Kinematics II — particular motions (accelerated, projectile, circular)

- Uniform rectilinear motion
- Uniformly accelerated rectilinear motion
- Falling objects
- Projectile motion
- Uniform circular motion

Special motions in the lab and the body

The standard motions of physics show up throughout biotechnology and physiology.

Uniformly accelerated motion + drag → a particle settles at a terminal velocity (sedimentation).

Projectile motion → charged droplets are deflected to sort cells in a flow cytometer.

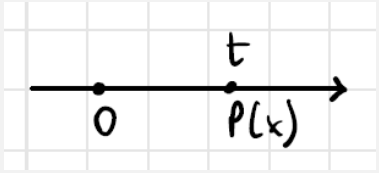
Uniform circular motion → the spinning rotor of a centrifuge, and blood in a curved vessel.

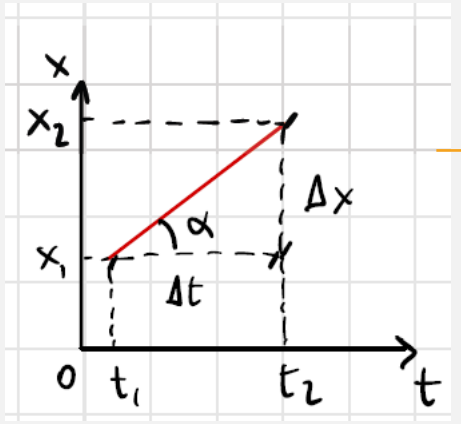
Free fall and constant acceleration also describe the physics of falling and impact.

UNIFORM RECTILINEAR MOTION

Motion of a particle with constant velocity: $\vec{v} = cost$

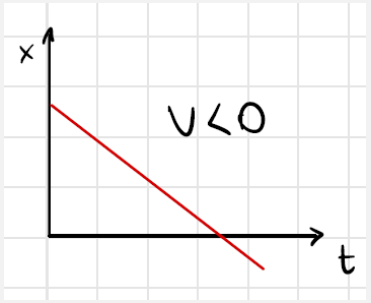
At any point, the acceleration is zero: $\vec{a} = 0$





$$v = \frac{\Delta x}{\Delta t} = \tan\alpha > 0$$

The slope of the straight line represents the velocity of the particle



Let us consider t_1 the initial instant ($t_1 = 0$) and t_2 a generic instant ($t_2 = t$), x_0 the abscissa of the initial position and x_2 a generic position ($x_2 = x$)

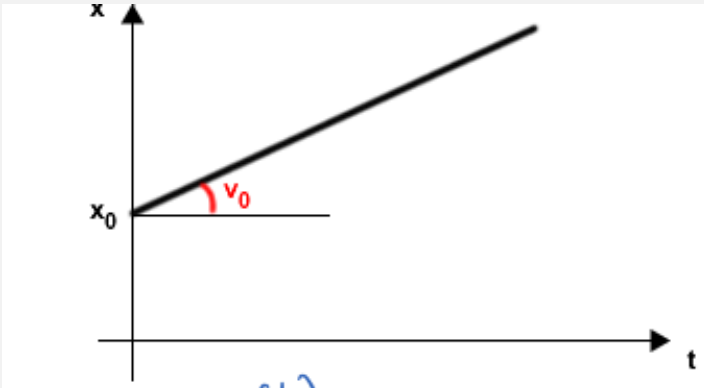
$x - x_0 = vt \rightarrow \Delta x = v\Delta t$

Position equation of uniform rectilinear motion

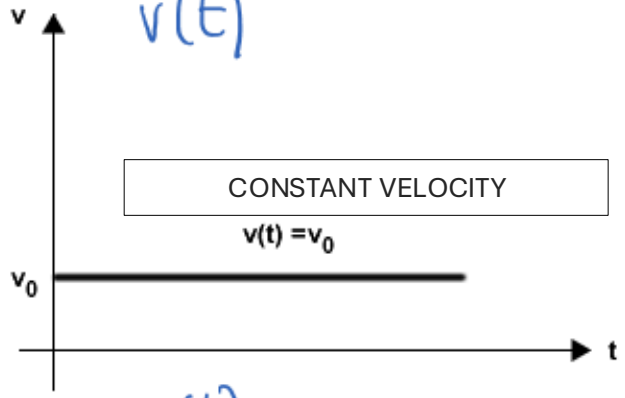
$$x = x_0 + vt$$

$$\Delta x = v\Delta t$$

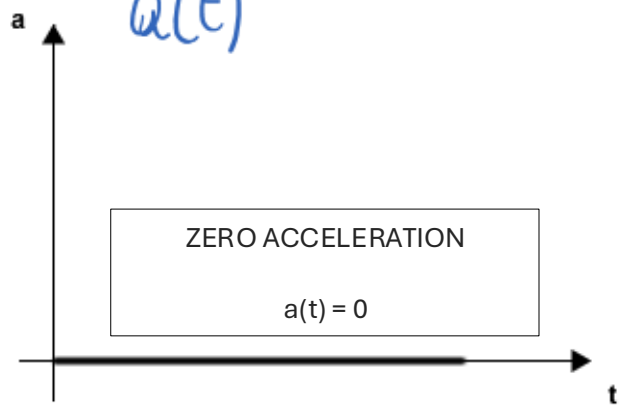
x(t)



v(t)

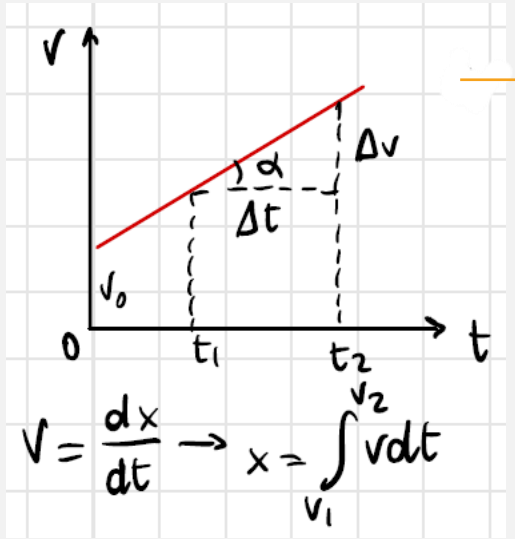


a(t)



UNIFORMLY ACCELERATED RECTILINEAR MOTION

Motion of a particle with constant acceleration: $\vec{a} = cost$



$a = \tan\alpha > 0$ → accelerated motion

The slope of the straight line represents the particle's velocity

First fundamental equation:

$$a_m = a = \frac{v - v_0}{t - 0}$$

↳ $t_0 = 0$

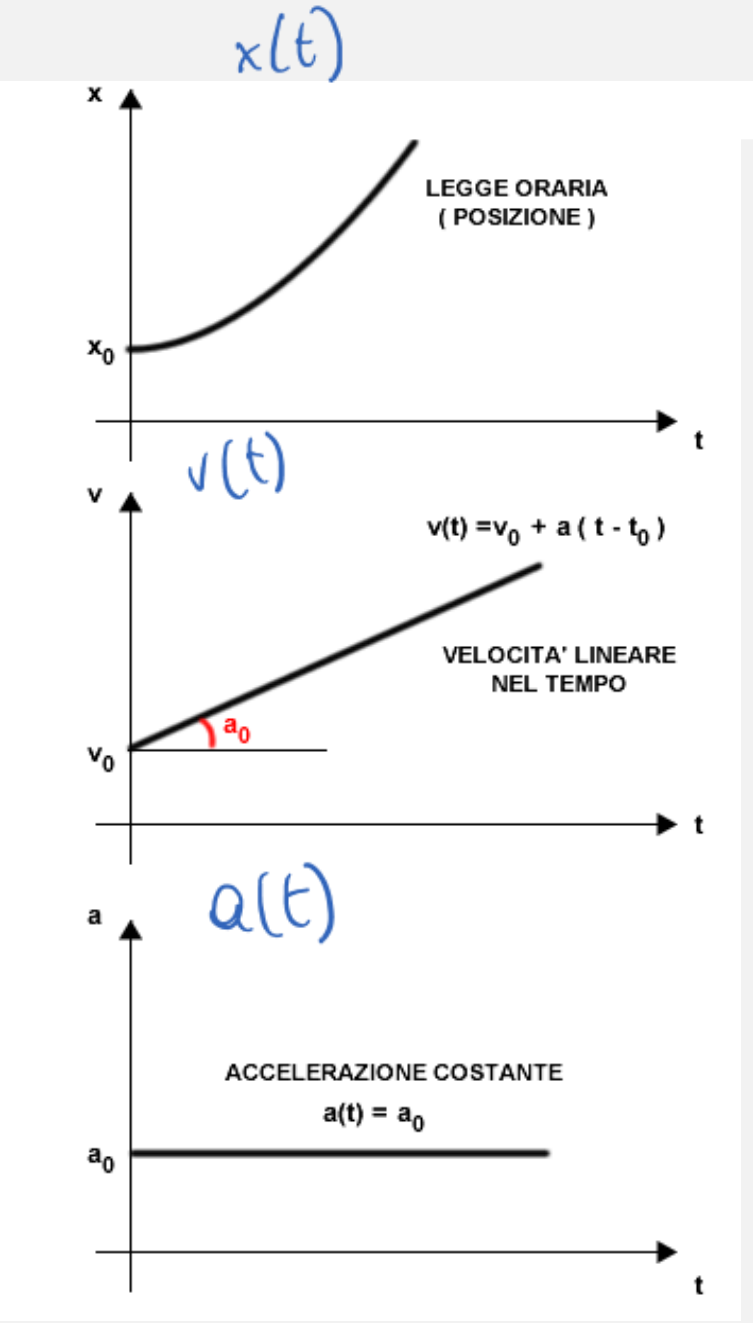
First kinematics equation

$$v = v_0 + at$$

Second fundamental equation:

$$v_m = \frac{x - x_0}{t - 0}$$

$$x = x_0 + v_m t$$



UNIFORMLY ACCELERATED RECTILINEAR MOTION

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \rightarrow \Delta v = a\Delta t$$

$$\Delta v = v_2 - v_1 \rightarrow v_2 = v_1 + \Delta v = v_1 + a\Delta t$$

$$\Delta v = v - v_0 \rightarrow v = v_0 + \Delta v = v_0 + a\Delta t$$

A body moving at velocity v_1 at time t_1 and is accelerated with constant a, it will reach velocity $v_2 = v_1 + a(t_2 - t_1)$ at time t_2

$$v_m = \frac{v_0 + v}{2}$$

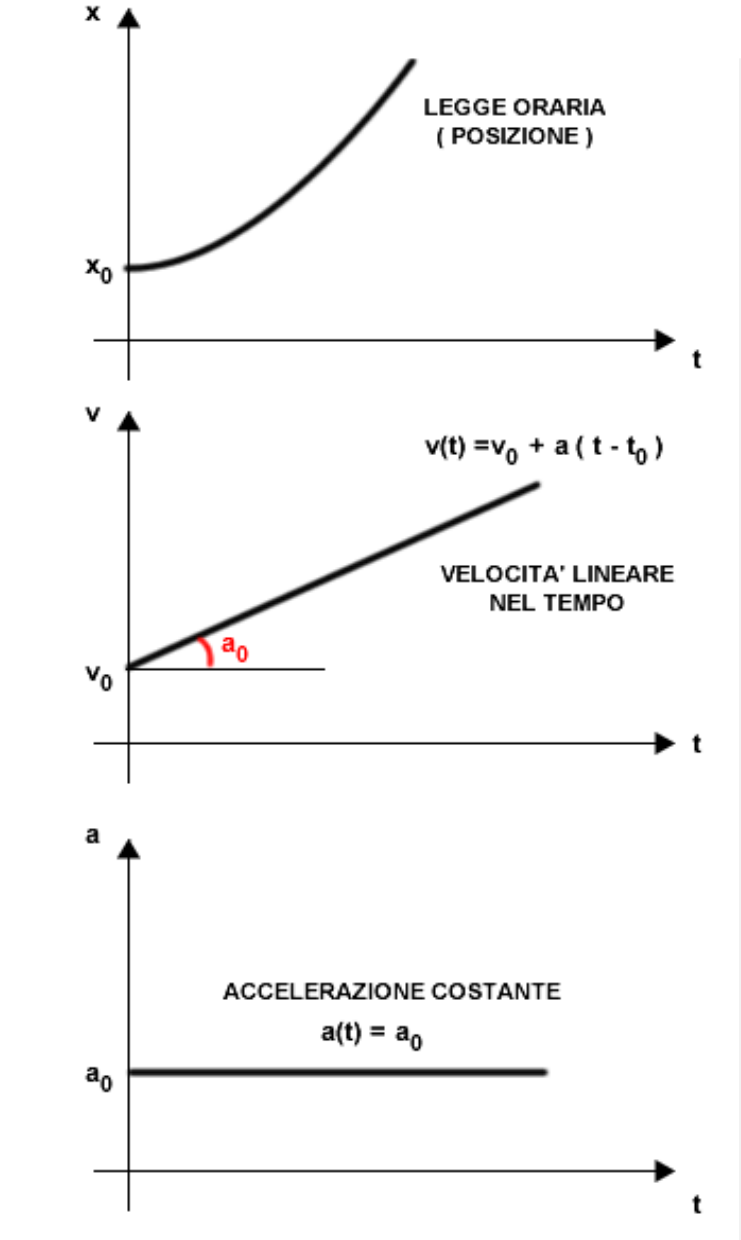
$$\Delta x = v_m \Delta t = \frac{1}{2}(v_0 + v)(t - t_0) = \frac{1}{2}(v_0 + v)\Delta t$$

$$v = v_0 + a\Delta t \rightarrow \Delta x = \frac{1}{2}(v_0 + v_0 + a\Delta t)\Delta t = \frac{1}{2}(2v_0\Delta t) + \frac{1}{2}a\Delta t^2$$

