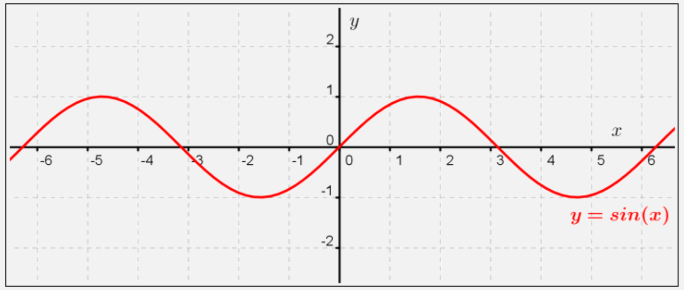
- **DA FUNZIONI ELEMENTARI A FUNZIONI SEMI-ELEMENTARI**

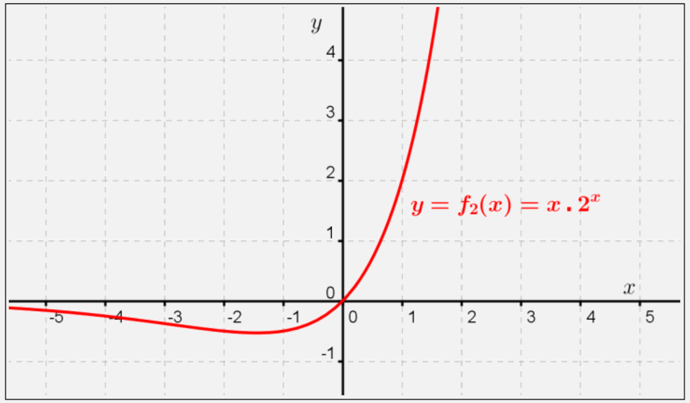
(trasformazioni elementari)

Funzione opposta → ribaltamento verticale $g(x) = -f(x)$

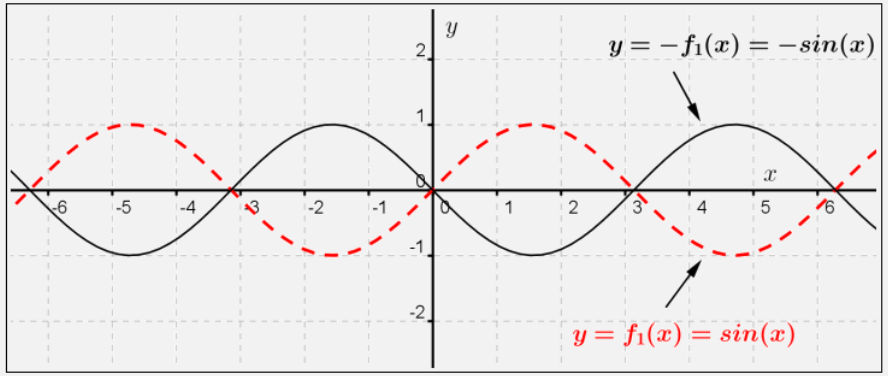
$y = f_1(x) = \sin x$

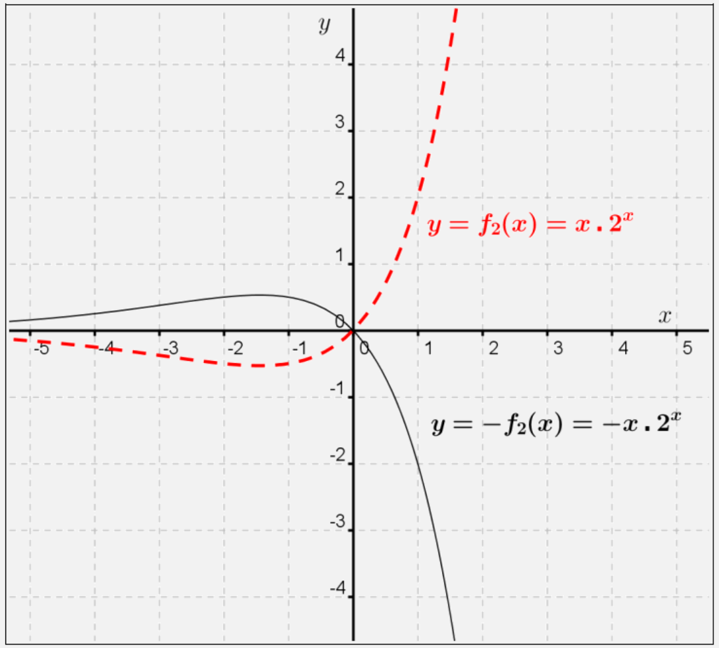


$y = f_2(x) = x \cdot 2^x$



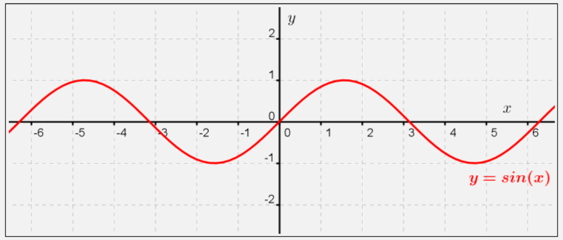
Il grafico ottenuto è formato da punti che hanno le ordinate di segno opposto:

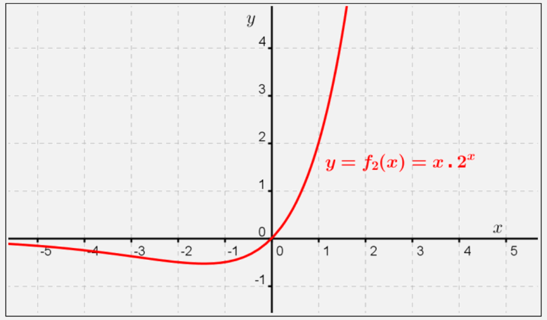




Ribaltamento orizzontale $g(x) = f(-x)$

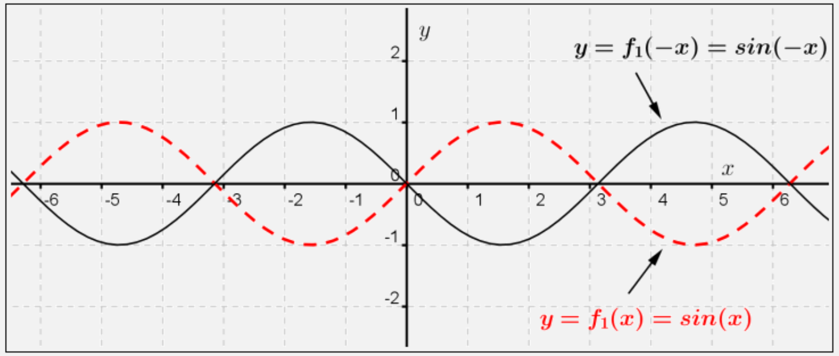
$y = f_1(x) = \sin x$

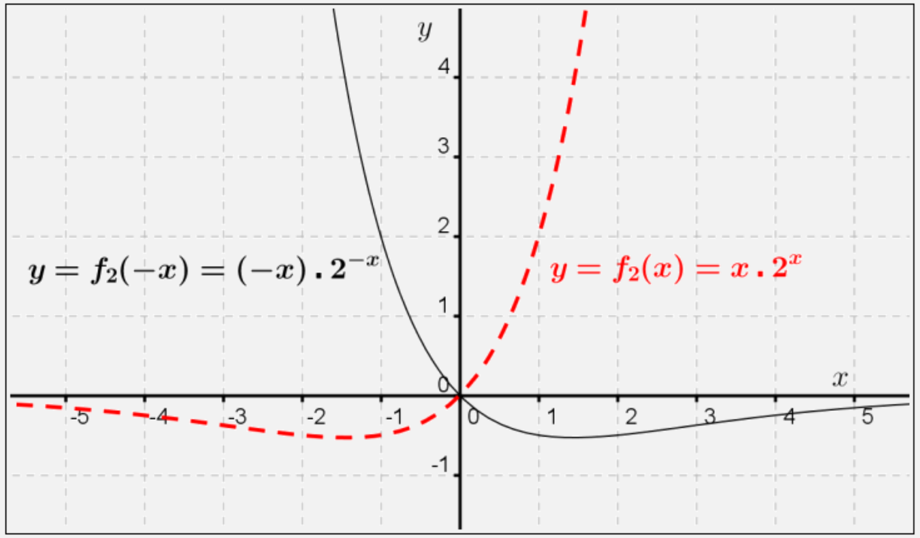




$y = f_2(x) = x \cdot 2^x$

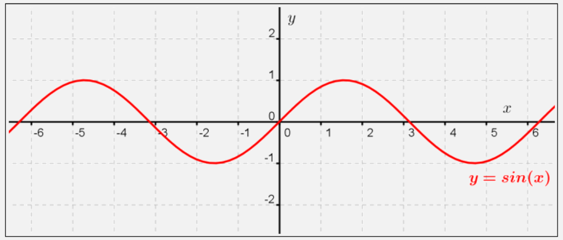
Il grafico ottenuto è formato da punti che hanno le ascisse di segno opposto:

