

# The model

- ▶ Also called: Agency Theory or Economic Theory of Incentives
- ▶ It deals with incentives: reasons and motivations for having someone else do something she would not be doing otherwise
- ▶ It deals with the ways to attain efficiency and maximize the surplus from a social relation when individual aims do diverge.

# The model

- ▶ The first actor is a Principal (henceforth  $P$ ).
- ▶ The second actor is an agent (henceforth  $A$ )
- ▶  $P$  owns an organization (say, a firm)
- ▶  $P$  does not have time or competencies to personally carry out those tasks and jobs that create value for her firm
- ▶  $P$  can however hire an agent  $A$  to work for her in producing value that she will appropriate and pay him a wage.
- ▶ At this point students are called to represent this situation in a perfectly competitive (labor) market.

# The model

- ▶ After hiring  $A$ , however,  $P$ 's and  $A$ ' interests do actually diverge (note: this would not happen in a perfectly competitive market)
- ▶  $P$  wants  $A$  to work as hard as he can and produce as much value as possible for the firm.  $P$  can either monitor  $A$  or motivate  $A$  (but both activities are either hard to carry out or costly)
- ▶ On the other hand, once  $A$  has been hired he has any reason to make his commitment and effort as small as possible
- ▶ As  $A$ 's actions are partly hidden or not observable, moral hazard takes place: a form of post-contractual opportunistic behavior whose main consequence is a loss of efficiency

# Cornerstones

The model is about nothing but motivation.

Motivation means incentives

Beating MH and restoring efficiency using appropriate contracts

Non observability and uncertainty will be our most fierce enemies

# Building Blocks: Production Technology

1. **Contribution**. Have it denoted with  $y$ . It is  $A$ 's contribution to value for  $P$ . E.g. harvest, increase in stocks prices.  
Contribution is assumed to be easy to observe, measure, and easy to be used in court
2. **Effort**. Have it denoted with  $e$ . This is “how hard  $A$  works”, “how much effort he puts in his work”. Effort is non observable, hardly measurable, not usable in court. We will assume that  $e$  can take on three values higher to lower:  $e_H$ ,  $e_M$ ,  $e_L$ .
3. **Uncertainty**. Have it denoted with  $\epsilon$ . This is “the effects of uncertain events that have an influence on  $y$  but do not depend on  $A$ 's effort”. E.g. it rains, bad economic trend. . .

# Building Blocks: Production Function

Our production function will thus be:

$$y = f(e, \epsilon)$$

# Building Blocks: Production Technology

We will initially adopt a most simplified model of uncertainty.

1. Let  $y_1 < \dots < y_n$  be possible outcomes of  $A$ 's work (ranked worst to best)
2. Let us assume that only one  $y_i$  will be realized
3. Let  $p_i^H, p_i^M, p_i^L$  the probabilities associated with  $y_i$  being realized when  $A$  works with  $e_H, e_M, e_L$
4. It then follows that the expected result if  $A$  works with effort  $e_M$  will be:  $E^M(y) = \sum_{i=1}^n p_i^M y_i$
5. Finally, we will assume that effort is productive, i.e. higher effort gets you higher expected contributions (but remember uncertainty: prob that contribution is miserable with highest effort is small but non null!)

# Building Blocks: Contracts

That is: how to have  $A$  work (hard) for  $P$ .

1. Problem  $e$  can't be observed (this is what we call an *asymmetry of info*)
2. Thus: contracts can't be based on  $e$
3. Solution: have contracts be based on  $y$  i.e. have  $y$  be a proxy for  $e$
4. That is, we will have:  $w = w(y)$ .



# Building Blocks: Contracts

We will consider a particular class of contracts: linear contracts.

1. These will set wage to be formed by a fixed (i.e. not depending on  $y$ ) plus a variable part (i.e. depending on  $y$ ).
2. Contracts will thus have the form:  $w = a + by$
3.  $by$  is what we call an *incentive*

## Building Blocks: $P$ 's Results

For the sake of simplicity, we will initially assume that both  $P$  and  $A$  are risk neutral (thus they are only interested in result's expected value and uncertainty has no cost for them). We will soon drop this assumption.

1.  $P$  she gets  $y$  but has to pay  $w$ . Thus her expected result is:

$$\pi(a, b) = E(y - w) = E(y) - E(w) = E(y) - a - b \cdot E(y)$$

2. Thus, assuming risk neutrality:

$$\pi(a, b) = e - a - b(e) = (1 - b)e - a$$

3. That is:  $P$ 's result is given by the value created by  $A$  minus what  $P$  has to pay to incentivate him.

## Building Blocks: $A$ 's Results

These will depend on the following consideration: if  $A$  accepts the contract he will receive a wage  $w$  but he will have to put effort in his work and thus bear some costs.

1. Let  $c(e)$  be  $A$ 's cost function.  $c(\cdot)$  value corresponds to its monetary cost.
2.  $c(e)$  is an increasing, convex function.  $c'(e)$  will thus be increasing.
3.  $A$ 's expected result will thus be given by the following utility function (assume a separable function for the sake of simplicity):

$$U_A(w, e) = U(w) - V(e)$$

4. As we assumed  $A$  to be risk neutral, we will have:

$$U_A(w, e) = a + b \cdot y - V(e)$$

## Building Blocks: $A$ 's Results

1. If  $A$  does not accept the contract, he will receive his reservation utility  $U_A(0) = U_0$ .
2. That is: the result of his best alternative option or the utility he derives from leisure.

# Temporal Sequence

1.  $P$  proposes to  $A$  a contract  $w = a + b \cdot y$
2.  $A$  subscribes to the contract iff  $U_A(w, e) \geq U_0$
3. If  $A$  subscribes to the contract, then he decides between  $e_H$ ,  $e_M$ ,  $e_L$  without having  $P$  be able to observe his choice.
4.  $P$  and  $A$  (and possibly a court) do execute the contract.
5.  $P$  gets  $\pi(a, b) = y(e) - a - b \cdot y$  and  $A$  receives  $U_A(w, e) = a + b \cdot y - V(e)$

Have a close look at Figures 1.1, 1.2, 1.3 on MSPC pp. 8, 9, 10 and the discussion therein.

# Let us have everything formalized

$P$ 's problem:

$$\max \sum_{i=1}^n p_i^H (y_i - w(y_i))$$

# Let us have everything formalized

Participation constraint:

$$\sum_{i=1}^n p_i^H [U(w(x_i)) - V(e_H)] \geq U_0$$

## Let us have everything formalized

Incentive compatibility constraint:

$$\sum_{i=1}^n p_i^H [U(w(x_i)) - V(e_H)] \geq \sum_{i=1}^n p_i^M [U(w(x_i)) - V(e_M)]$$

$$\sum_{i=1}^n p_i^H [U(w(x_i)) - V(e_H)] \geq \sum_{i=1}^n p_i^L [U(w(x_i)) - V(e_L)]$$



## A neat (tough) problem

It might well happen that the solution to this problem be too costly for  $P$ .

Total utility for  $P$  might not indeed be maximized by  $e^H$  but rather by  $e^M$ .

This happens when the difference between  $y$  and  $w$  (i.e. her profits) is not maximized by  $e^H$  but rather by  $e^M$  or even by  $e^L$ .

This is called *productivity constraint*. It requires that the increase in  $y$  obtained as a result of an increase in  $e$  be higher than the increase in  $b$  that  $P$  has to pay in order to have a higher  $e$ .

Formal solution to this problem is real hard. We will deal with it only by some examples.