The Dutch Book

The "Dutch Book" argument, tracing back to independent work by F.Ramsey (1926) and B.deFinetti (1937), offers prudential grounds for action in conformity with personal probability.

A Dutch Book is a set of bets bought or sold at such prices as to guarantee a net loss. An agent is susceptible to a Dutch Book, and her credences said to be ‘incoherent’, if there exists such a set of bets, bought or sold at prices that she deems acceptable (by the lights of her credences).

First some gambling terminology:

If you pay $r for the right to receive $s if A is true, you are said to have made a bet on A with a betting quotient of r/s, and a stake s. Here may be positive, zero, or negative; and s may be positive or negative.

Dutch book arguments normally assume that for any proposition A, there is a number p(A) such that you are willing to accept any bet with betting quotient p(A). As the notation indicates, p(A) is thought of as your subjective probability for A.

Dutch book arguments purport to do this by showing that if p does not satisfy the axioms of probability, then you will be willing to accept bets that necessarily give you a sure loss. A set of bets with this property is called a Dutch book.

Dutch Book Theorem: if a set of betting prices violate the probability calculus, then there is a Dutch Book consisting of bets at those prices.

The argument for probabilism involves the normative claim that if you are susceptible to a Dutch Book, then you are irrational.

The sense of ‘rationality’ at issue here is an ideal, suitable for logically omniscient agents rather than for humans; ‘you’ are understood to be such an agent.

Classic Dutch Book arguments for the numerical axioms that we have a mathematical characterization of the probability calculus.

Here’s an example:

A bookmaker has offered the following odds and attracted one bet on each horse, making the result irrelevant. The implied probabilities, i.e. probabilità of each horse winning, add up to a number greater than 1.

Horse number	Offered odds	Implied probability	Bet Price	Bookie Pays if Horse Wins
1	Even	1 / 1+1= 0,5	$ 100	$100 stake +$100
2	3 to 1 against	1 / 3+1 =0,25	$ 50	$50 stake + $150
3	4 to 1 against	1 / 4+1 = 0,2	$ 40	$40 stake + $160
4	9 to 1 against	1 / 9+1 = 0,1	$ 20	$20 stake + $180
		Total: 1.05	Total: $210	Always: $200

Whichever horse wins in this example, the bookmaker will pay out $200 (including returining the winning stake), but the punter has bet $210, hence making a loss of $10 on the race.

However, if Horse 4 was withdrawn and the bookmaker doesn’t adjust the other odds, the implied probabilities would add up to 0.95.

In such a case, a gambler could lock in (guarantee himself) a profit of $10 by betting $100, $50 and $40 on the remaining three horses, respectively, and not having to stake $20 on the withdrawn horse, which now cannot win.

The fallacy

From the fact that you are willing to accept each of two bets that together would give a sure loss, the argument infers that you are willing to give away money to a bookie. This assumes that if you are willing to accept each of the bets, you must be willing to accept both of them. But that assumption is surely false in the present case.

This, then, is the fallacy in the Dutch book argument: it assumes that bets that are severally acceptable must also be jointly acceptable; and as our example shows, this is not so.

BIBLIOGRAPHY

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