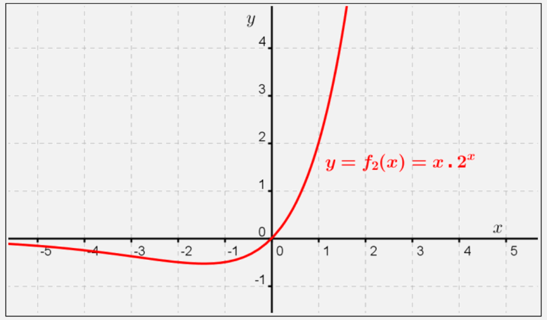




Traslazione verticale $g(x) = f(x) + k$

$y = f_1(x) = \sin x$

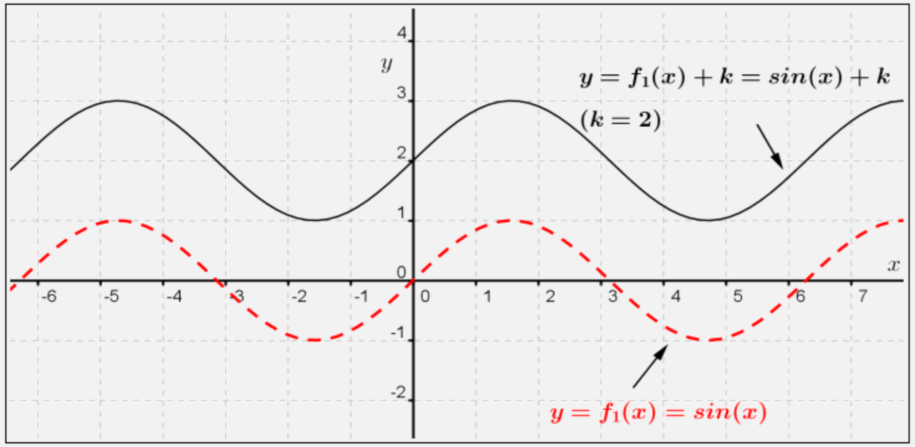


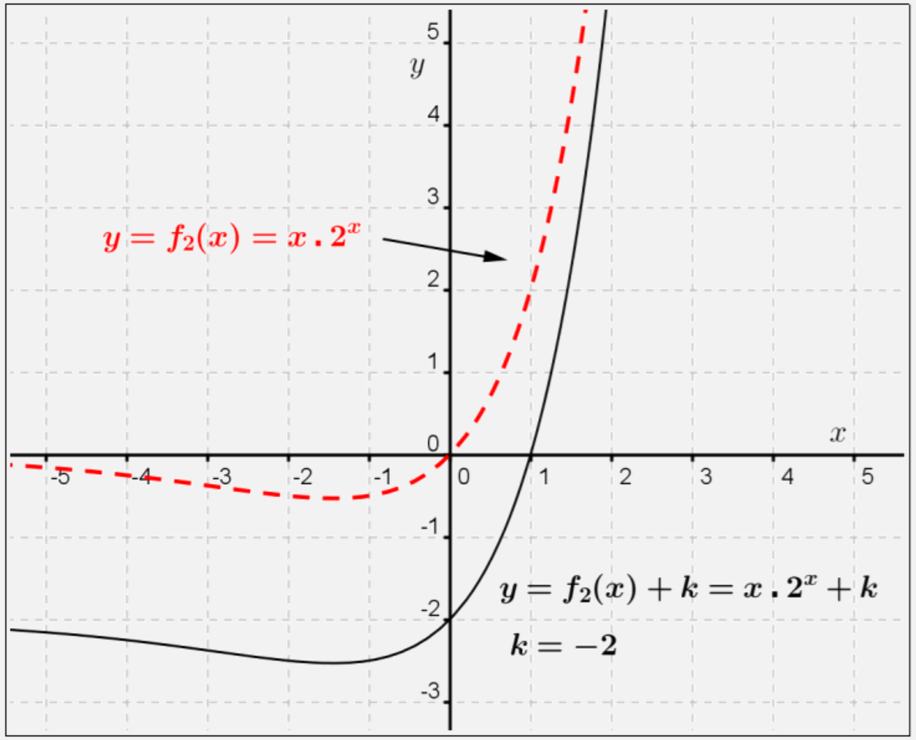


$y = f_2(x) = x \cdot 2^x$

Il grafico ottenuto è formato da punti che hanno le ordinate spostate di k.

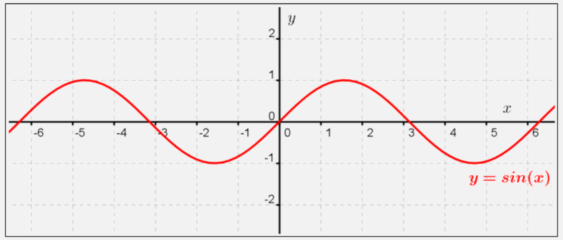
- Se $k > 0$ la traslazione è verso l'alto
- Se $k < 0$ la traslazione è verso il basso

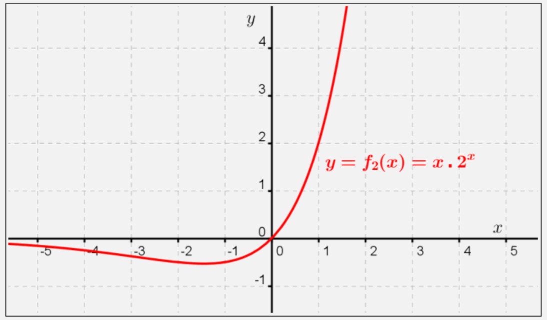




Traslazione orizzontale $g(x) = f(x + k)$

$y = f_1(x) = \sin x$

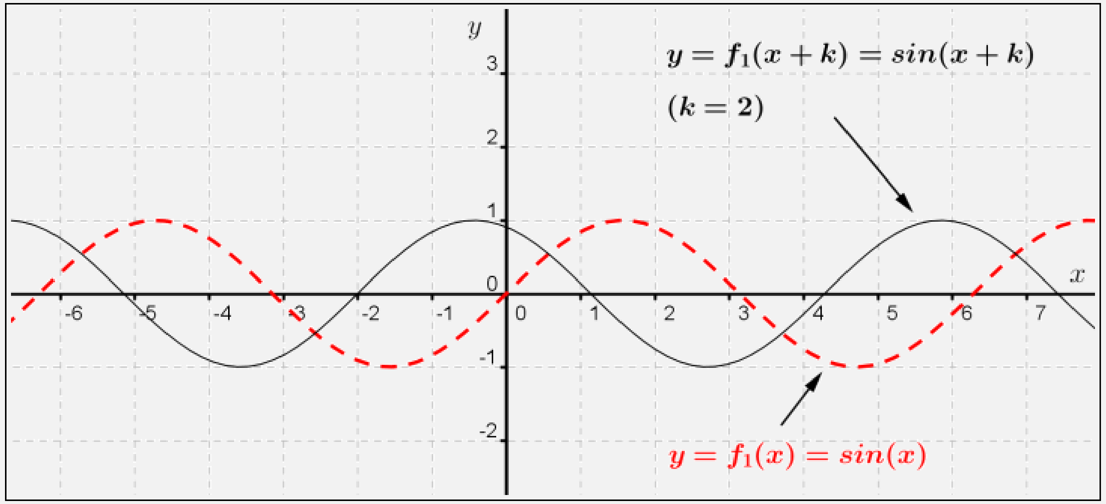


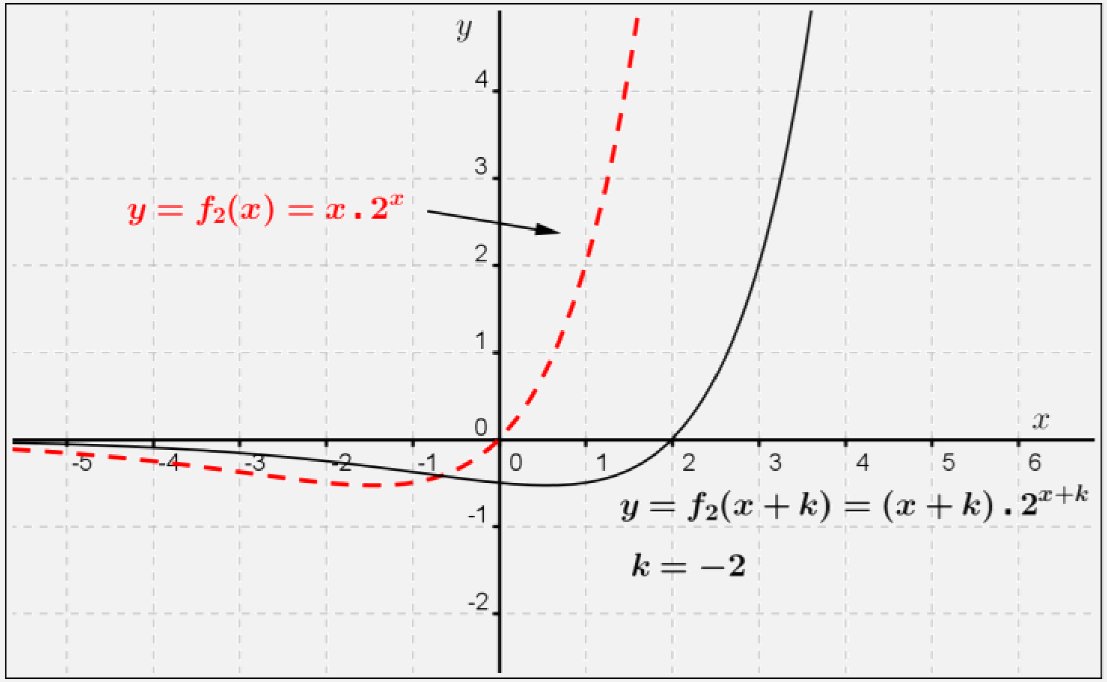


$y = f_2(x) = x \cdot 2^x$

Il grafico ottenuto è formato da punti che hanno le ascisse spostate di k.