Second equation of kinematics

$$\Delta x = v_0\Delta t + \frac{1}{2}a\Delta t^2$$

$$x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$$



UNIFORMLY ACCELERATED RECTILINEAR MOTION

Important mathematical relations

$$\Delta v = v_f - v_i = a\Delta t$$

Fourth kinematics equation

$$\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$$

$$\left(\frac{v_f + v_i}{2}\right)\Delta t$$

$$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$x_f - x_i$

$$x = x_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

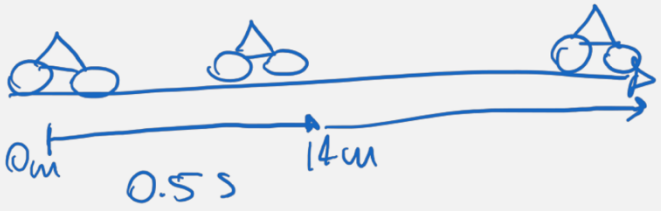
Third kinematics equation

$$v_f^2 - v_i^2 = 2a\Delta x$$

SPECIAL MOTIONS

Example

Braking distance. Calculate the total stopping distance for an initial speed of 28 m/s and assuming that the acceleration of an automobile is $a=-6.0\ m/s^2$. Assume that the reaction time is 0.50 s.



$v = \text{cost} = 28\ m/s$
$a = 0\ m/s^2$
$t = 0.5\ s$

$a = \text{cost} = -6\ m/s^2$
$v_i = 28\ m/s$
$v_f = 0\ m/s$

① $t = 0.5\ s$
$v_i = 28\ m/s$
$v_f = 28\ m/s$
$a = 0\ m/s^2$
$x_i = 0\ m$

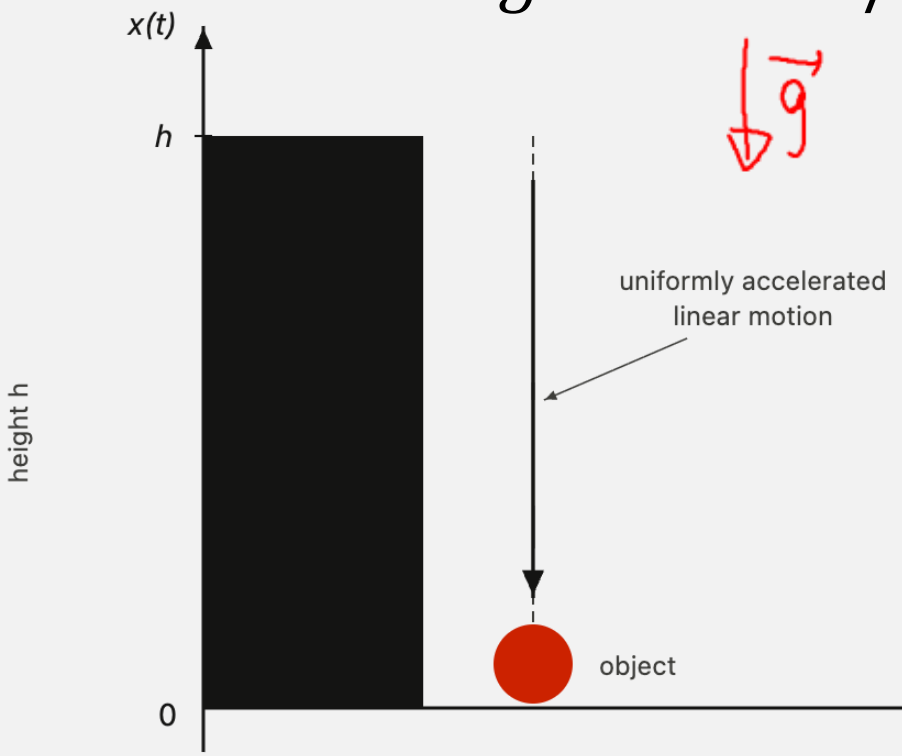
$\rightarrow \Delta x = v\Delta t$
$x_f - x_i = v(t_f - t_i)$
$x_f = v \cdot t_f$
$x_f = 28\ m/s \cdot 0.5\ s = 14\ m$

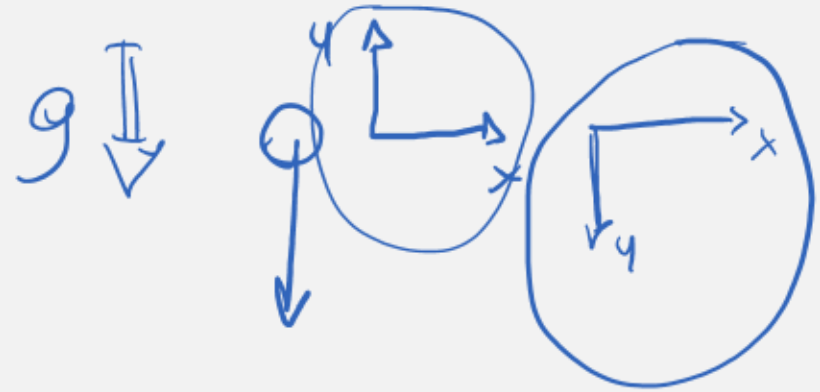
② x_f?
$x_i = 14\ m$
$v_i = 28\ m/s$
$v_f = 0\ m/s$
$a = -6\ m/s^2$

$\rightarrow v_f^2 - v_i^2 = 2a\Delta x$
$v_f^2 - v_i^2 = 2a(x_f - x_i) \rightarrow$
$x_f - x_i = \frac{v_f^2 - v_i^2}{2a} \rightarrow x_f = x_i + \frac{v_f^2 - v_i^2}{2a}$
$= 14\ m + \frac{0\ m^2/s^2 - (28\ m/s)^2}{2(-6\ m/s^2)}$
$\leftarrow$ 49 m

FALLING OBJECTS

$$\vec{g} = 9.80\, m/s^2$$





$$\Delta h = h_f - h_i = v_i \Delta t + \frac{1}{2} g \Delta t^2$$

$$h_f - h_i = v_i (t_f - t_i) + \frac{1}{2} g (t_f - t_i)^2$$

1

$$h_f = h_i + v_i t_f + \frac{1}{2} g t_f^2$$

$$v_f^2 - v_i^2 = 2g\Delta h \longrightarrow v_f^2 = v_i^2 + 2g(h_f - h_i) \longrightarrow$$

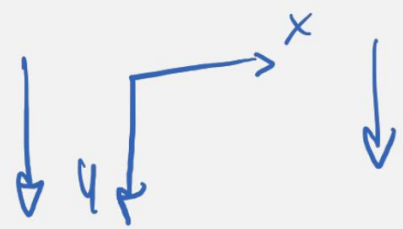
2

$$v_f = \sqrt{v_i^2 + 2g(h_f - h_i)}$$

FALLING OBJECTS

Example

Suppose that a ball is dropped from a tower 70 m high. How far will it have fallen after 1 s, 2 s, and 3 s? Assume h is positive downward and neglect air resistance.

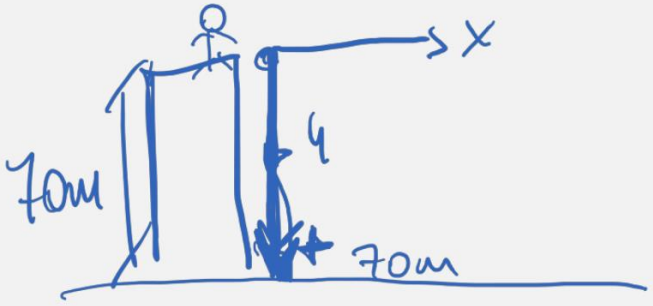


$h_i = 0\,m$

$h_f = 70\,m$

$v_i = 0\,m/s$

$g = 9.8\,m/s^2$



$t_1 = 1\,s$

$t_2 = 2\,s$

$t_3 = 3\,s$

(1) $h_f = \cancel{h_i} + \cancel{v_i t} + \frac{1}{2} g t^2 = \frac{1}{2} \cdot 9.8\,m/s^2 \cdot 1\,s^2 = 4.9\,m$

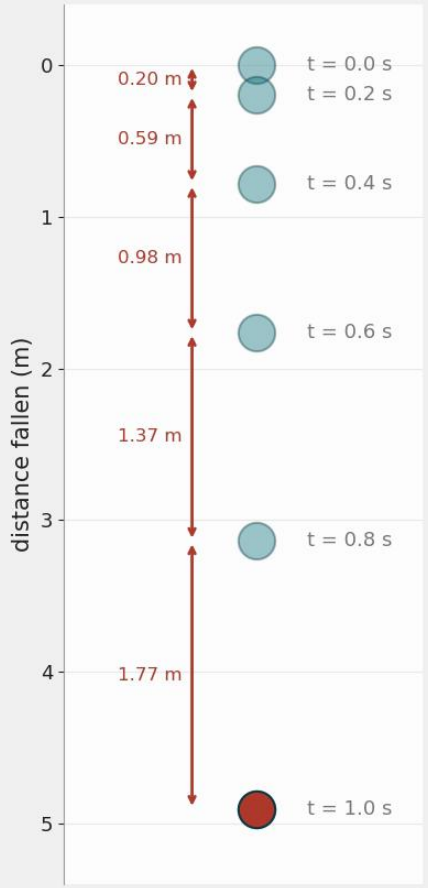
(2) $h_f = \frac{1}{2} \cdot 9.8\,m/s^2 \cdot (2s)^2 = 19.6\,m$

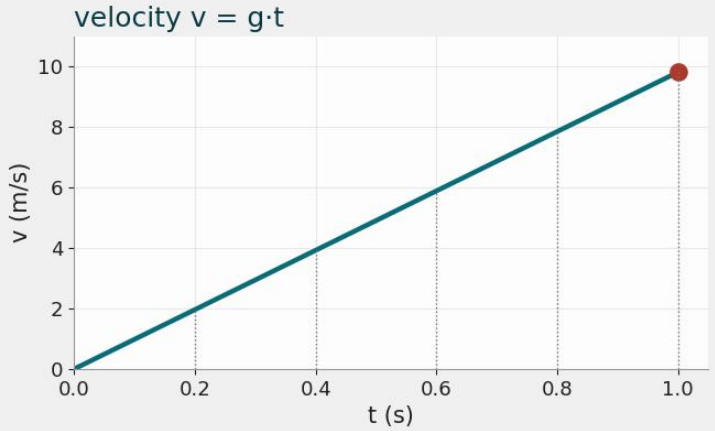
(3) $h_f = \frac{1}{2} \cdot 9.8\,m/s^2 \cdot (3s)^2 = 44.1\,m$

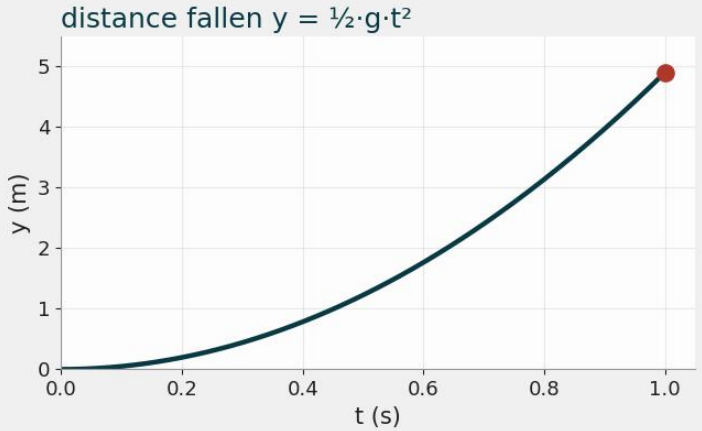
Animation — free fall and accelerated motion

