Linear Contracts and Incentives Intensity

1. Linear contracts

A linear contract has the form:

$$w(y) = s + by \tag{1}$$

where b is a measure of incentives intensity: the larger b the larger the link between remuneration and performance.

The linear schema in Eq.1 is general enough as to represent three most different situations as long as b varies:

1. with $b = 0$ wage will not depend on y at all, i.e. a fixed wage. It is noteworthy that in this case risk allocation is optimal, the agent is totally insured from variations in y and incentives have no role at all. (Try and represent this situation in an Edgeworth Box whose dimensions are two possible levels for y. See Perez Castrillo and Macho Stadler pp.)

2. with $b = 1$ wage will totally depend on y. In this case, the agent will be a *residual claimant* of y, risk will be wholly allocated to him/her and incentives do have maximal strength. (Try the same exercise for this case as well).

3. with $0 < b < 1$ we are in a *profit sharing* situation: incentives are stronger as b aproximates 1 and risk follows accordingly. (Try the same exercise).

2. Optimal incentives

We will now try and understand what is an optimal incentive scheme, that is: the optimal value of b. With that value we will refer the a value of b that maximizes the total surplus of the agency relation in such a way that both incentives effects and risk allocation are taken in due consideration.

As we always assume that e be not observable, we will consider the following production function:

$$y = \pi e + \epsilon \tag{2}$$

where π stands for effort's productivity and ϵ is a random term with $E(\epsilon) = 0$ and $Var(\epsilon) = \sigma^2$.

Under uncertainty, agent's utility can be measured on the ground of the certainty equivalent i.e. the difference between expected value and risk premium. Based on Eq.1 we can actually prove that the agent's certainty equivalent u_c can be expressed as:

$$u_c = s + b\pi e - c(e) - (1/2)rb^2\sigma^2 \tag{3}$$

In Eq.3 the first two terms represent expected wage, $c(e)$ represents effort's cost function and the last term represents the risk premium. With the parameter r we represent a measure of risk aversion ($r = 0$ is neutrality and higher levels of r indicates increasing levels of risk aversion). We will assume that the cost function for effort be given by $c(e) = \gamma e^2/2$.

On the ground of incentives offered by the principal, the agent chooses an effort level maximizing the equivalente certo with respect to e. From the first order condition:

$$\frac{\partial u_c}{\partial e} = 0$$

we obtain:

$$b\pi = c'(e) = \gamma e \tag{4}$$

Equation 4 establishes that the marginal benefit relative to a greater effort must be equal to effort's marginal cost (i.e. an increase in e determines an increase in y equal to π and thus a higher expected wage equal to $b\pi$).

From Eq. 4 we can also determine the agent's incentive reaction function (call it $e(b)$):

$$e = b\pi/\gamma. \tag{5}$$

Now, the principal's expected profit is equal to:

$$\Pi = E(y - w) = (1 - b)y - s = (1 - b)\pi e - s \tag{6}$$

The total surplus S generated by the agency relation will be given by the sum of the agent's certainty equivalent (Eq.3) and principal's expected profits (Eq.6). We thus sum u_c and Π and get:

$$S = \Pi + u_c = \pi e - \frac{\gamma e^2}{2} - \frac{rb^2\sigma^2}{2} \tag{7}$$

We can thus define a first best contract' main features. If e is observable the principal and agent can negotiate effort's optimal level. Maximizing S in Eq.

7 with respect to e, first best effort is such that $\pi = \gamma e$, that is: $e_f = \pi/\gamma$. It is noteworthy that effort's optimal level e_f can indeed be obtained via a linear contract by setting $b = 1$ (as one easily realizes from Eq. 7). This is the case in which the agent becomes a residual claimant on y.

As we perfectly know, under conditions of asymmetric information a contract is not viable that is based on effort' level but, as a matter of fact, the agent can receive incentives by relating his remuneration to output level. At the very same time, we also know that this has the undesired effect of allocating some risk to the agent and this effect must be taken into consideration as one designs an incentivizing contract.

According to this perspective, the main tool a principal can rely on is the intensity of incentives i.e. the b parameter in $w = s+by$. As a consequennce, our main focus is on determining the optimal level of incentives b^*. This optimal level will be such that it can actually maximize social surplus while considering the agent's reaction function (i.e. Eq.5). By substituting Eq.5 in Eq.7 we have:

$$S = \pi\left(\frac{b\pi}{\gamma}\right) - \left(\frac{\gamma}{2}\right)\left(\frac{b\pi}{\gamma}\right)^2 - \frac{rb^2\sigma^2}{2} \tag{8}$$

By deriving S with respect to b and equating to zero, we get:

$$\frac{\partial S}{\partial b} = \frac{\pi^2}{\gamma} - \frac{b\pi^2}{\gamma} - rb\sigma^2 = 0 \tag{9}$$

From Eq.9 we can finally determine b^* (i.e. the optimal value for b):

$$b^* = \frac{1}{1 + \frac{r\gamma\sigma^2}{\pi^2}} \tag{10}$$

Maximizing total surplus requires that wage' sensibility to performance be establishes relative to Eq.10. As a matter of fact, b^* will be higher:

1. the smaller is σ^2, that is: the smaller is uncertainty in production. It is noteworthy that as σ^2 gets smaller accuracy in performance measurement increases and a strict correlation of wages to performance is way more convenient (this happens as risks on agent will be very small);

2. the smaller is the agent's risk aversion r. If bearing risk is not costly for the agent strong incentives are a good idea because compensating the agent for risk becomes relatively cheaper;

3. the smaller is the marginal cost of effort γ. That is: incentives tend to be stronger the slower the disutility of effort grows as agent chooses a higher level of effort;

4. the larger is effort's marginal productivity π. That is: it is optimal to give strong incentives whenever one gets large increases in output as effort increases.

On the contrary, a contract characterized by weak incentives ($b \to 0$) and low risk for the agent will be prevailing if: i) uncertainty is high; ii) risk aversion is high: iii) effort's disutility quickly grows; iv) production's sensibility to effort is limited.