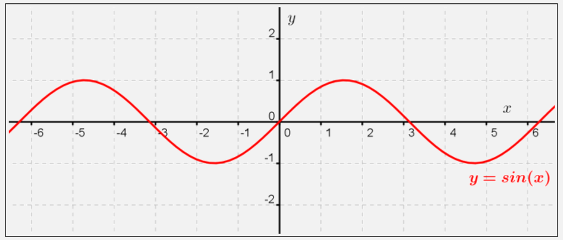
- Se $k > 0$ la traslazione è verso sinistra
- Se $k < 0$ la traslazione è verso destra

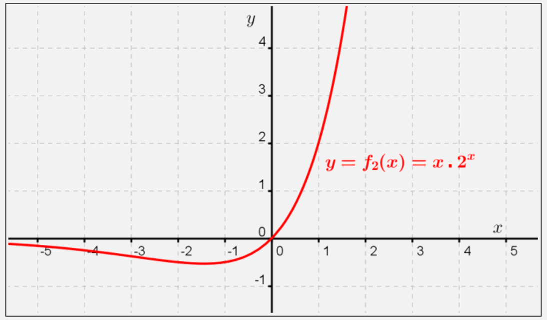




Dilatazione/Contrazione verticale $g(x) = k \cdot f(x), k > 0$

$y = f_1(x) = \sin x$

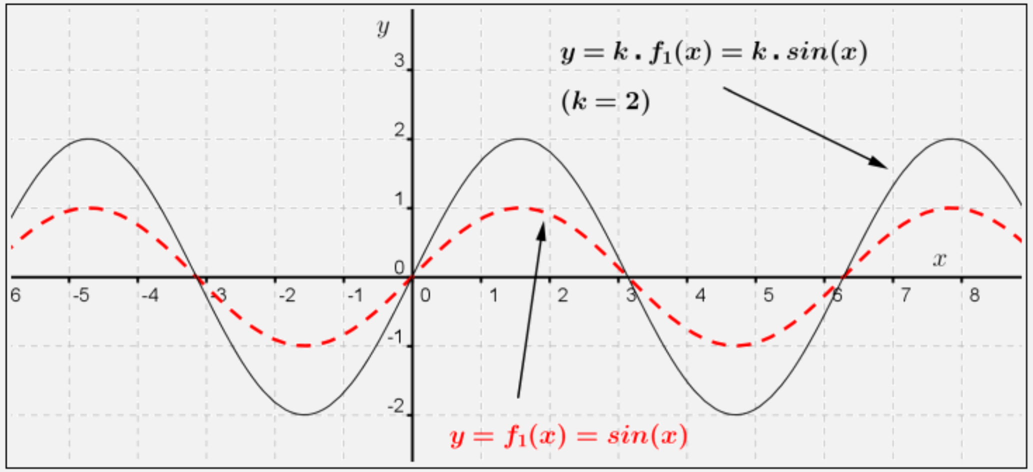


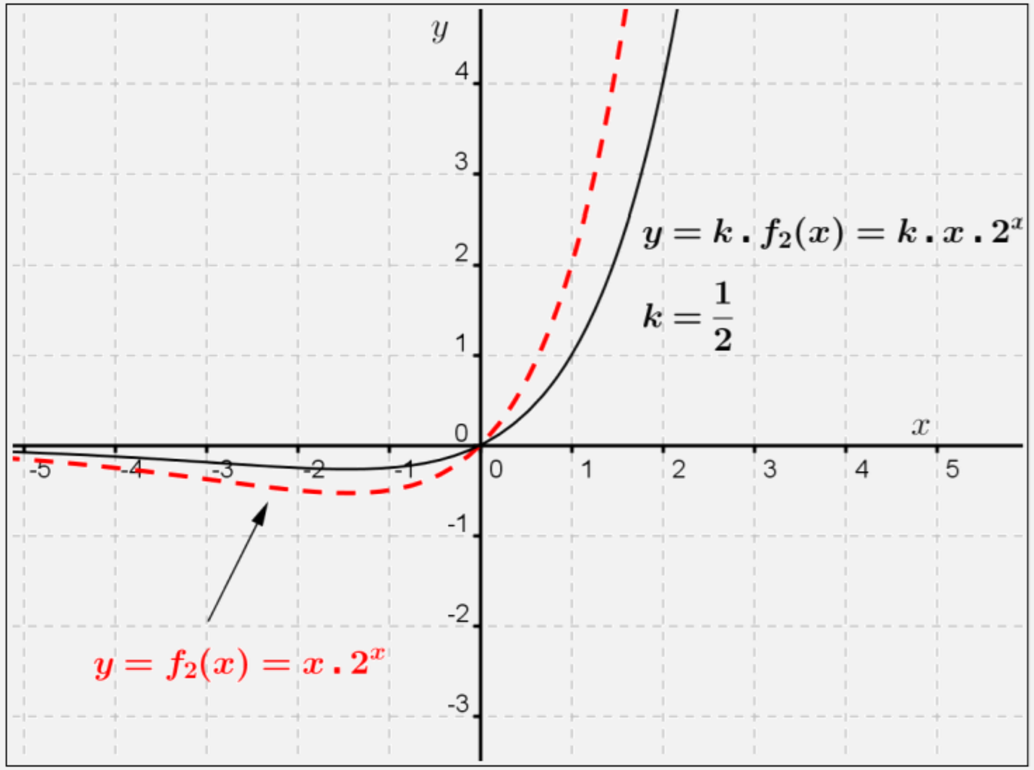


$y = f_2(x) = x \cdot 2^x$

Il grafico ottenuto è formato da punti che hanno le ordinate moltiplicate per k.

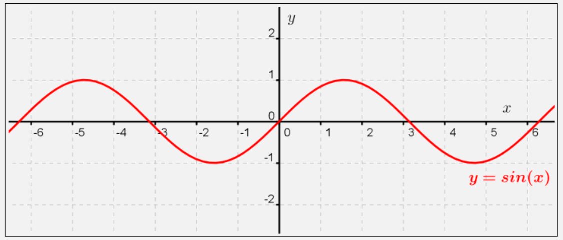
- Se $k > 1$ si ha una dilatazione
- Se $0 < k < 1$ si ha una contrazione

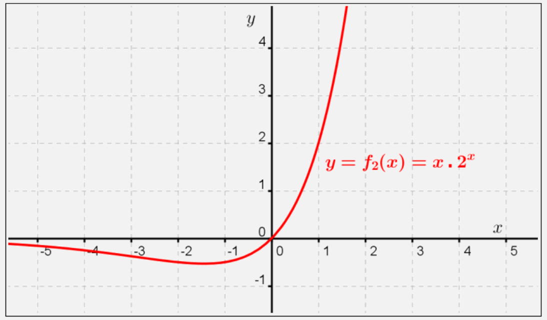




Dilatazione/Contrazione orizzontale $g(x) = f(x \cdot k), k > 0$

$y = f_1(x) = \sin x$

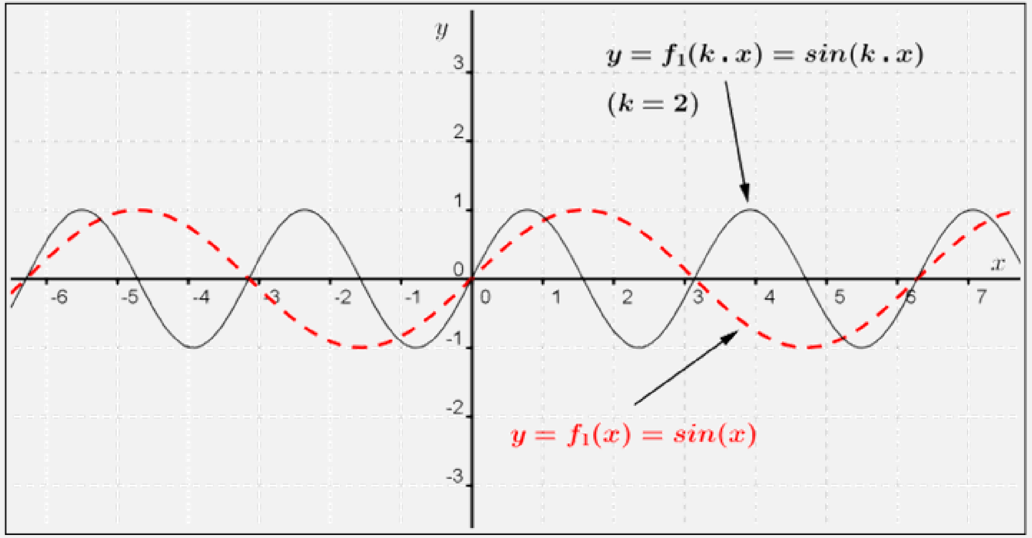


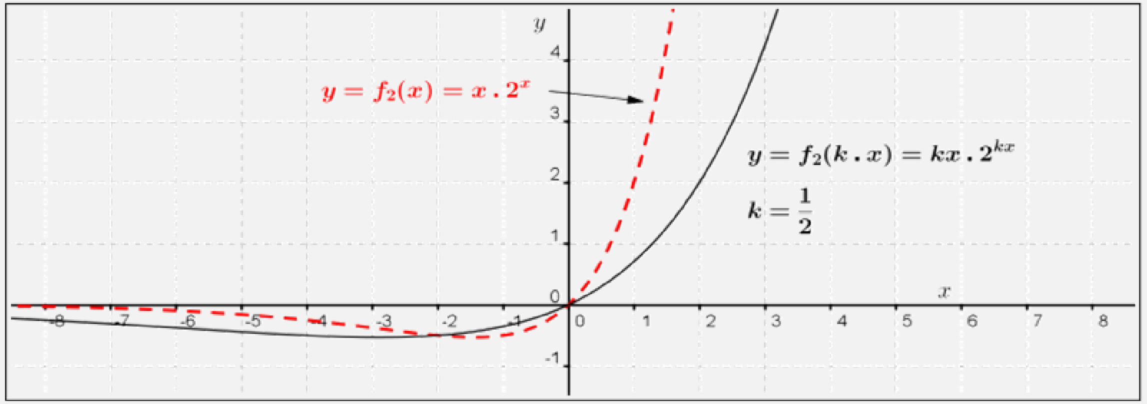


$y = f_2(x) = x \cdot 2^x$

Il grafico ottenuto è formato da punti che hanno le ascisse mutate di un fattore k.