Uniformly accelerated motion: free fall

A ball falls from rest with constant acceleration a = g = 9.81 m/s² (snapshots every 0.2 s)







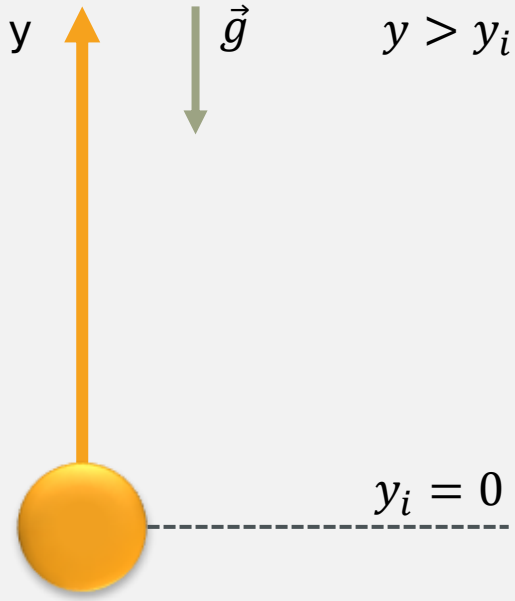
What we observe

Δv = g·Δt = 1.96 m/s
every 0.2 s

t = 1.00 s
v = 9.81 m/s
y = 4.91 m

Distances in equal intervals
are in the ratio 1 : 3 : 5 : 7 : 9

OBJECTS THROWN UPWARD



$$y_i = 0$$
$$t_i = 0 \rightarrow t - t_i = t$$
$$v_f = v_{y\,max} = 0$$

$$v_f - v_i = at = -gt$$
$$y = y_i + v_i t + \frac{1}{2}at^2 = v_i t - \frac{1}{2}gt^2$$

$$v_f = 0 \rightarrow 0 - v_i = -gt$$

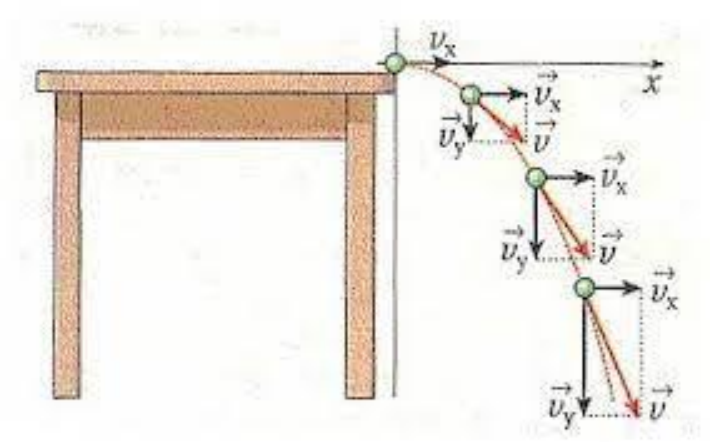
→ I determine *ts* such that *ts=vig*

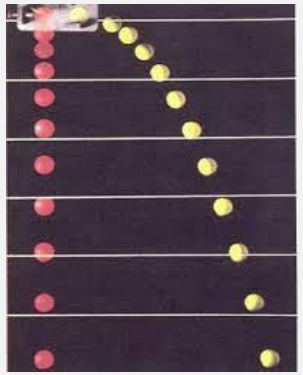
$$y(t_s) = v_i t_s - \frac{1}{2}gt_s^2 \rightarrow y(t_s) = v_i \frac{v_i}{g} - \frac{1}{2}g\frac{v_i^2}{g^2} = \frac{v_i^2}{g} - \frac{1}{2}\frac{v_i^2}{g} = \frac{1}{2}\frac{v_i^2}{g}$$

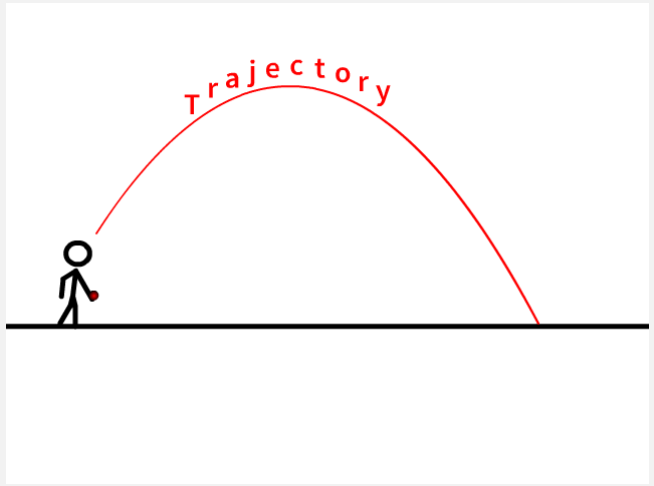
→ Substituting back in *ts* the expression for *vi* as a function of *ymax*, I obtain:

$$v_i^2 = 2gy_{max} \rightarrow t_s = \frac{\sqrt{2gy_{max}}}{g} = \sqrt{\frac{2y_{max}}{g}}$$

PARABOLIC (OR PROJECTILE) MOTION







$$\Delta v = v_f - v_i = a\Delta t$$

$$\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$$

$$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$v_f^2 - v_i^2 = 2a\Delta x$$

AXIS x → $a_x = 0$

$$\Delta v_x = 0 \ (v_x = cost)$$

$$\Delta x = v_x\Delta t$$

AXIS y → $a_y = cost$

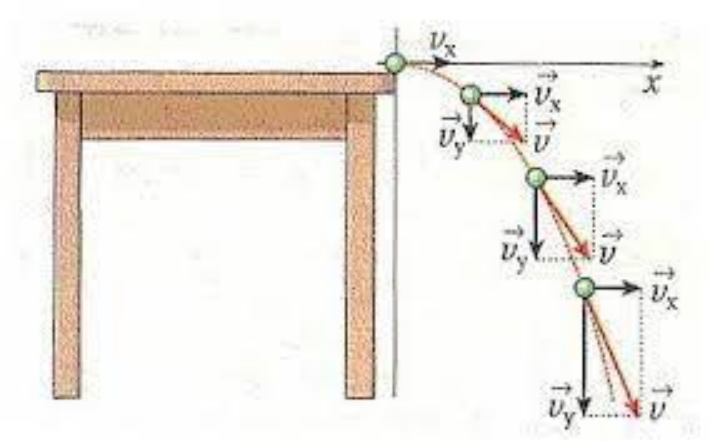
$$\Delta v_y = a_y\Delta t = -g\Delta t$$

$$\Delta y = \frac{1}{2}(v_{fy} + v_{iy})\Delta t$$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$v_{fy}^2 - v_{iy}^2 = 2a_y\Delta y$$

PARABOLIC MOTION



$\vec{a} = \vec{g}$

y

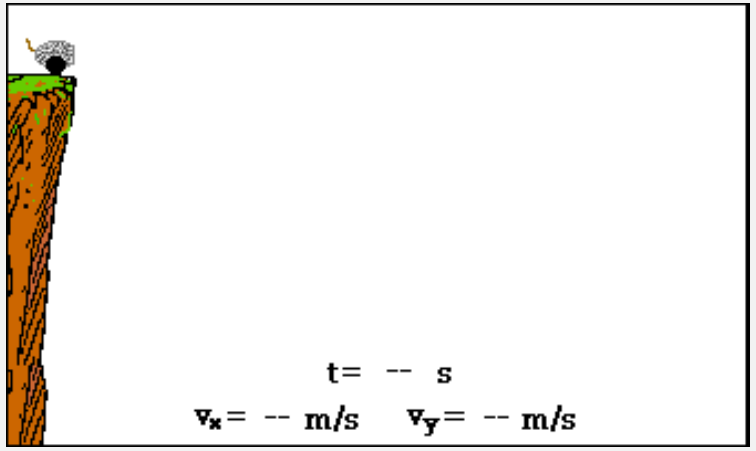
x

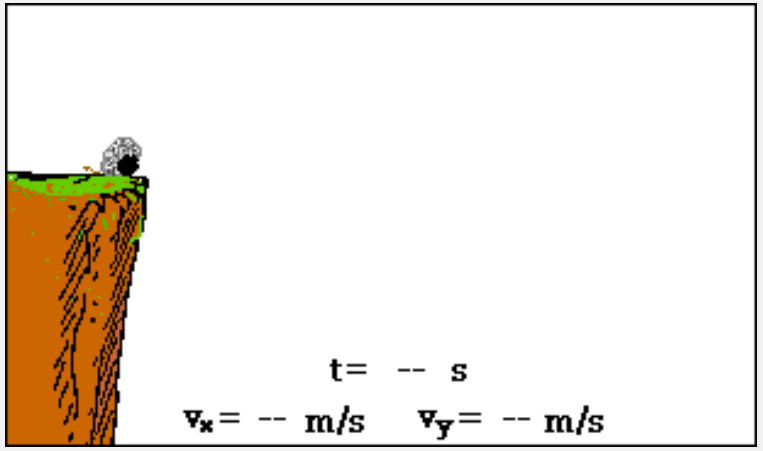
The ball leaves the table ($t = 0$): the ball is subject to gravitational acceleration:

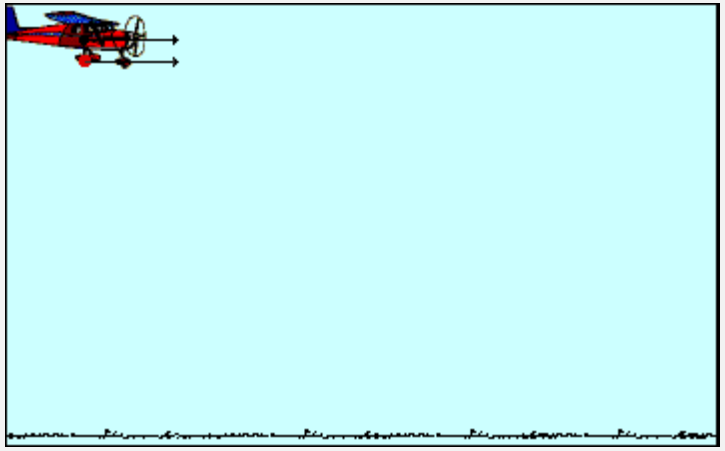
- v_y is initially 0, then increases
- $a_y = g$

$$\Delta v_y = a_y \Delta t \rightarrow v_y = -gt$$

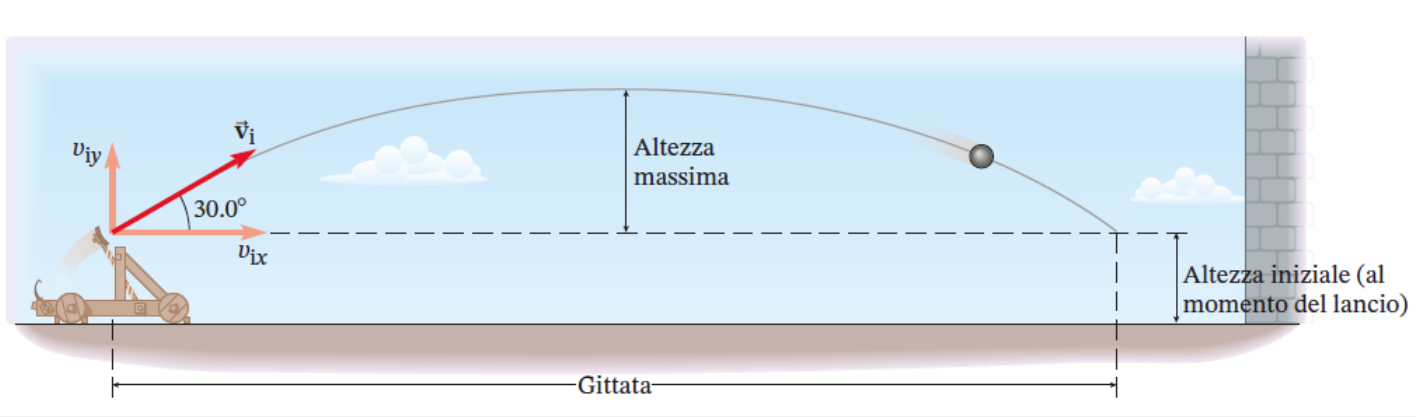
Vertical displacement (y) → $\Delta y = v_{iy}\Delta t + \frac{1}{2} a_y (\Delta t)^2 \rightarrow \Delta y = -\frac{1}{2} g (\Delta t)^2$ (setting y_i=0)

