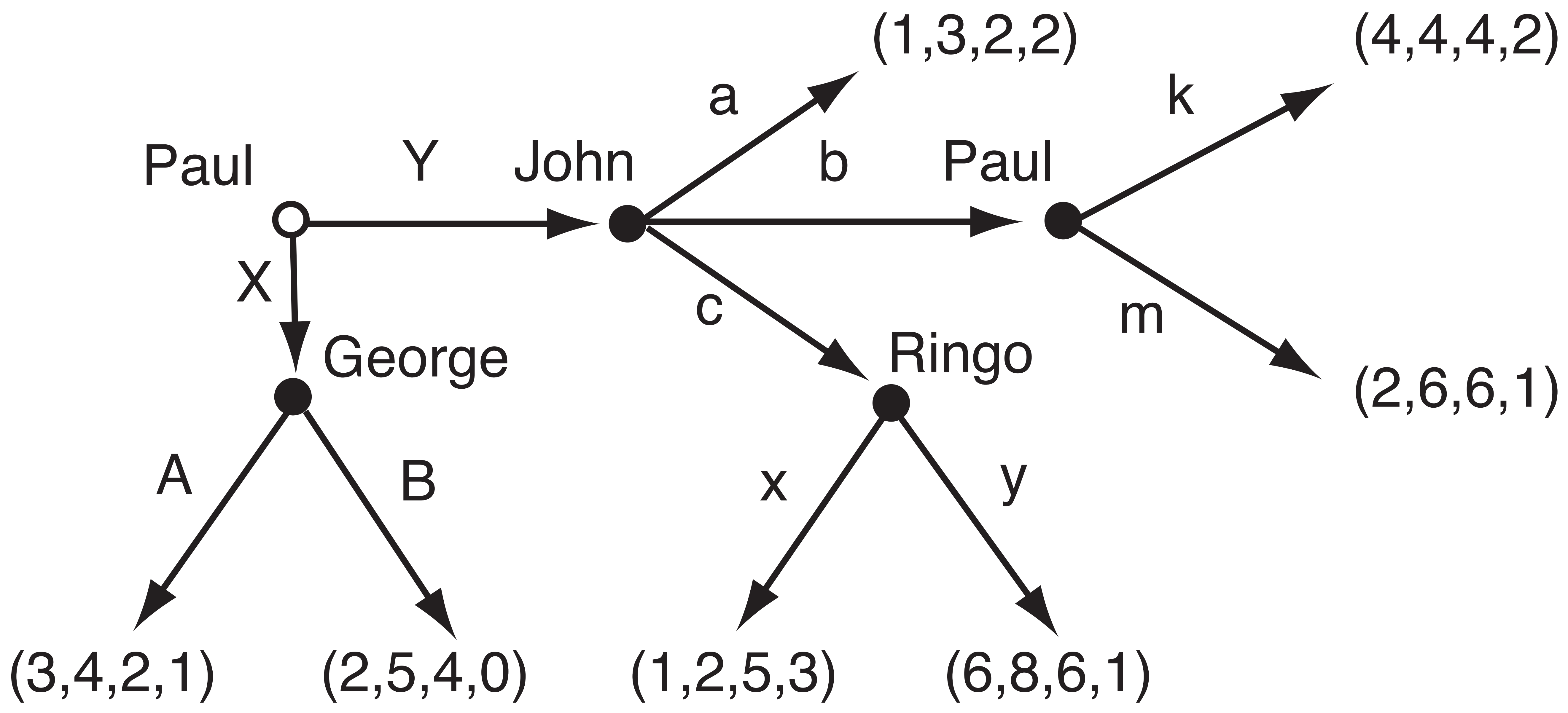
And now Paul reasons: "If I choose Y, John will reason as above and choose b, and I will then choose k, and so I'll get 4.

My other option is X.

With this choice, George is given the move, and I anticipate that he'll choose B (4 for George), since A gives George only 2.

So, if I choose Y, I anticipate getting 4.

If I choose X, I anticipate getting 2. My best choice is Y."