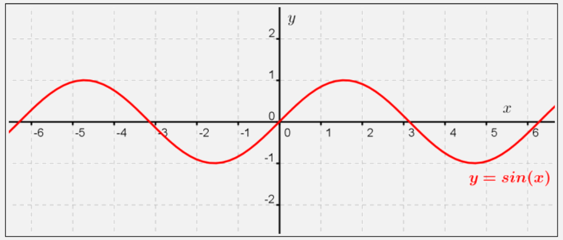
- Se $k > 1$ si ha una contrazione
- Se $0 < k < 1$ si ha una dilatazione

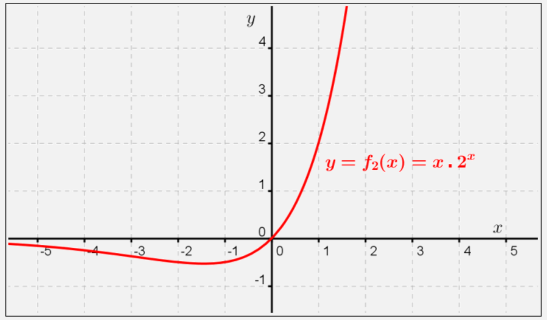




Valore assoluto della funzione $g(x) = |f(x)|$

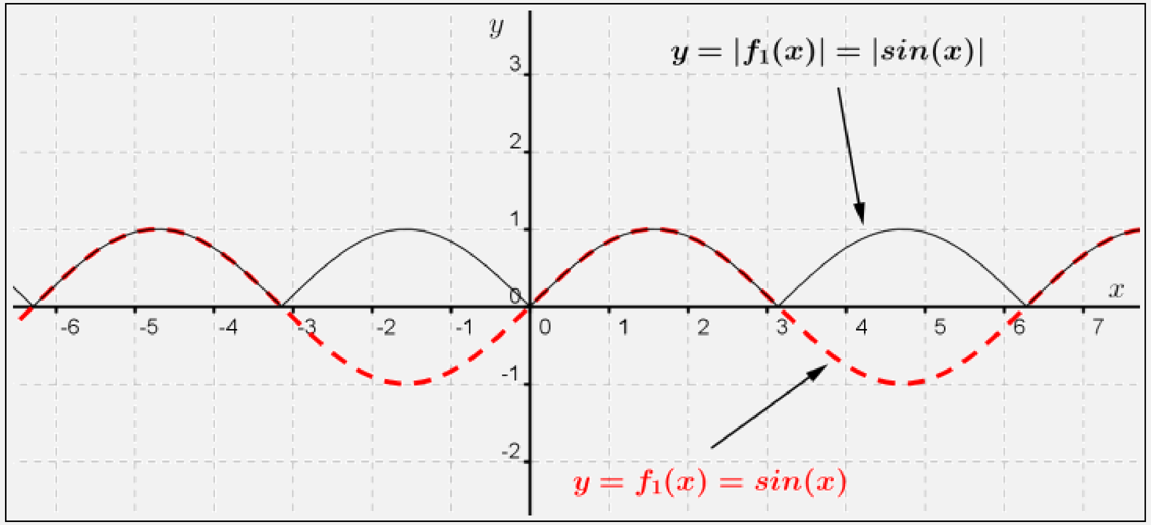
$y = f_1(x) = \sin x$

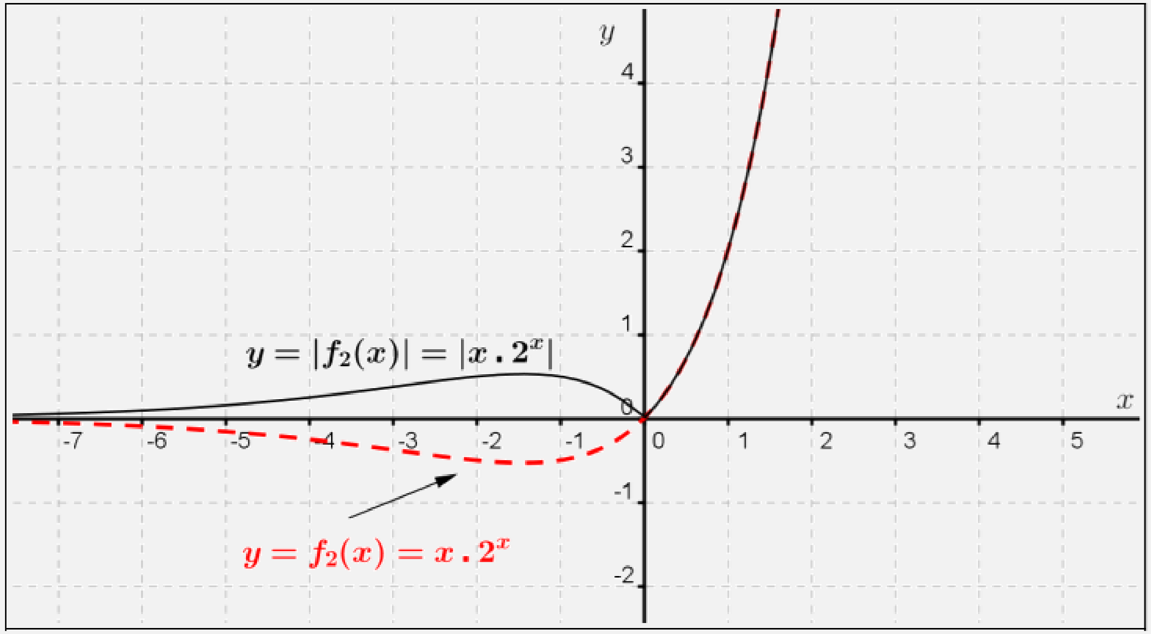




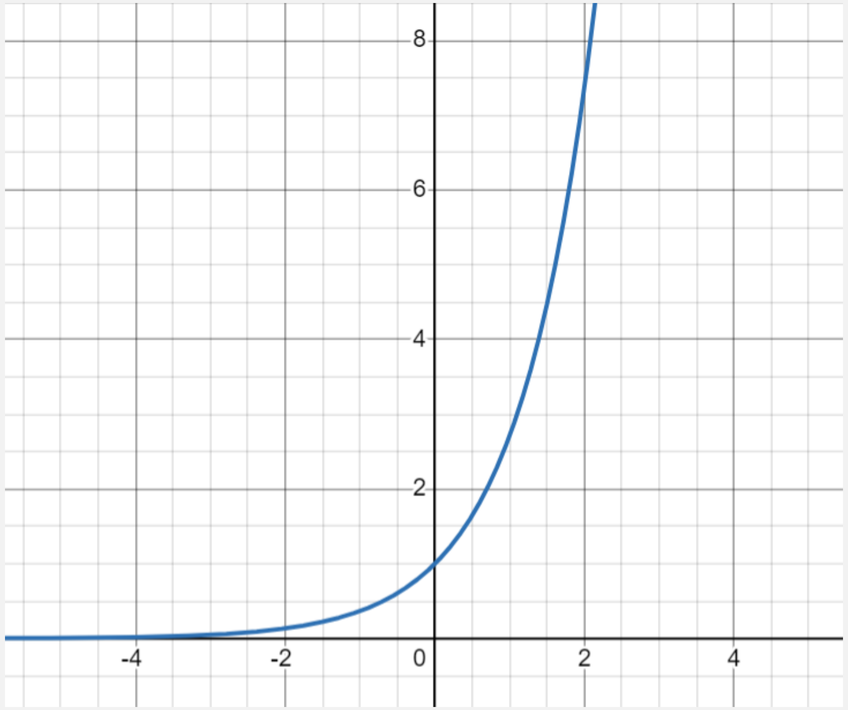
$y = f_2(x) = x \cdot 2^x$

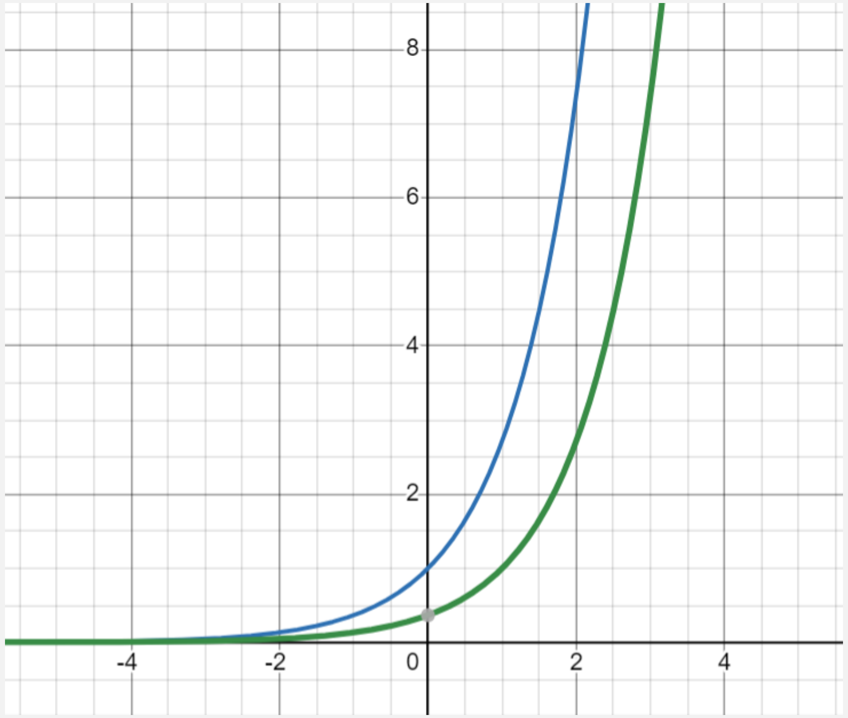
Il grafico ottenuto è formato da punti le cui ordinate sono il modulo di quelle del grafico iniziale.
La parte positiva della funzione rimane invariata; quella negativa viene ribaltata nel semi-piano delle ordinate positive.