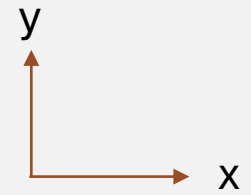






PARABOLIC MOTION

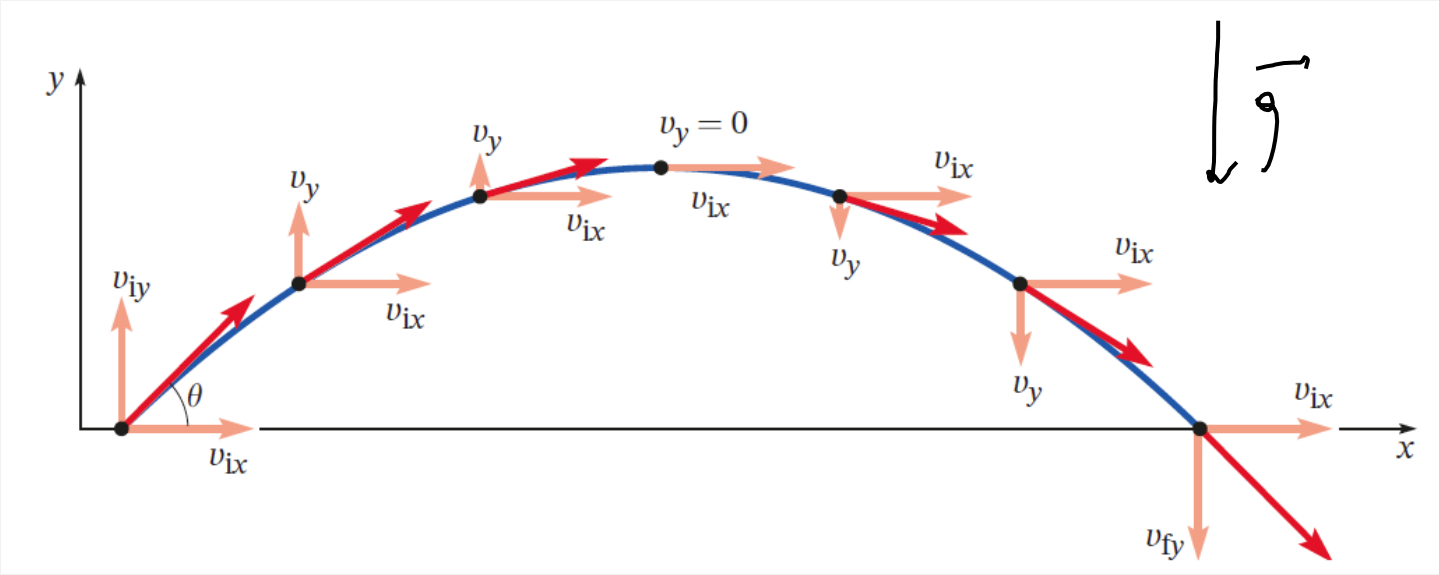




Here the velocity has a vertical component v_{iy}

The body is launched with an initial velocity $\vec{v_i}$ → this forms an angle θ with the horizontal axis:

$$v_{ix} = v_i cos\theta \text{ and } v_{iy} = v_i sin\theta$$

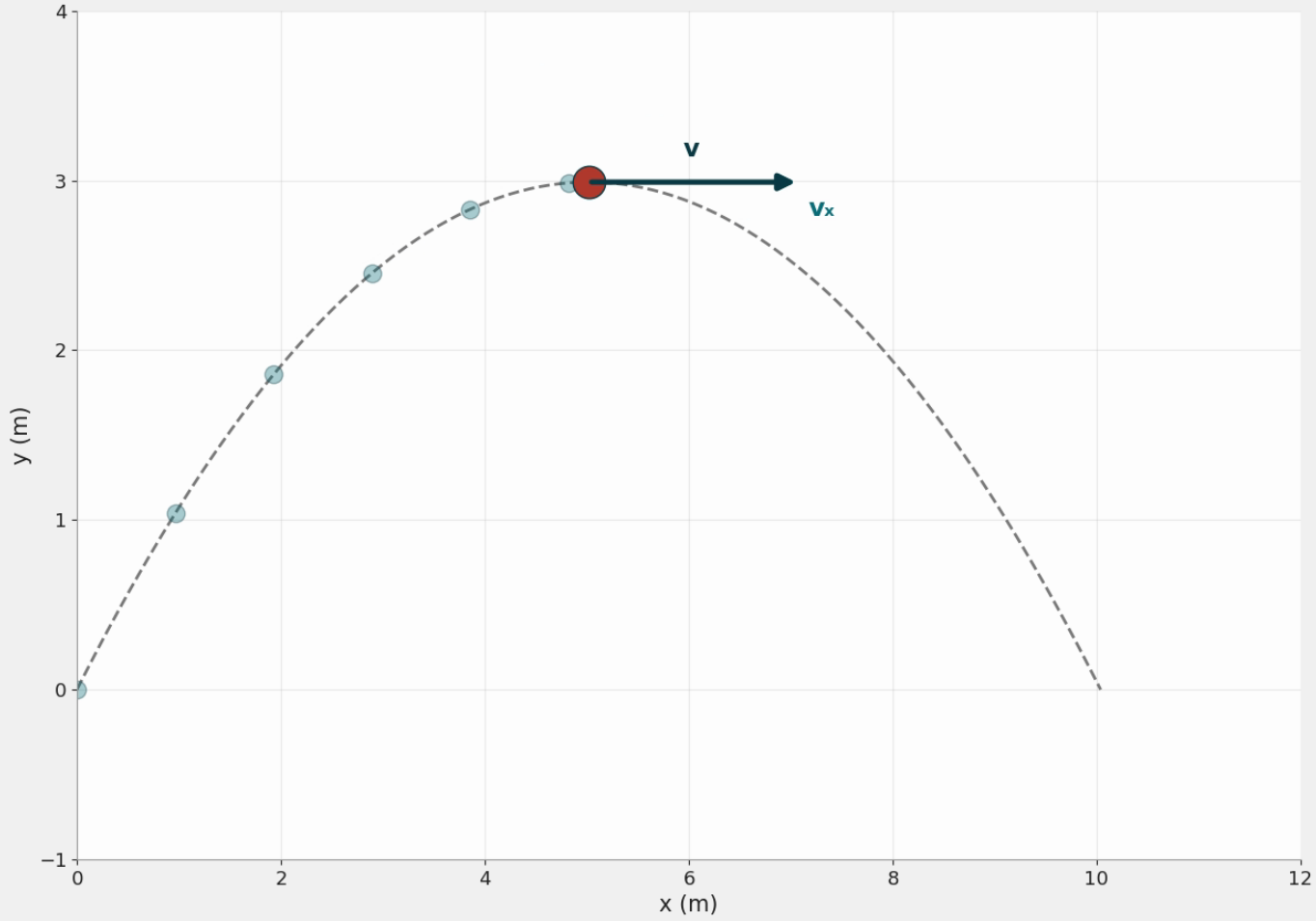


The angle θ is called the angle of elevation.
During the flight, the only force acting is the weight force: a_x=0 and v_x=const
Vy is initially positive, decreases during the ascent, becomes zero at the maximum height, and finally increases in magnitude during the descent, but with a negative direction.

Animation — projectile motion

Projectile motion: two independent motions

Horizontal: uniform motion. Vertical: uniformly accelerated motion with a = −g



Oblique launch

v_0 = 10 m/s, θ = 50°

$v_x = v_0 \cos\theta$ = 6.43 m/s (constant)

$v_{y0} = v_0 \sin\theta$ = 7.66 m/s

t = 0.78 s

$v_y = v_{y0} - gt$ = 0.00 m/s

At the apex $v_y = 0$:

$y_{max} = v_{y0}^2/2g$ = 3.0 m

Example — projectile motion sorts cells (flow cytometry)

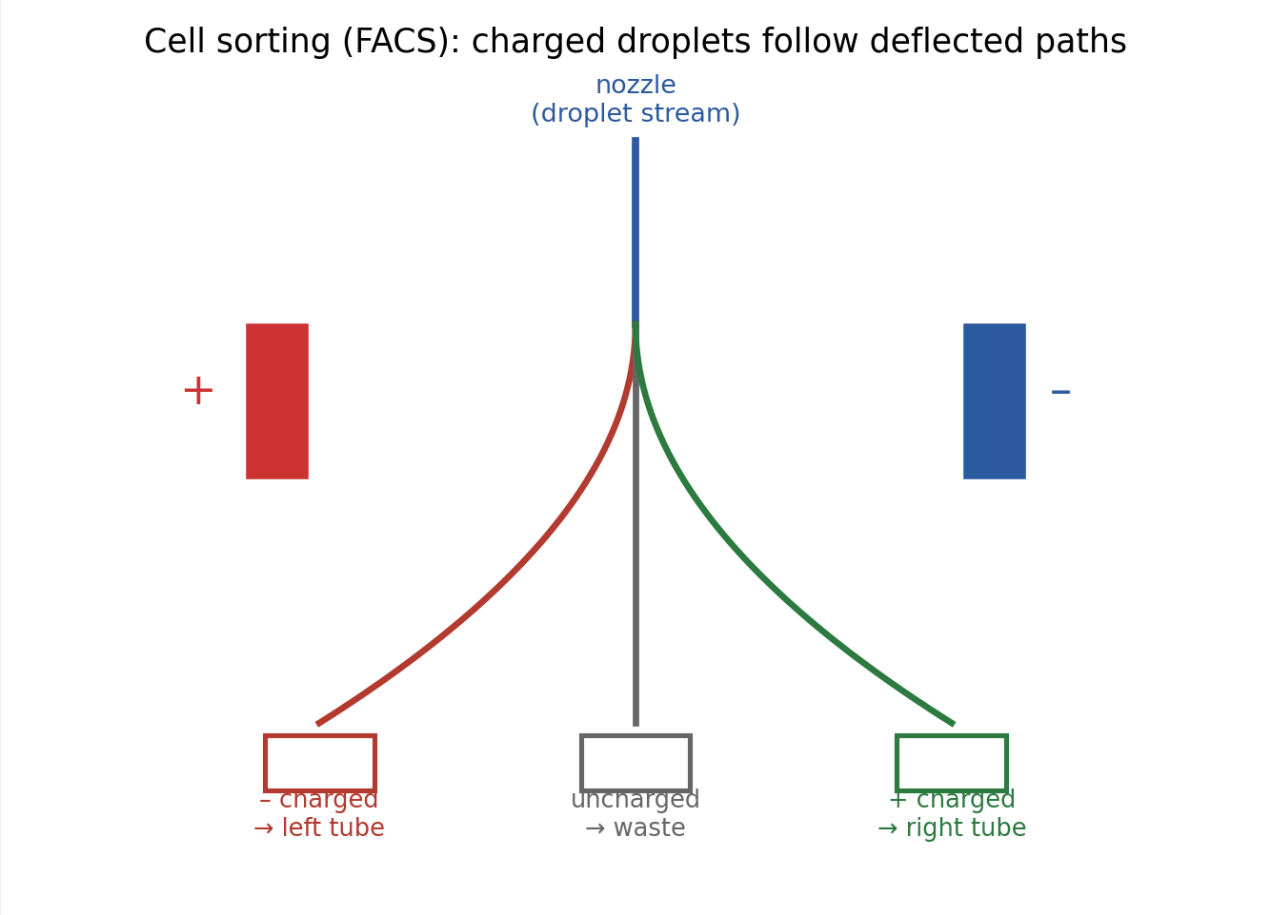
In a cell sorter (FACS) the sample breaks into tiny charged droplets, each carrying one cell.

Between charged plates each droplet follows a projectile path — sideways deflection plus vertical fall:

the electric force sets a sideways acceleration; inertia carries it downward

Droplets land in different tubes according to their charge — sorting cells one by one.

The same projectile equations describe inkjet bioprinting and droplet dispensing.