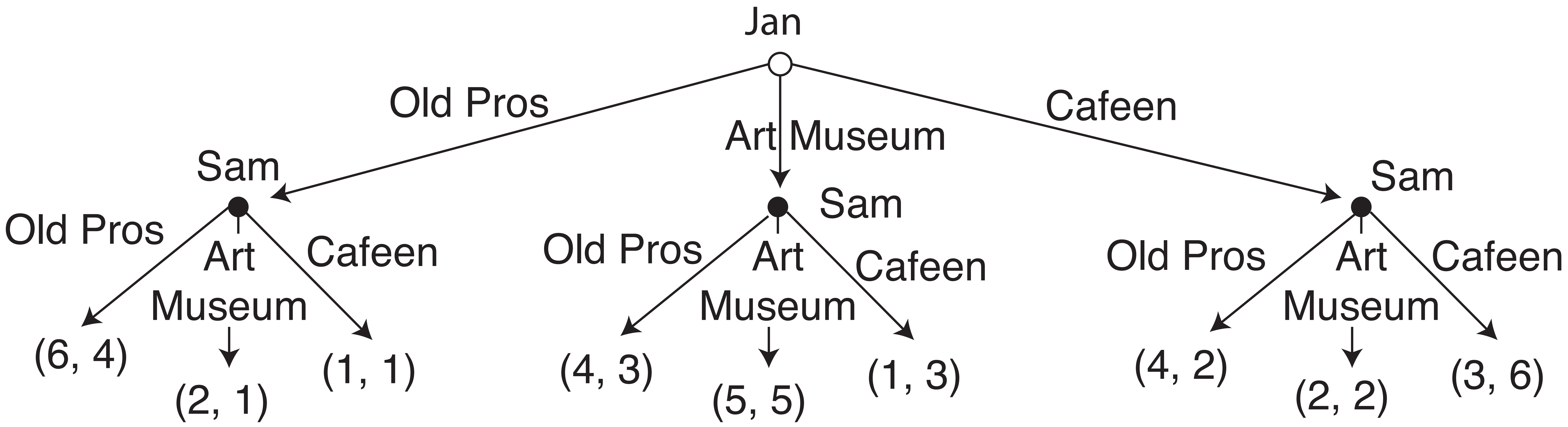
Backward Induction

So, if we think that the players see the game as we do and reason in the manner just suggested, we predict Paul will choose Y, John will choose b, and Paul will choose k.

This is *backward induction* applied to this simple game.

Backward Induction

Try Backward Induction for the Sam and Jan game as an exercise:



Backward Induction

- The key to applying backward induction is that there are no information sets
- What do you do if you reach a node where the player moving is indifferent between two or more options, but it matters to other players which choice is made?
- Finally, this technique works as long as the game can consist of no more than some finite number of moves by players.

Should we believe in backward induction?

Problem: backward induction is weak in revealing players' preferences.

Have a look at a famous game.

Have the game played in class.

Should we believe in backward induction?

The ultimatum game

There are two players, A and B.

A moves first and chooses to be either greedy or fair.

If A is fair, each side gets $5.

If A is greedy, B must:

a) accept A's greed, giving $9 to A and $1 to B

b) or reject A's offer, which gives $0 to both.

Should we believe in backward induction?

The ultimatum game

- Represent the game on the blackboard in extensive form
- Actually play the game
- Have the students make predictions
- Social preferences

Should we believe in backward induction?

The Centipede game

- Players 1 and 2,
- One dollar is put on the table.
- Player 1 has the first move; she can either take the dollar, leaving Player 2 with nothing, or say "I pass."
- If she passes, a second dollar is put on the table, and it is Player 2's turn to take the money, leaving Player 1 with nothing, or to say "I pass." If Player 2 passes, a third dollar is put on the table, and we go back to Player 1. And so on, and so on.
- This continues until either one of the players takes the money off the table, ending the game, or the amount of money on the table reaches $10.

Should we believe in backward induction?

The Centipede game

- When $10 is reached, it is Player 2's turn:
- He can take the $10 or say "Not yet."
- If he says "Not yet," four five-dollar bills are added to the pot, for a total of $30
- now Player 1 makes the final choice: She can say "I want it all" or she can say "$15 for each."
- And whatever she says is how the game ends.

Should we believe in backward induction?

The Centipede game

Backward induction says that Player 1 will take the $1 at the start of the game

Is this what you would do if you were Player 1, playing against one of your classmates?

Should we believe in backward induction?

The Centipede game

Even if money is everything to the players with very high probability, a lot of stages in the game where the potential benefits to the players grow if they trust each other, can overthrow the logic of backward induction.

Backward induction is premised on every player knowing *for certain* what will happen for the rest of the game. That can be a fairly heroic premise.

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Nash Equilibrium

John Nash

Nash Equilibrium

Economists employ dominance and iterated dominance, both strict and weak, whenever they can.

But, in many economic contexts this does not get all the way to a predicted outcome.

And in extensive-form games, backward induction can be at least difficult and, in some cases, impossible to apply; information sets can intefere.

In such cases, the analysis turns to Nash equilibria.

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Nash Equilibrium

Let's go back to Sam and Jan:

		Jan's choice		
		Old Pros	Art Museum	Cafeen
Sam's choice	Old Pros	6,4	4,3	4,2
	Art Museum	2,1	5,5	2,2
	Cafeen	1,1	1,3	3,6

Nash Equilibrium

The strategy profile in which Sam chooses Old Pros and Jan chooses Old Pros is a Nash equilibrium. Each is playing a best response to what the other person is doing.

		Jan's choice		
		Old Pros	Art Museum	Cafeen
Sam's choice	Old Pros	6,4	4,3	4,2
	Art Museum	2,1	5,5	2,2
	Cafeen	1,1	1,3	3,6

Nash Equilibrium

Suppose Sam chooses the art museum. Jan's best response is the art museum, to which Sam's best response is the art museum. *This is another Nash equilibrium*.

		Jan's choice		
		Old Pros	Art Museum	Cafeen
Sam's choice	Old Pros	6,4	4,3	4,2
	Art Museum	2,1	5,5	2,2
	Cafeen	1,1	1,3	3,6

Nash Equilibrium

And suppose Sam chooses Cafeen. Jan's best response is Cafeen. But Sam's best response to this is Old Pros. So Sam choosing Cafeen is not part of a Nash Equilibrium.

		Jan's choice		
		Old Pros	Art Museum	Cafeen
Sam's choice	Old Pros	6,4	4,3	4,2
	Art Museum	2,1	5,5	2,2
	Cafeen	1,1	1,3	3,6

Nash Equilibrium

Let's have a formal definition:

For a strategic-form game, a Nash equilibrium is a strategy profile such that no player, by changing his or her part of the strategy profile unilaterally, can improve his or her payoff.