Esempio. $f(x) = |e^{x-1} - 2|$?





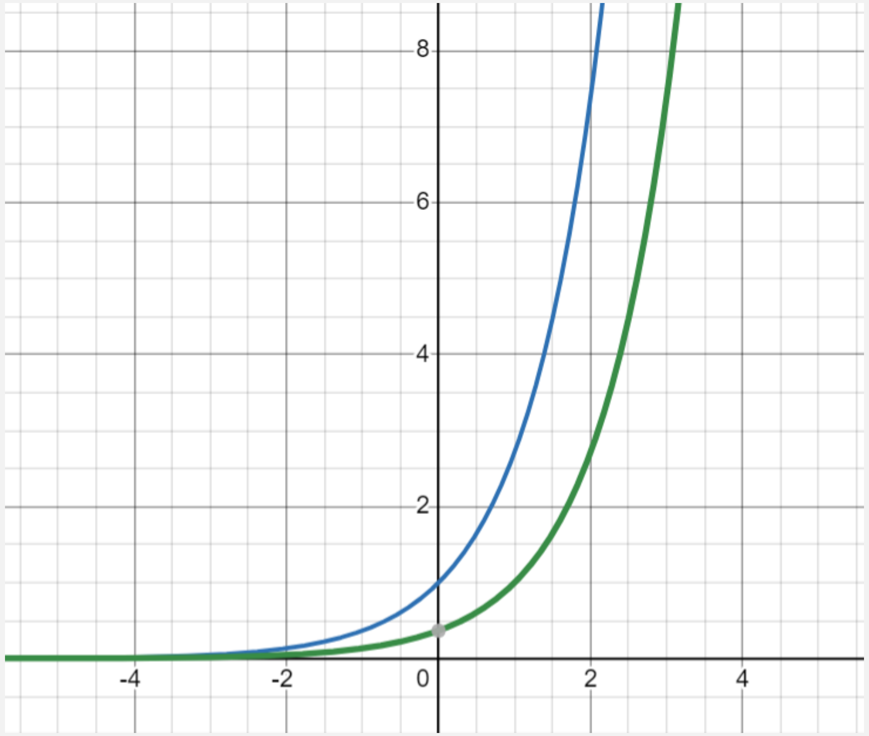
$f(x) = e^{x}$

Punti importanti: $(0,1), (1, e)$

$f(x) = e^{x-1}$

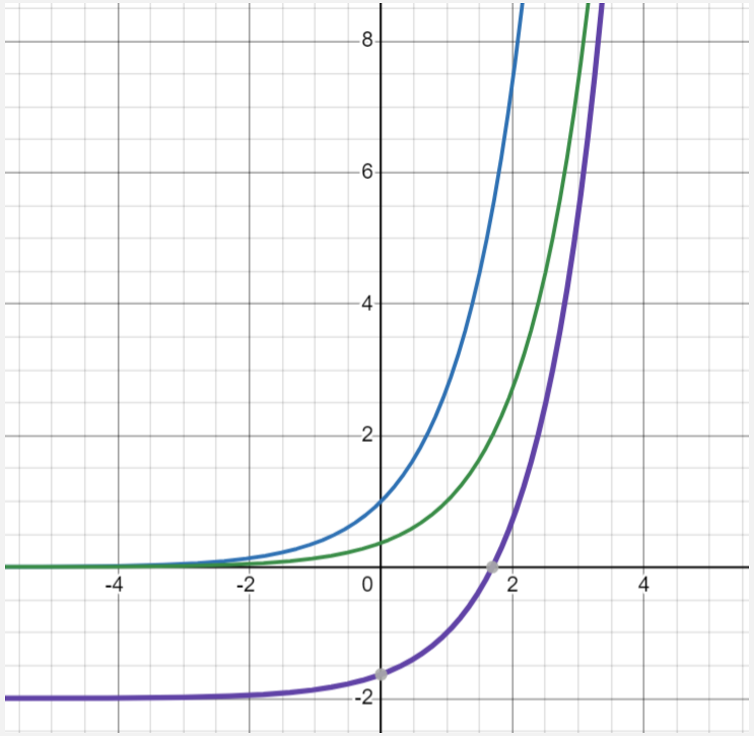
Punti importanti: $(1,1), (2, e)$

Esempio. $f(x) = |e^{x-1} - 2|$?



$f(x) = e^{x-1}$

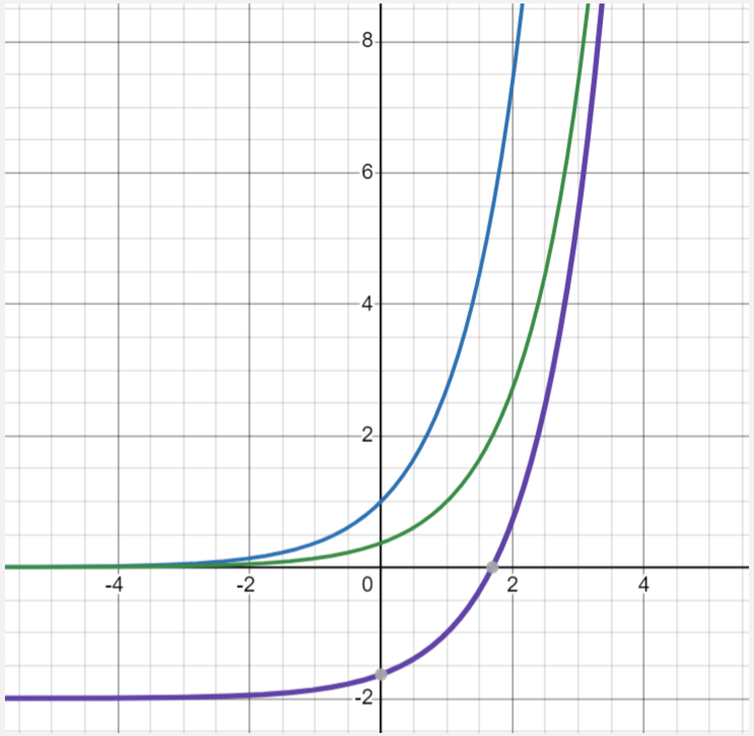
Punti importanti: $(1,1), (2, e)$



$f(x) = e^{x-1} - 2$

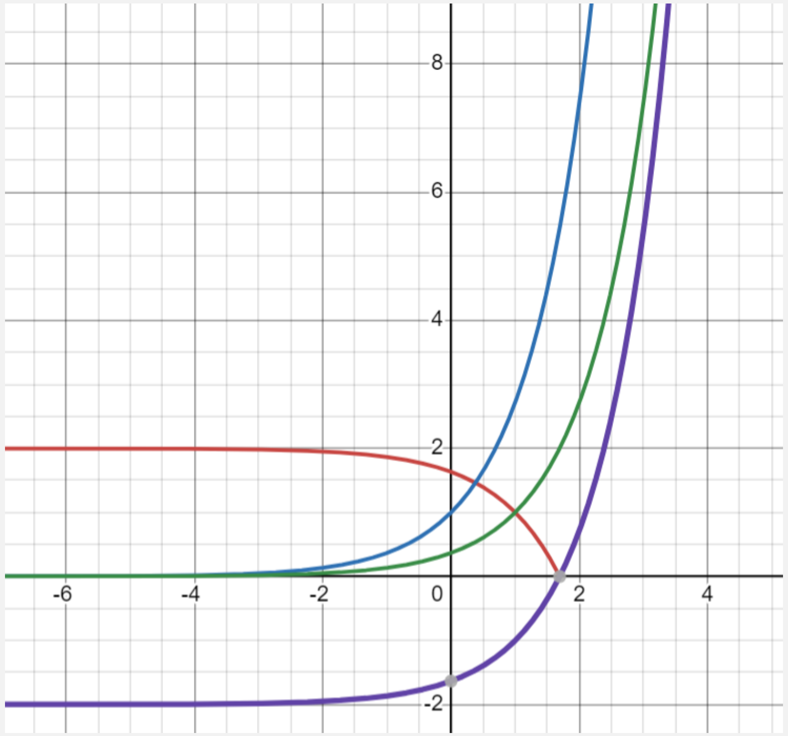
Punti importanti: $(1, -1), (2,0)$

Esempio. $f(x) = |e^{x-1} - 2|$?