In practice — a real cell sorter (FACS, Fluorescence-Activated Cell Sorting)

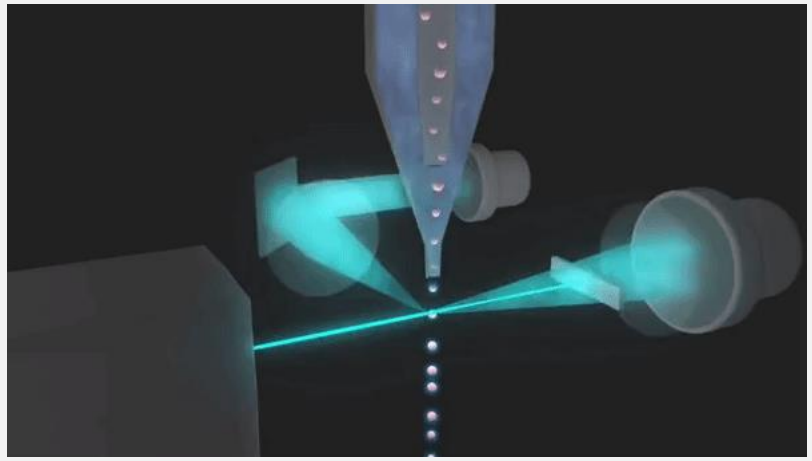
The stream breaks into tens of thousands of droplets per second, each ≤ one cell.

A laser reads every cell; selected droplets are charged just before the plates.

sideways deflection ∝ (charge × field) × time spent between the plates

Modern sorters purify millions of cells by type — for research and cell therapy.

The same droplet physics underlies inkjet and 3-D bioprinting.





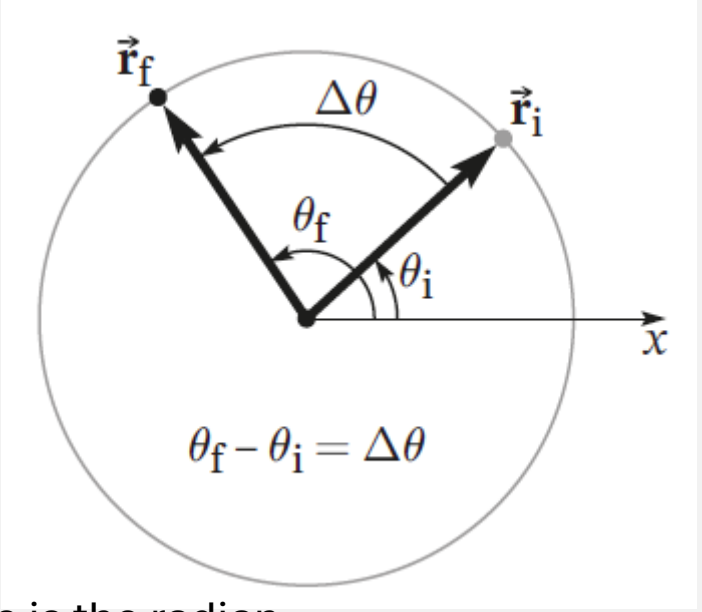
Cell sorter (MoFlo) — CC BY-SA 3.0

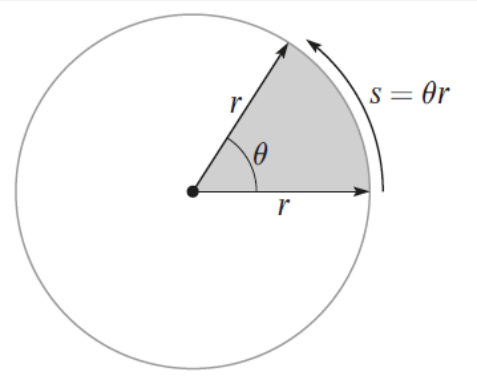
UNIFORM CIRCULAR MOTION

Definition of angular displacement: $\Delta\theta = \theta_f - \theta_i$

Definition of average angular velocity: $\omega_m = \frac{\Delta\theta}{\Delta t}$

Definition of instantaneous angular velocity: $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$



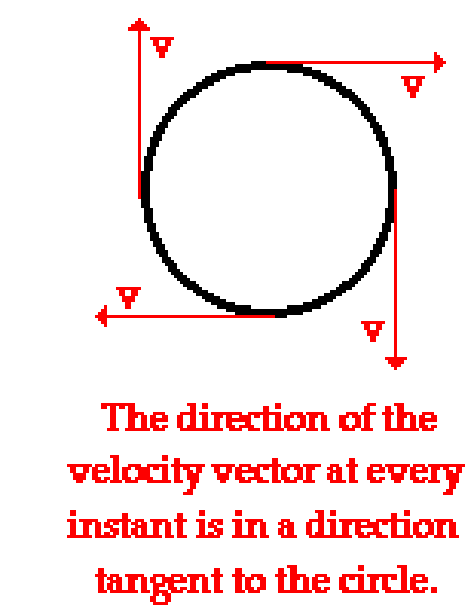


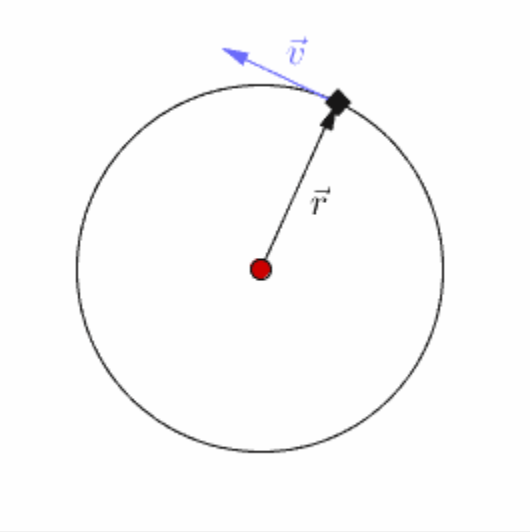
In many situations the most convenient measure of angle is the radian.

$\theta(in\ radian) = \frac{s}{r}$ $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi rad$ $360° = 2\pi rad$

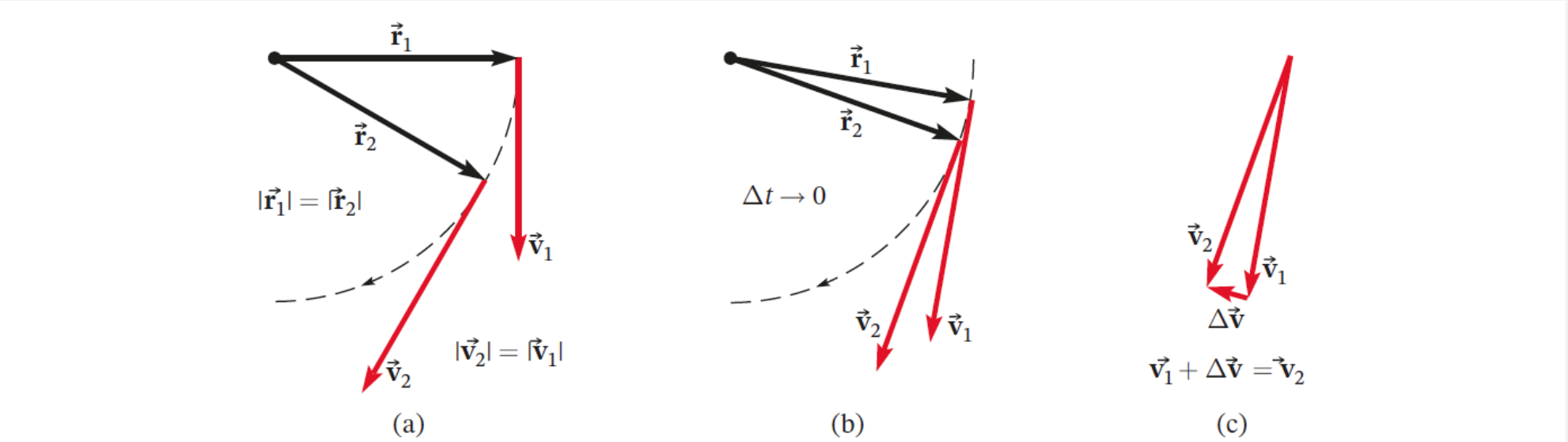
A particle moving with uniform circular motion has a velocity with constant magnitude, while its direction changes over time: at every instant of time, the direction of the instantaneous velocity is in fact tangent to the circular trajectory

Since the direction of the particle's velocity changes continuously,
the particle must possess a non-zero acceleration





UNIFORM CIRCULAR MOTION



Portion of a circular trajectory of radius r, along which a point-like body moves with uniform circular motion. At the points considered, the two velocity vectors are tangent to the trajectory and have the same magnitude

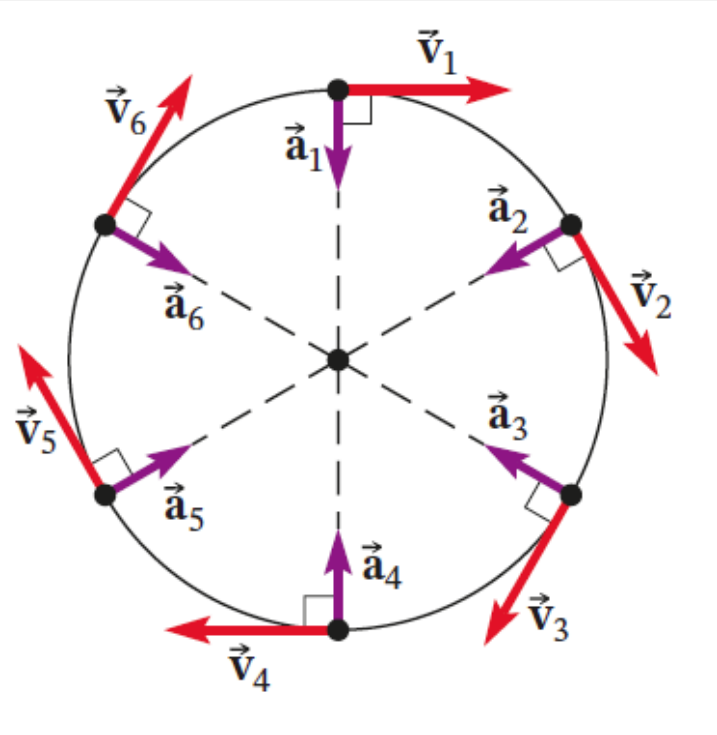
As the time interval between the two positions considered becomes progressively smaller, the two position vectors become increasingly close

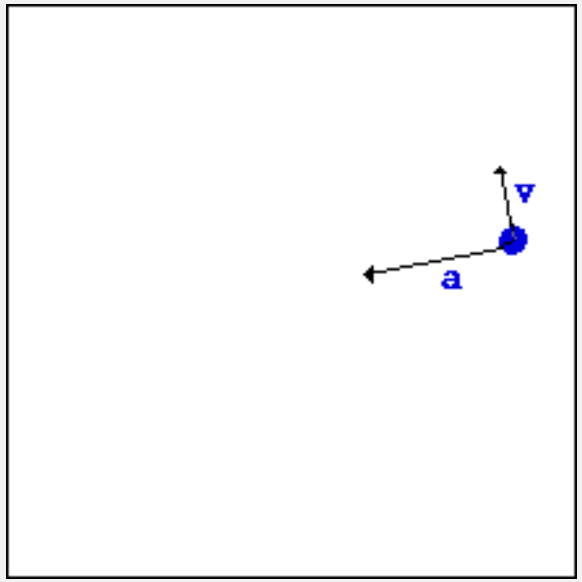
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

We determine the change in velocity Δv over a small time interval. When $\Delta t \to 0$, the angle between the two vectors tends to 0 and Δv becomes perpendicular to the velocity itself. Δv is directed along the radius of the circumference and oriented toward the center

UNIFORM CIRCULAR MOTION

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$



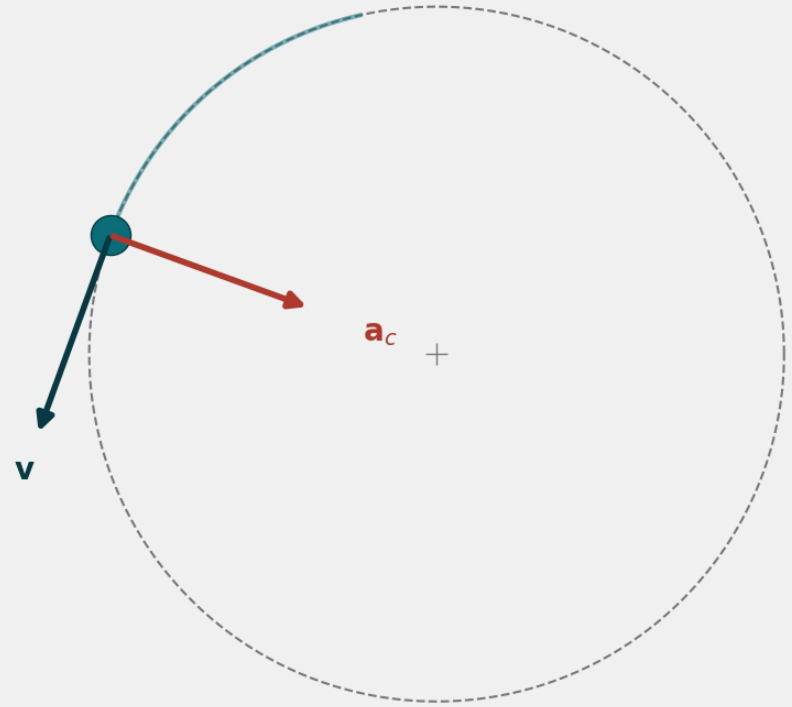


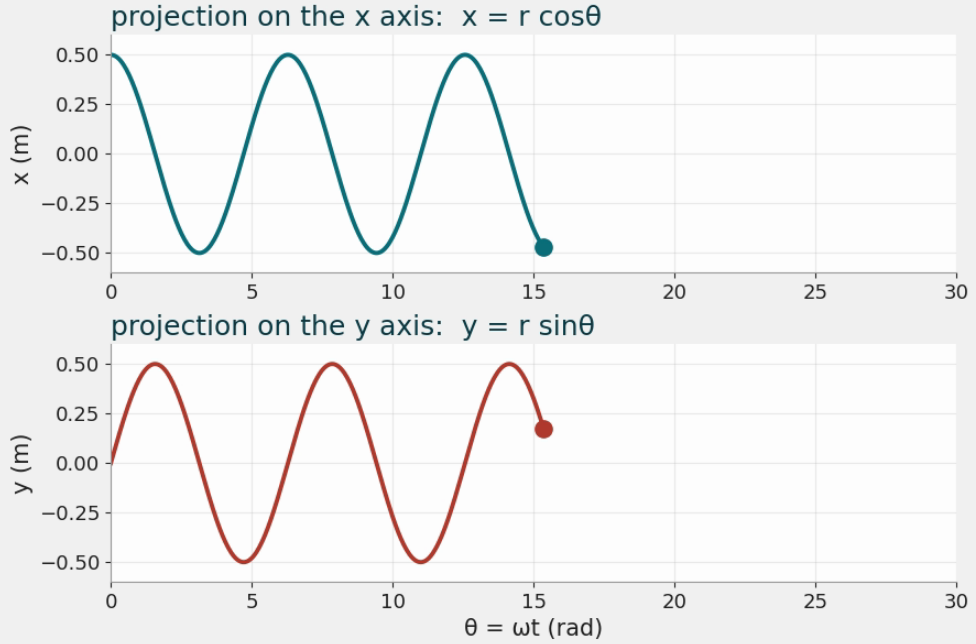
a has the same direction and orientation as Δv → therefore, the acceleration will also be radially directed toward the center of the circumference:
The acceleration of a body moving with uniform circular motion is called radial acceleration $\overrightarrow{a_r}$ or centripetal acceleration

Animation — uniform circular motion

Uniform circular motion

A particle moves on a circle of radius r = 0.50 m with ω = 2.0 rad/s





v changes direction:

there is an acceleration towards the centre

$a_c = v^2/r = \omega^2 r$ **= 2.0 m/s²**

v and a_c are always perpendicular:
a_c does not change the magnitude of v

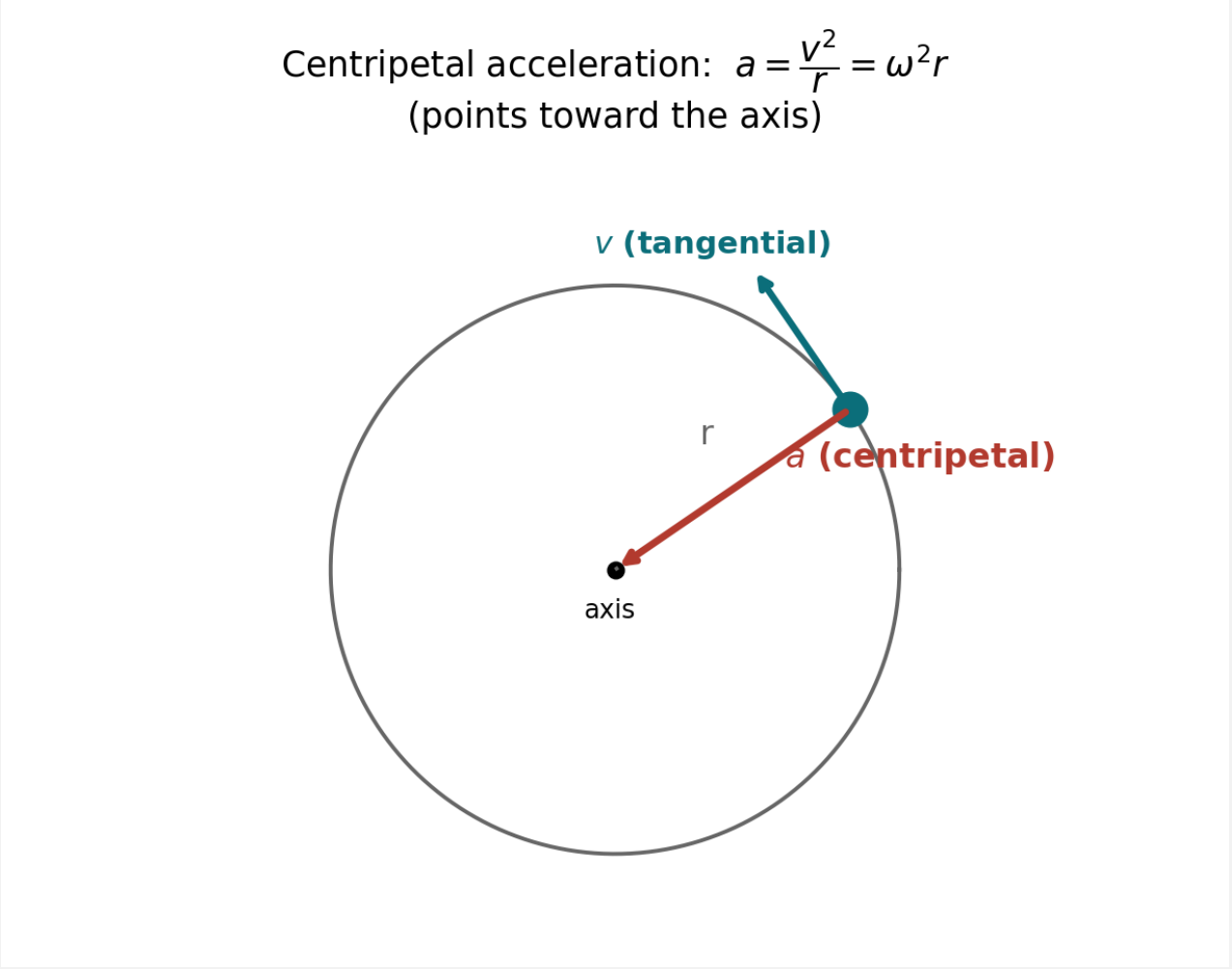
Example — centripetal acceleration in a centrifuge

A sample spun in a centrifuge moves in a circle at constant speed — yet it is accelerating.

Its velocity constantly changes direction, giving an acceleration toward the axis:

$$a = v^2/r = \omega^2 r$$

This large acceleration (thousands of × g) drives particles to sediment by density.



In practice — the g-force in a centrifuge

A microcentrifuge at 13 000 rpm with rotor radius r = 9 cm:

$$\omega = 2\pi \cdot 13000/60 \approx 1361 rad/s; a = \omega^2 r \approx 1,7\times10^5 m/s^2$$

that is

$$a/g \approx 1,7\times10^5/9,81 \approx 17000\times g$$

Such acceleration pellets cells and organelles in minutes.

Ultracentrifuges exceed 100 000 rpm to separate proteins and ribosomes.



Laboratory centrifuge — CC0

Example — circular motion in a centrifuge rotor

Tubes in a rotor travel in circles at a constant angular velocity ω.

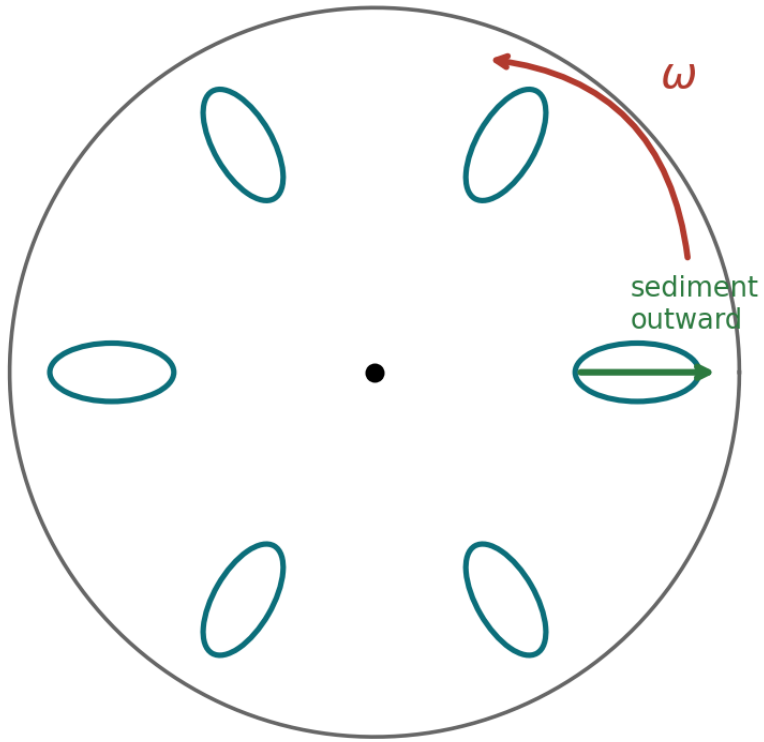
Each sample continually changes direction, so it accelerates toward the axis (centripetal):

$\omega = 2\pi N / 60$ (from rpm), $a = \omega^2 r$

The resulting centrifugal effect drives particles outward, to sediment by density.

A higher ω or a larger radius r means a stronger separating force.

Centrifuge rotor (top view): tubes spin at angular velocity ω



In practice — choosing a spin

Protocols specify the force in × g, not rpm, so they transfer between rotors.

convert with: $RCF = 1.118\times10^{-5} \cdot r(cm) \cdot N(rpm)^2$

At the same rpm, a larger radius r gives a larger force at the tube bottom.

Differential centrifugation spins in steps: nuclei, then mitochondria, then microsomes.

Swinging-bucket vs fixed-angle rotors change the sedimentation path length.



Ultracentrifuge rotor — Cygaretka, CC BY-SA 3.0

Non-Uniform Circular Motion

If the angular velocity changes in time we introduce the angular acceleration: $\alpha = d\omega/dt$ (rad/s^2).

With constant α the angular analogues of uniformly accelerated motion hold:

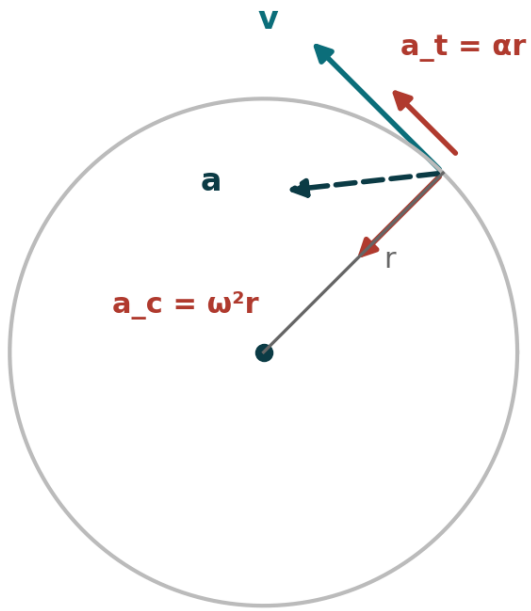
$\omega = \omega_0 + \alpha t \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \omega^2 = \omega_0^2 + 2\alpha\theta$

number of turns = θ / 2π; ω = 2πN/60 (N in rpm).

The acceleration has two components: **tangential** $a_t = \alpha r$ (changes the magnitude of v) and **centripetal** $a_c = \omega^2 r$ (changes the direction):

$a = \sqrt{(a_t^2 + a_c^2)}$

Non-uniform circular motion



Ultracentrifugation and the Svedberg Coefficient

In a rotor the acceleration is $\omega^2 r$: it grows with the distance from the axis. In the Stokes regime the sedimentation velocity is proportional to it:

$$v = s\,\omega^2\,r$$

s is the sedimentation coefficient (unit: seconds; 1 svedberg = 1 S = 10^{-13} s). It depends on the particle (mass, shape, density), not on the machine.