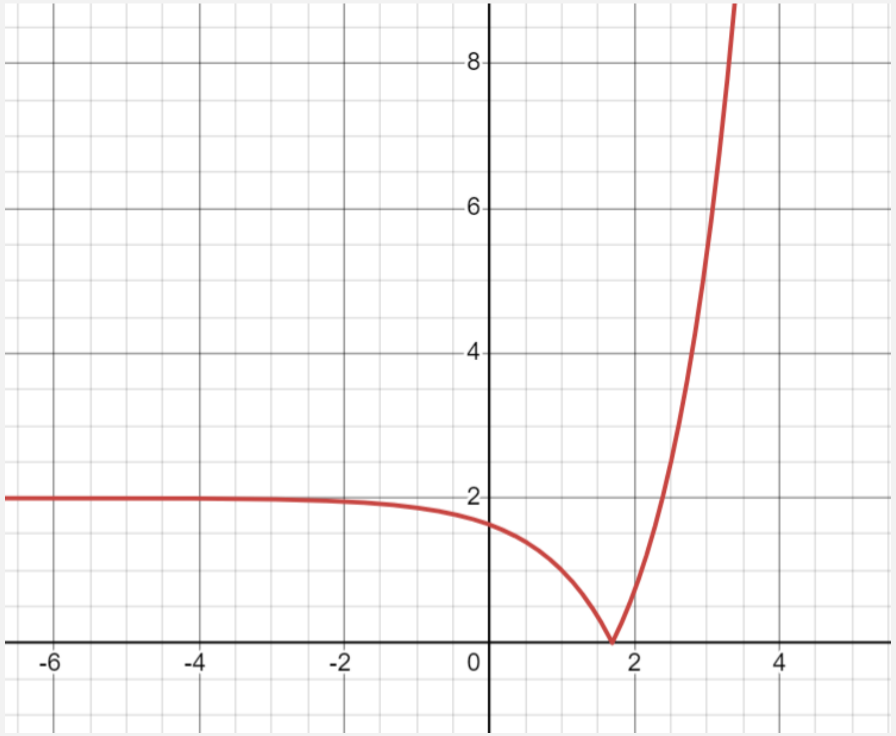
$f(x) = e^{x-1} - 2$

Punti importanti: $(1, -1)$, $(2,0)$



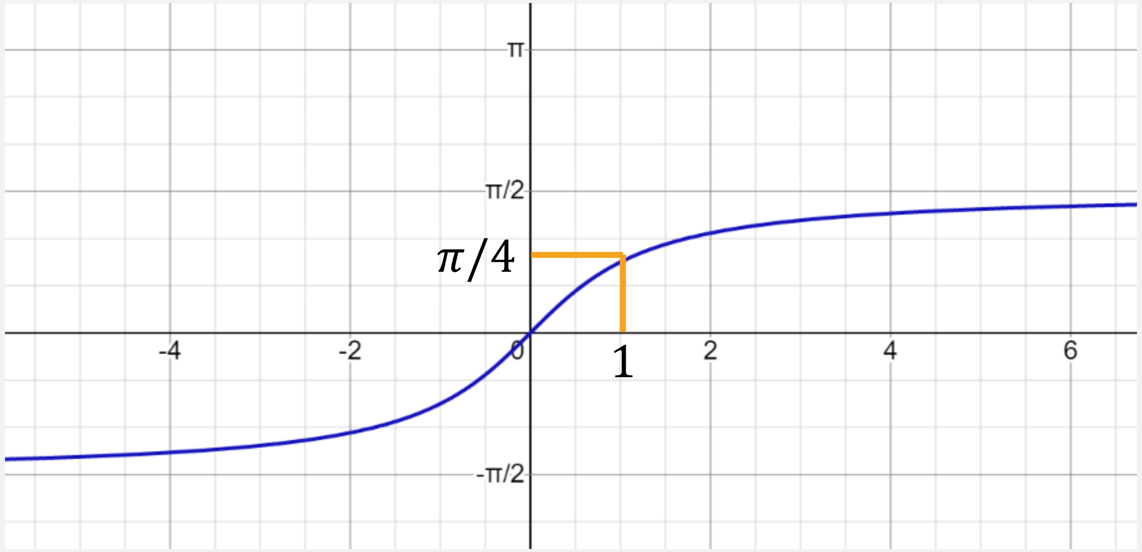
$f(x) = |e^{x-1} - 2|$

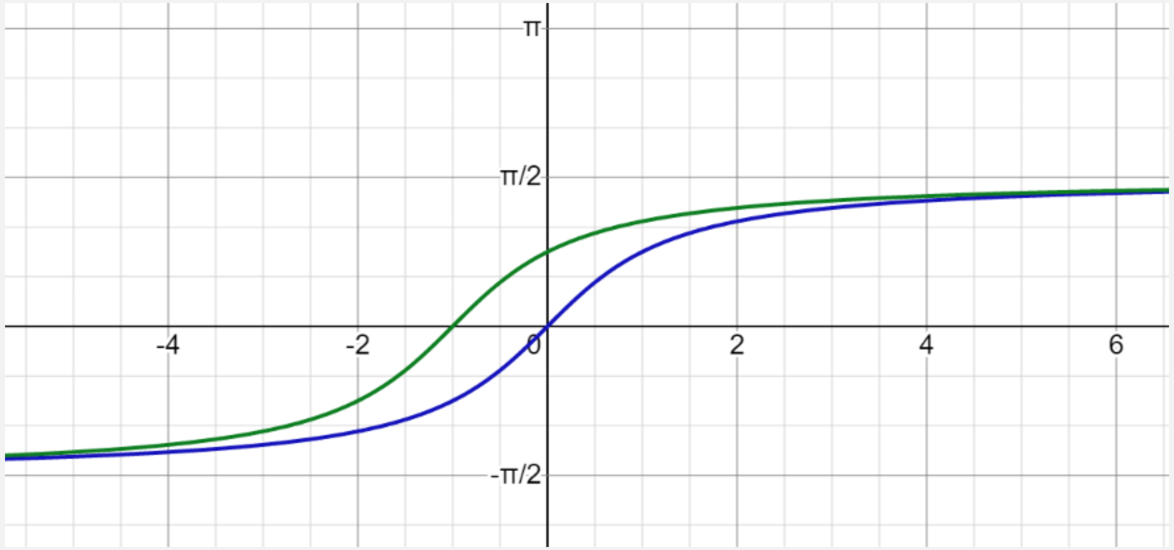
Punti importanti: $(1,1)$, $(2,0)$



$f(x) = |e^{x-1} - 2|$

Esempio. $f(x) = |\text{arctg}(x+1)| + \pi$

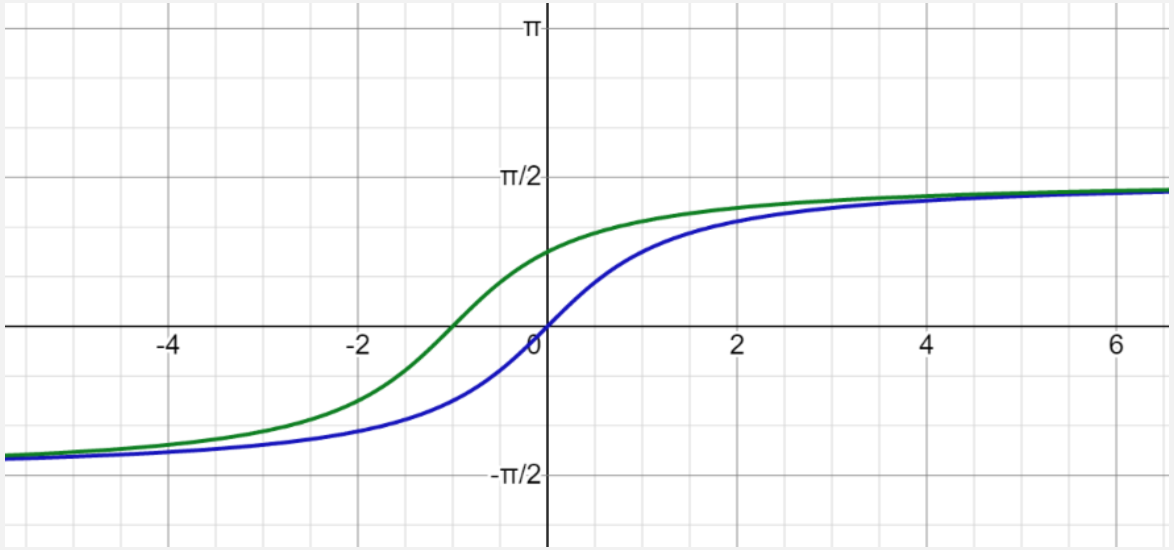




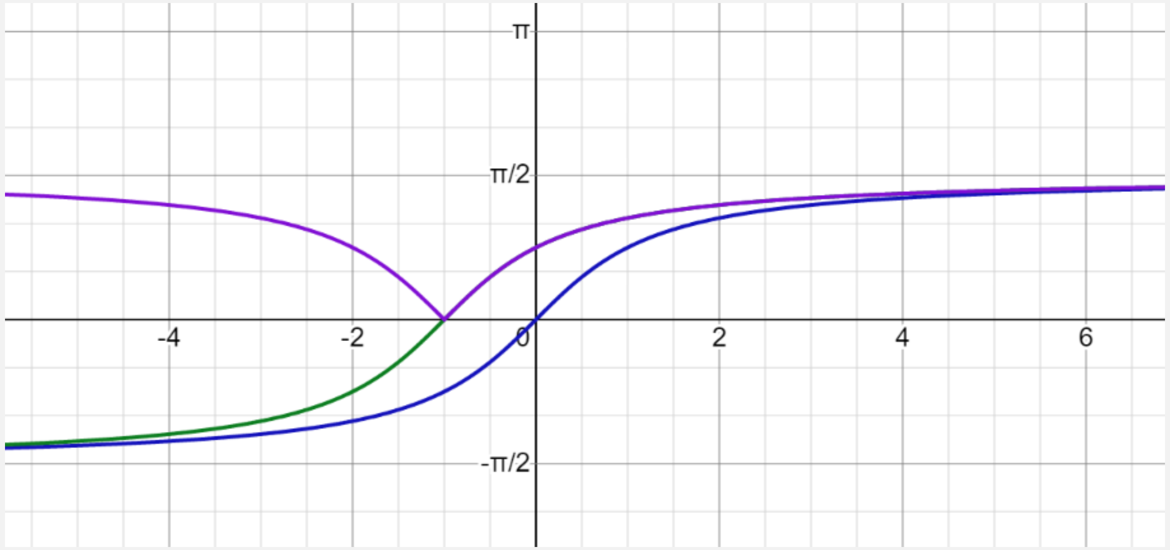
$f(x) = \arctan x$

$f(x) = \arctan(x+1)$

Esempio. $f(x) = |\text{arctg}(x+1)| + \pi$

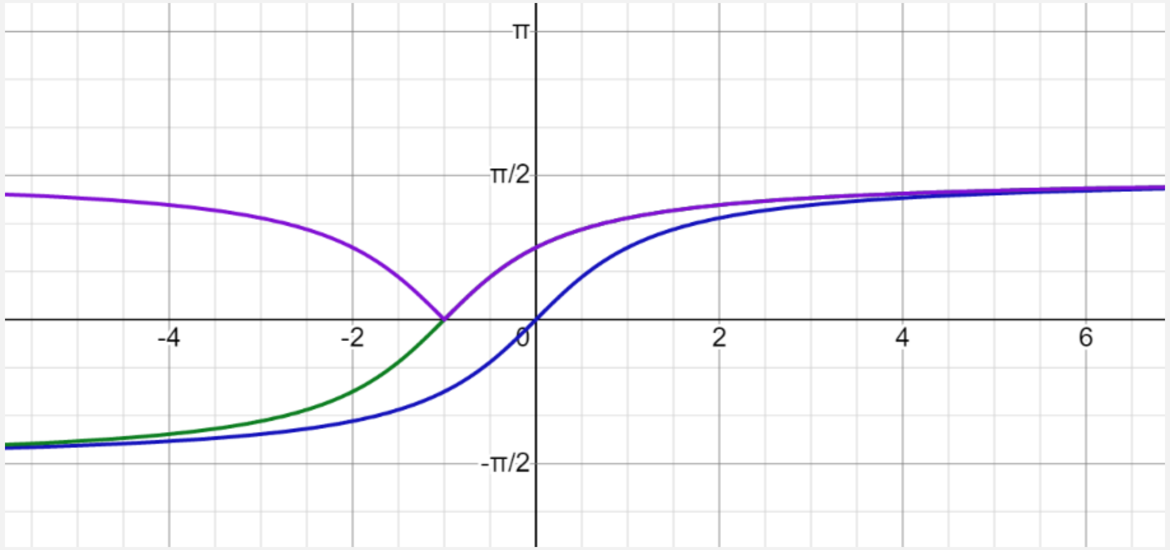


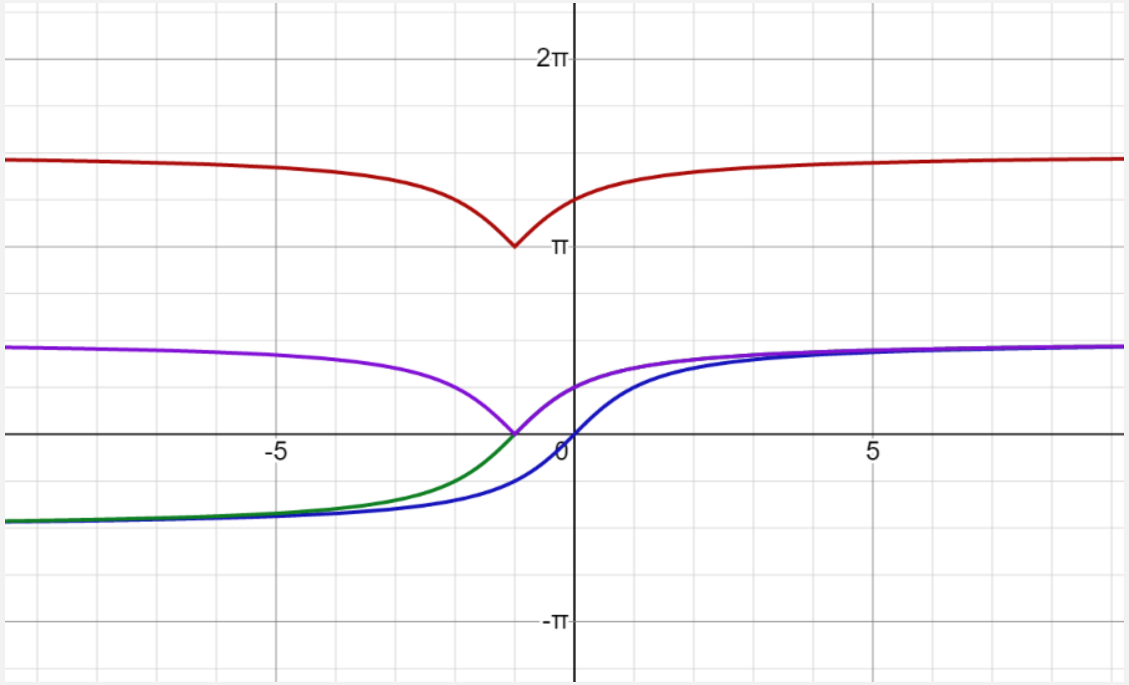
$f(x) = \arctan(x+1)$



$f(x) = |\arctan(x+1)|$

Esempio. $f(x) = |\text{arctg}(x+1)| + \pi$