Since $v = dr/dt = s\omega^2 r$, the position grows exponentially:

$$r(t) = r_0\,e^{s\omega^2 t} \qquad t = \ln(r/r_0) / (s\omega^2)$$

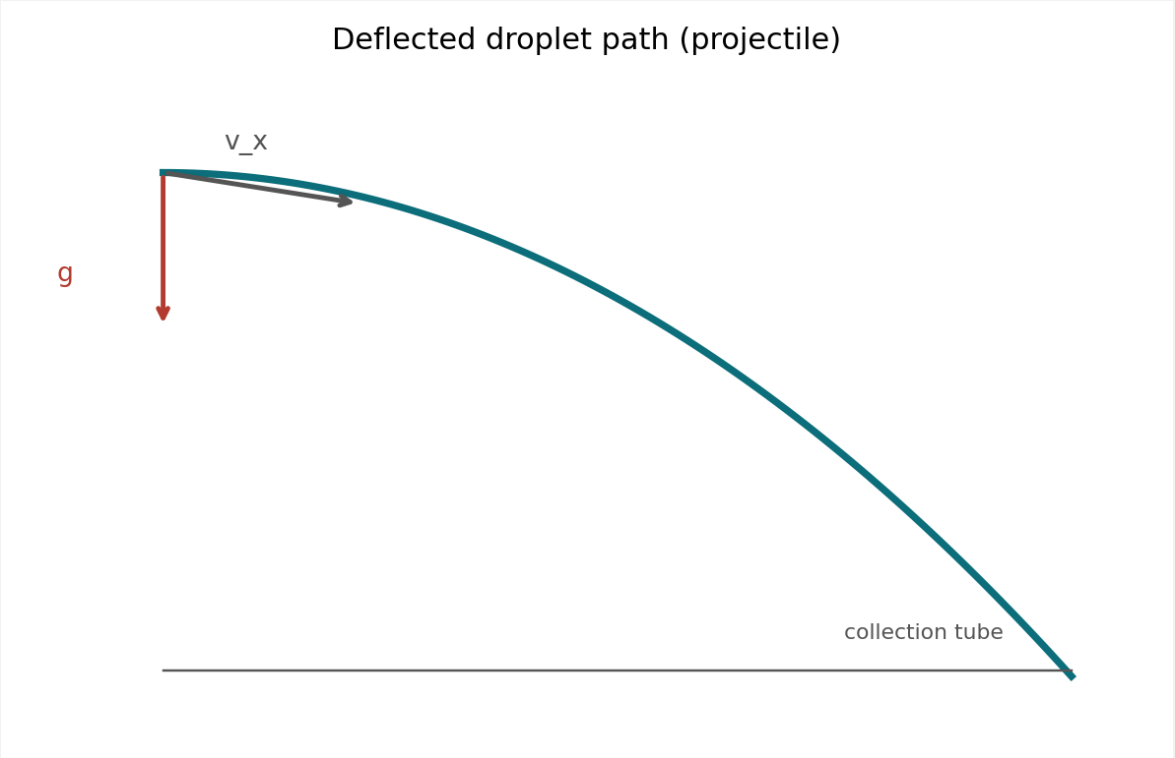
Ribosomes are named after s: 80S in eukaryotes (subunits 60S + 40S), 70S in bacteria (50S + 30S). S values are not additive (60 + 40 ≠ 80): they also depend on shape.

Exercise 2 — sorting droplets (projectile motion)

Droplets fall at v_y = 12 m/s and, past the plates, carry a sideways v_x = 0.40 m/s.

The collection tubes sit 6 cm below the plates. Find (a) the fall time;

(b) the sideways landing distance; (c) the spacing between + and − charged tubes.



Exercise 2 — solution

Step 1 — idea. In projectile motion the motions along x and y are independent and last the same time t: along x the component is constant (v_x = 0.40 m/s); along y the droplet starts with v_y = 12 m/s downwards and accelerates with g.

Step 2 — fall time (a). Along y, with d = 6 cm = 0.06 m downwards:

$$d = v_y t + \tfrac{1}{2} g t^2 \quad \rightarrow \quad 4.905\, t^2 + 12\, t - 0.06 = 0$$

$$t = [-12 + \sqrt{(12^2 + 4 \cdot 4.905 \cdot 0.06)}] / (2 \cdot 4.905) = \mathbf{4.99\times10^{-3}\ s \approx 5.0\ ms}$$

Check: $\tfrac{1}{2} g t^2 = 4.905 \cdot (5.0\times10^{-3})^2 = 1.2\times10^{-4}$ m = 0.12 mm, only 0.2 % of 60 mm. Gravity is negligible and $t \approx d / v_y = 0.06 / 12 = 5.0$ ms is enough.

Step 3 — lateral displacement (b). In the same time t the horizontal component is uniform motion:

$$x = v_x \cdot t = 0.40 \cdot 5.0\times10^{-3} = 2.0\times10^{-3}\ \text{m} = \mathbf{2.0\ mm}$$

Step 4 — distance between the tubes (c). Droplets with opposite charge are deflected in opposite directions: they land at +2.0 mm and −2.0 mm from the vertical. The distance between the tubes is the sum:

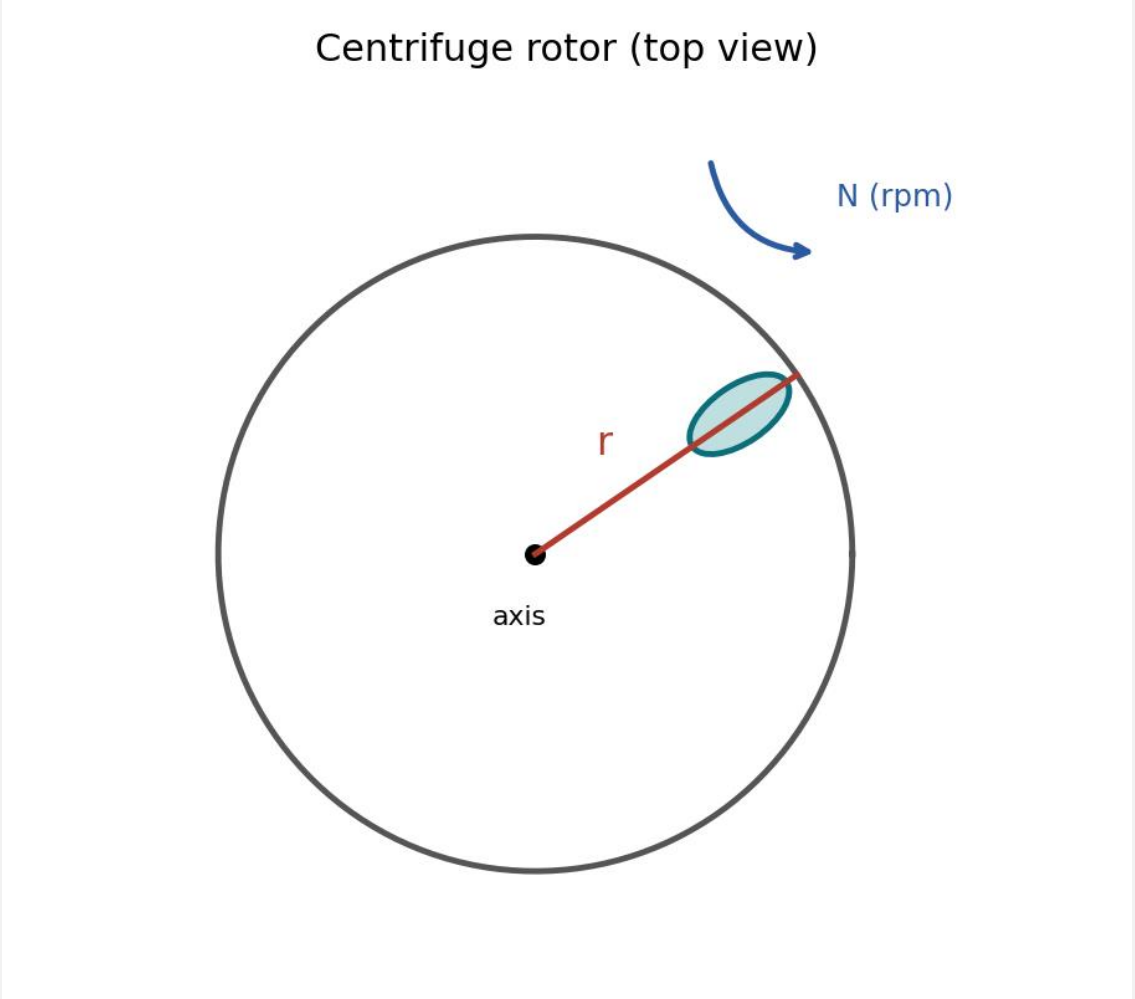
$$2 \cdot 2.0\ \text{mm} = \mathbf{4.0\ mm}$$

Exercise 3 — a centrifuge spin (circular motion)

A rotor of radius r = 10 cm spins at N = 5 000 rpm.

Find (a) the angular velocity; (b) the period; (c) the centripetal acceleration and its ×g;

(d) the number of revolutions in a 5-minute spin.



Exercise 3 — solution

Step 1 — angular velocity (a). Convert from rpm to rad/s: 1 turn = 2π rad and 1 min = 60 s.

$\omega = 2\pi N / 60 = 2\pi \cdot 5000 / 60 =$ **524 rad/s**

Step 2 — period (b). It is the time of one turn: with 5000 turns per minute,

$T = 60 / N = 60 / 5000 =$ **0.012 s** (equivalent to $2\pi/\omega = 0.0120$ s)

Step 3 — centripetal acceleration (c). With r = 10 cm = 0.10 m:

$a = \omega^2 r = 524^2 \cdot 0.10 = 2.74\times10^4$ m/s² → $2.74\times10^4 / 9.81 \approx$ **2800 × g**

Check: $v = \omega r = 52.4$ m/s and $a = v^2/r = 52.4^2 / 0.10 = 2.75\times10^4$ m/s²: the same value.

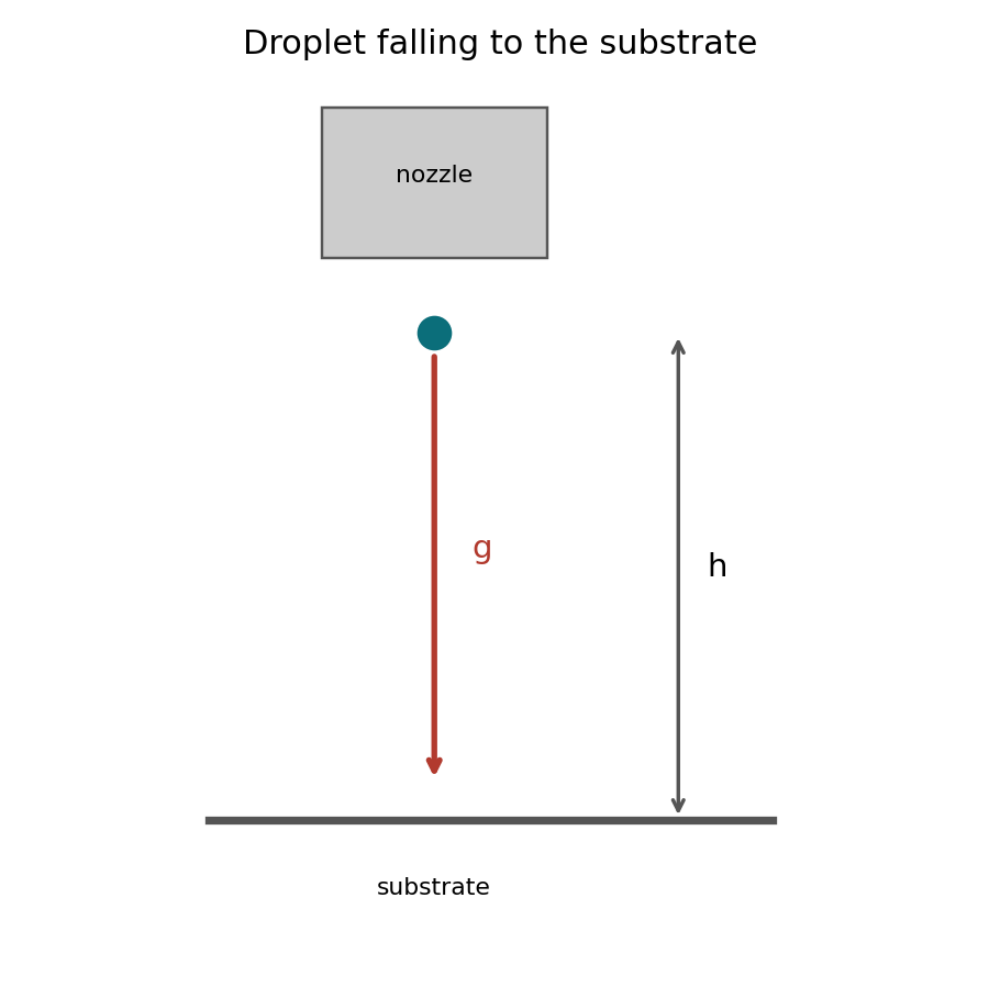
Step 4 — number of turns (d). In 5 min the rotor makes $N \cdot t = 5000$ rpm · 5 min = **25 000 turns**.