$f(x) = |\arctan(x+1)|$

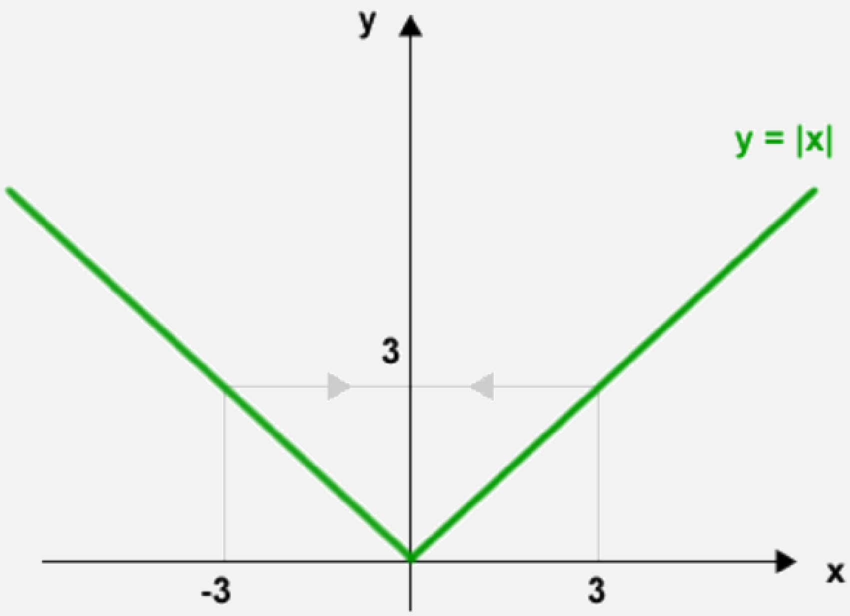
$f(x) = |\arctan(x+1)| + \pi$

- **FUNZIONI DEFINITE A TRATTI**

Caso più semplice: $f(x) = |x|$

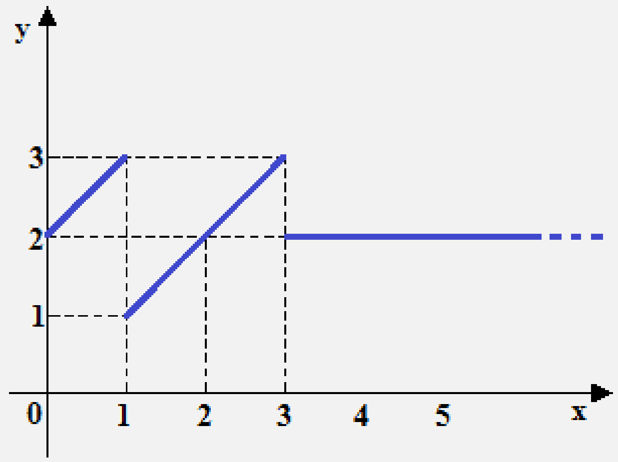
La funzione è composta da due espressioni: la prima (x) si considera solo per i valori non negativi; la seconda $(-x)$ solo per i valori negativi della x

$$y = |x| = f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

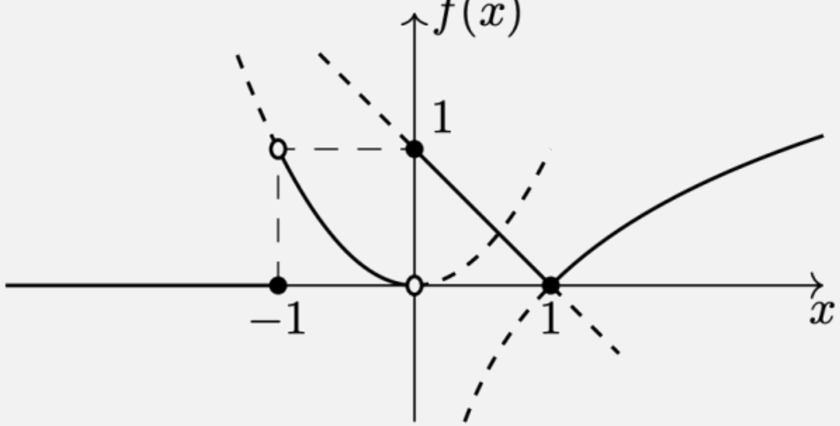


Per ogni x positiva o nulla, la funzione è crescente.
Per ogni x negativa, la funzione è decrescente.

$$f(x) = \begin{cases} x+2, & 0 \le x \le 1 \\ x, & 1 < x \le 3 \\ 2, & 3 < x \le 5 \end{cases}$$

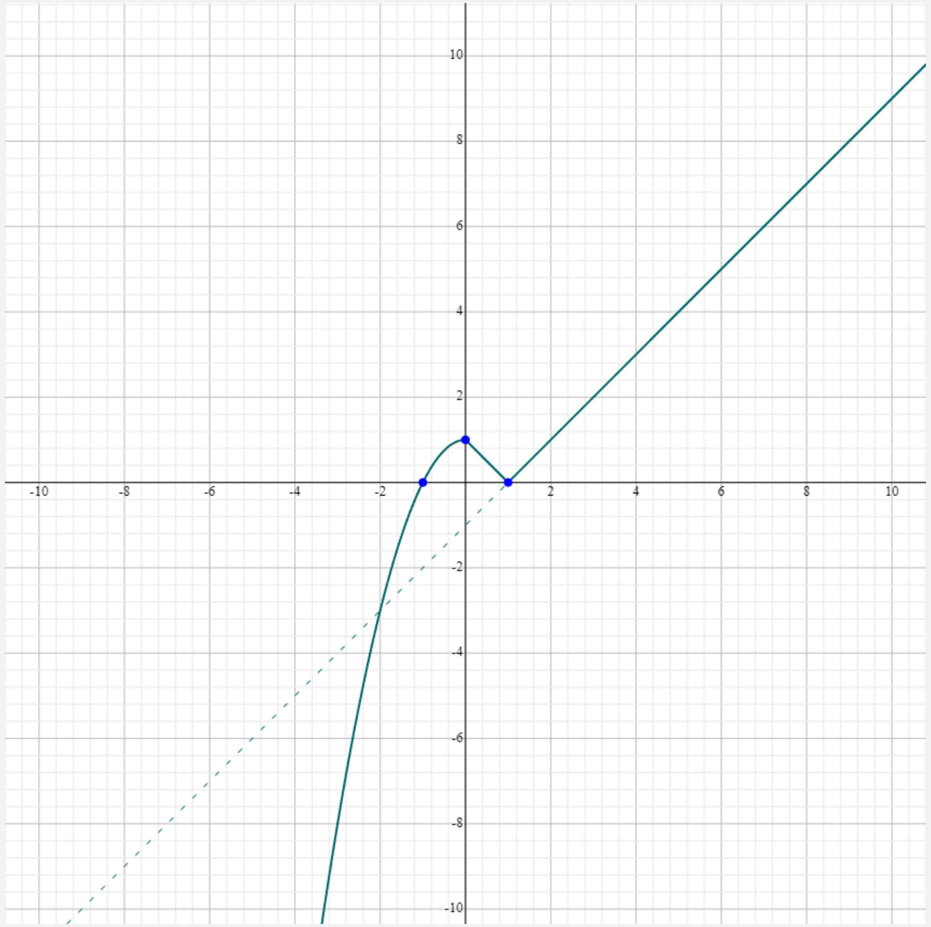


$$f(x) = \begin{cases} 0, & x \le -1 \\ x^2, & -1 < x < 0 \\ 1-x, & 0 \le x < 1 \\ \ln x, & x \ge 1 \end{cases}$$



$$f(x) = \begin{cases} 1 - x^2, & x \leq 0 \\ |x - 1|, & x > 0 \end{cases}$$

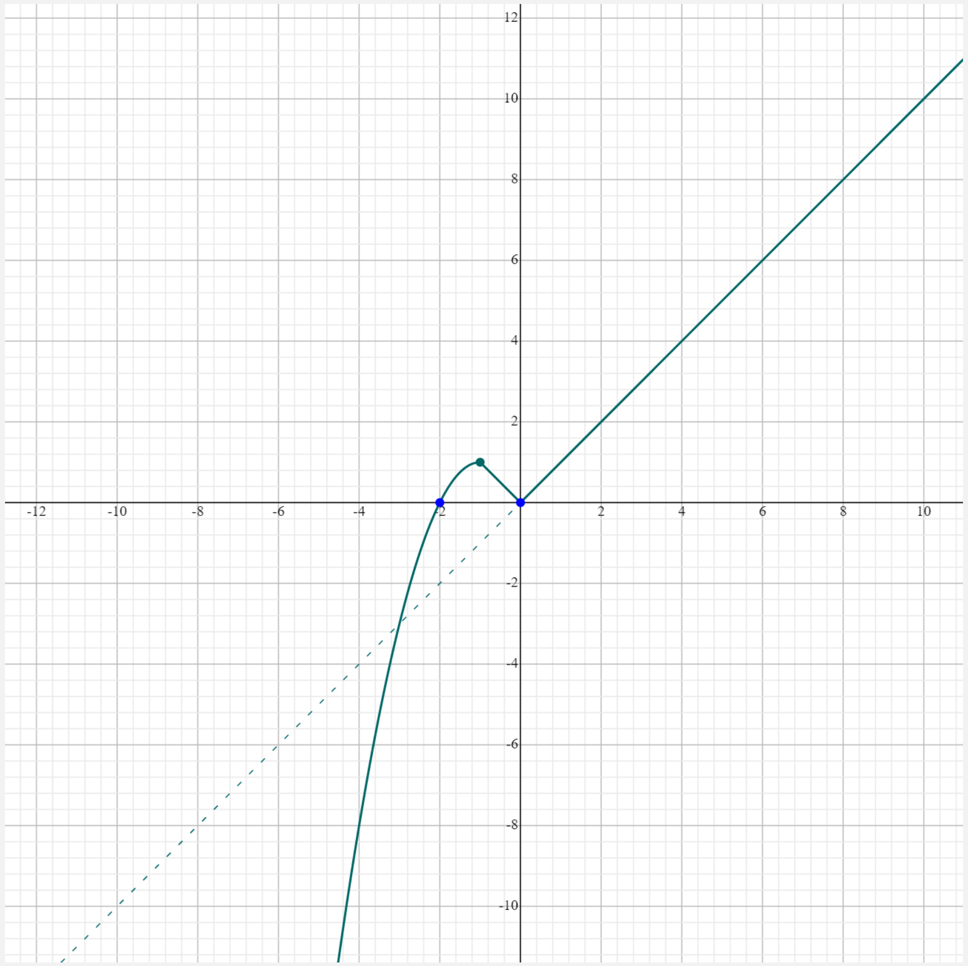
Determinare massimi/minimi assoluti/relativi per $x \in (-\infty, +4)$



- Dominio: $-\infty < x < \infty$
- Intersezioni X: $(-1,0), (1,0)$; Intersezioni Y: $(0,1)$
- Intervalli monotoni: crescente $-\infty < x < 0$; decrescente $0 < x < 1$
- Non esiste minimo assoluto
- Minimo relativo: $x = 1, y = 0$
- Massimo assoluto: $x = 4, y = 3$
- Massimo relativo: $x = 0, y = 1$

$$f(x) = \begin{cases} -x^2 - 2x, & x \leq -1 \\ |x|, & x > -1 \end{cases}$$

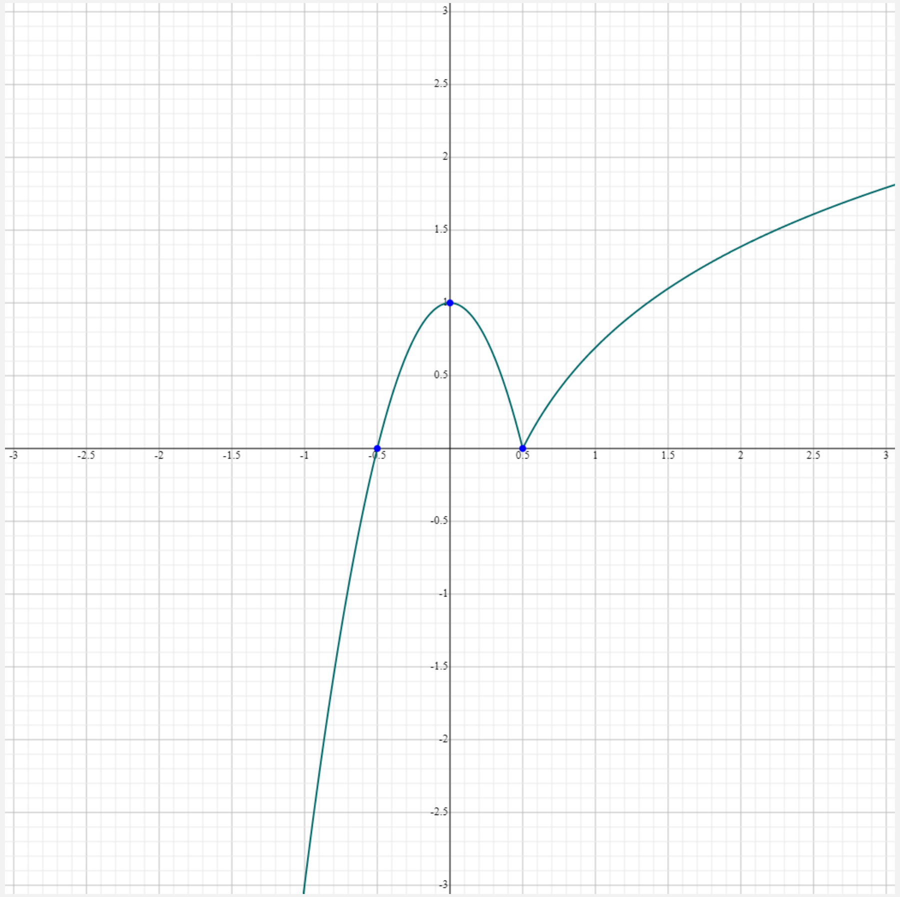
Determinare massimi/minimi assoluti/relativi per $x \in [-\infty, +3)$



- Dominio: $-\infty < x < \infty$
- Intersezioni X: $(-2,0)$, $(0,0)$; Intersezioni Y: $(0,0)$
- Intervalli monotoni: crescente $-\infty < x < -1$; decrescente $-1 < x < 0$
- Non esiste minimo assoluto
- Minimo relativo: $x = 0, y = 0$
- Massimo assoluto: $x = 3, y = 3$
- Massimo relativo: $x = -1, y = 1$

$$f(x) = \begin{cases} 1 - 4x^2, & x \leq \frac{1}{2} \\ \log_e 2x & x > \frac{1}{2} \end{cases}$$

Determinare massimi/minimi assoluti/relativi per $x \in [-1, +\infty)$



- Dominio: $-\infty < x < \infty$
- Intersezioni X: $\left(\frac{1}{2}, 0\right), (-\frac{1}{2}, 0)$; Intersezioni Y: $(0,1)$
- Intervalli monotoni: crescente $-\infty < x < 0$; decrescente $0 < x < \frac{1}{2}$
- Non esiste massimo assoluto
- Massimo relativo: $x = 0, y = 1$
- Minimo assoluto: $x = -1, y = -3$
- Minimo relativo: $x = \frac{1}{2}, y = 0$