Exercise 4 — a bioprinting droplet (free fall)

A droplet is released from rest and falls h = 3.0 mm to the substrate (g = 9.81, ignore air).

Find (a) the fall time; (b) the impact speed;

(c) the impact speed if instead it is ejected downward at 2.0 m/s.



Exercise 4 — solution

Step 1 — idea. Free fall from rest: $v_0 = 0$ and constant acceleration $a = g$ (air neglected). The height is $h = 3.0\ \text{mm} = 3.0\times10^{-3}\ \text{m}$.

Step 2 — fall time (a). From $h = \tfrac{1}{2} g t^2$:

$$t = \sqrt{(2h/g)} = \sqrt{(2 \cdot 0.0030 / 9.81)} = \sqrt{(6.12\times10^{-4})} = \mathbf{0.0247\ s \approx 25\ ms}$$

Step 3 — impact speed (b). $v = g t = 9.81 \cdot 0.0247 = \mathbf{0.243\ m/s}$.

Without going through the time: $v^2 = 2gh \rightarrow v = \sqrt{(2 \cdot 9.81 \cdot 0.0030)} = 0.243\ \text{m/s}$.

Step 4 — ejected downwards (c). With $v_0 = 2.0$ m/s the relation $v^2 = v_0^2 + 2gh$ gives:

$$v = \sqrt{(4.00 + 0.0589)} = \sqrt{4.059} = \mathbf{2.01\ m/s}$$

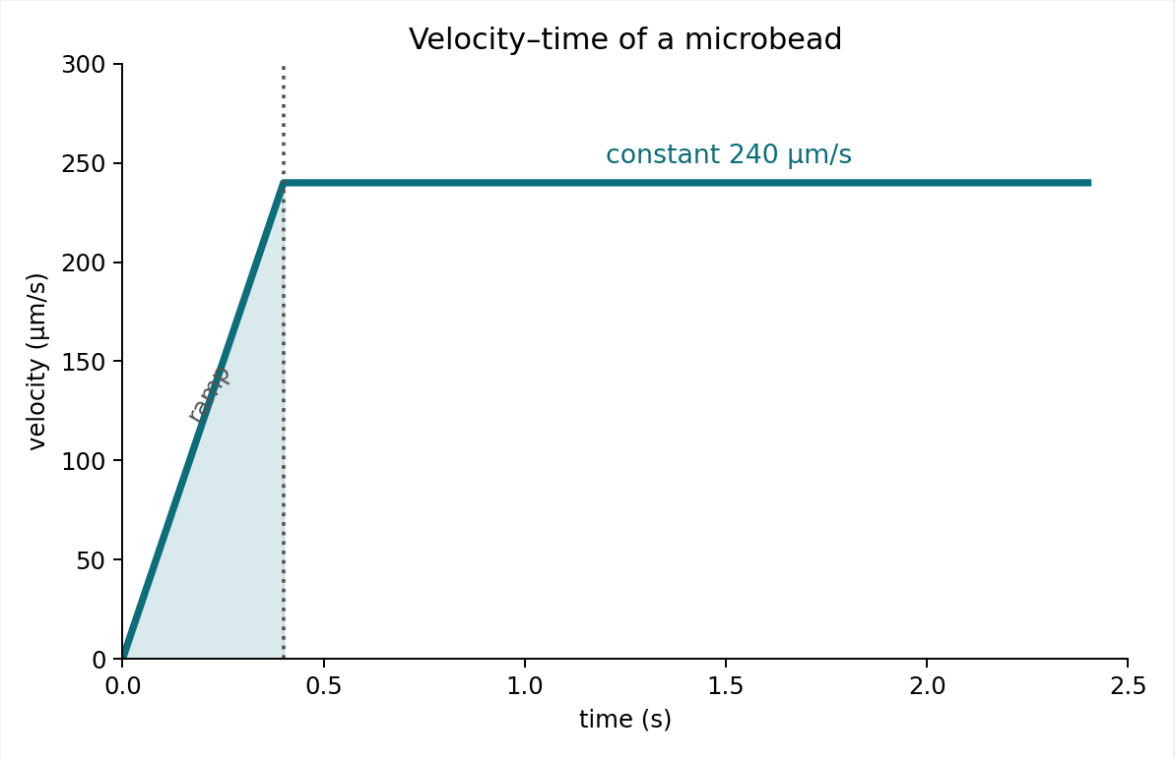
Meaning: the 3 mm fall adds only ≈ 0.01 m/s; the impact speed is dominated by the ejection.

Exercise 5 — a microbead speeding up then cruising

A microbead accelerates uniformly from rest to 240 µm/s over 0.40 s, then moves at 240 µm/s.

Find (a) the acceleration; (b) the distance during the speed-up;

(c) the extra distance in the next 2.0 s at constant speed.



Exercise 5 — solution

Step 1 — units. It is convenient to work in μm and s: v = 240 μm/s (= 2.4×10^{-4} m/s). The motion has two phases: accelerated (0.40 s) and uniform (2.0 s).

Step 2 — acceleration (a). The motion is uniformly accelerated from rest, so a is constant:

$a = \Delta v / \Delta t = (240 - 0) / 0.40 =$ **600 μm/s²** $= 6.0\times10^{-4}$ m/s²

Step 3 — distance during the acceleration (b). From the equation of motion with $v_0 = 0$:

$d = \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2} \cdot 600 \cdot 0.40^2 = \frac{1}{2} \cdot 600 \cdot 0.16 =$ **48 μm**

Check: the average speed in the ramp is (0 + 240)/2 = 120 μm/s, so d = 120 · 0.40 = 48 μm (the area of the triangle under the v(t) curve).

Step 4 — constant-speed phase (c). With constant v the distance is speed times time:

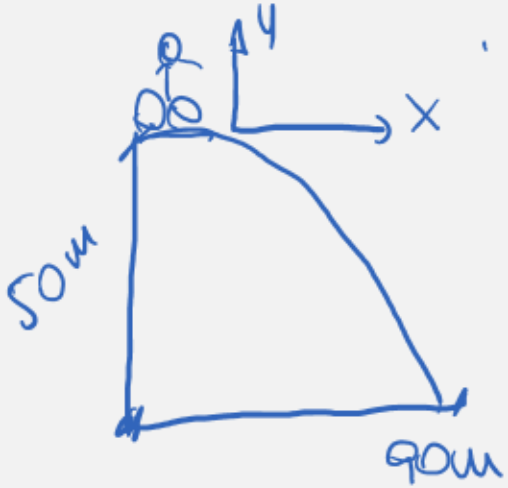
$d = v \cdot t = 240 \cdot 2.0 =$ **480 μm** (the area of the rectangle under v(t))

Step 5 — final check. Total distance = 48 + 480 = 528 μm in 2.4 s, i.e. 220 μm/s on average: a value between 0 and 240 μm/s, as expected.

PARABOLIC MOTION

Exercise

A stuntman launches himself horizontally on a motorcycle from a cliff 50 m high. At what speed does the motorcycle leave the edge of the cliff if it lands on the surrounding ground at a distance of 90 m from the base of the cliff?



$\Delta x = 90m$

$y_f = -50m$

v_{ix}? $\rightarrow$ $\Delta x = v_{ix}\Delta t \rightarrow v_{ix} = \frac{\Delta x}{\Delta t}$

$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$

Δy: $-50m$; v_{iy}: $0m/s$; a: $-9.8m/s^2$

$\rightarrow \Delta t = \sqrt{\frac{2\cdot(-50m)}{-9.8m/s^2}}$

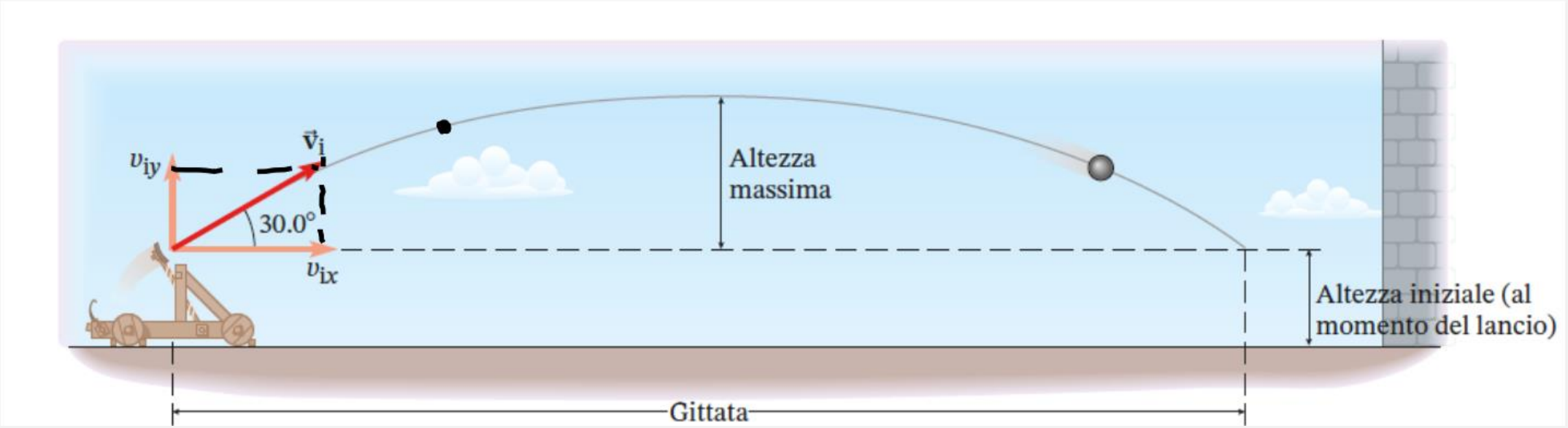
$= 3.19s$

$v_{ix} = \frac{\Delta x}{\Delta t} = \frac{90m}{3.19m/s} = 28.2m/s$

PARABOLIC MOTION

Exercise

A catapult launches a 32 kg stone with a speed of 50.0 m/s at an elevation angle of 30.0°. A) What is the maximum height reached by the stone? B) What is the flight time (the time interval during which the projectile remains in the air before falling back to the ground)? C) What is its range (the horizontal distance traveled by the projectile before falling back to the ground)?



Exercise — solution (a)

Step 1 — idea. In projectile motion ($g = 9.8$ m/s², air neglected) the motions along x and y are independent: along x the velocity is constant ($a_x = 0$), along y the acceleration is constant $a_y = -g$. The mass (32 kg) is not needed.

Step 2 — initial components. With $v_i = 50.0$ m/s and $\theta = 30.0°$:

$$v_{ix} = v_i \cos\theta = 50.0 \cdot 0.866 = 43.3 \text{ m/s} \qquad v_{iy} = v_i \sin\theta = 50.0 \cdot 0.500 = 25.0 \text{ m/s}$$

Step 3 — maximum height (A). At the top the vertical component vanishes, $v_y = 0$. Rise time:

$$v_y = v_{iy} - g\,t_r = 0 \quad \rightarrow \quad t_r = v_{iy} / g = 25.0 / 9.8 = 2.55 \text{ s}$$

$$h = v_{iy}\,t_r - \tfrac{1}{2}\,g\,t_r^2 = v_{iy}^2 / (2g) = 25.0^2 / (2 \cdot 9.8) = \mathbf{31.9\ m}$$

Equivalent with the average vertical velocity $(0 + v_{iy})/2$: $h = \tfrac{1}{2} \cdot 25.0 \cdot 2.55 = 31.9$ m.

Exercise — solution (b–c)

From the previous steps: $v_{ix} = 43.3$ m/s, $v_{iy} = 25.0$ m/s, $t_r = 2.55$ s.

Step 4 — flight time (B). The stone lands at the launch height ($\Delta y = 0$): rise and descent last the same time, so

$$t = 2\,t_r = 2 \cdot 2.55 = \mathbf{5.10\ s} \qquad (\text{or from } 0 = v_{iy}\,t - \tfrac{1}{2}\,g\,t^2 \;\rightarrow\; t = 2\,v_{iy}/g)$$

Step 5 — range (C). Along x the motion is uniform for the whole flight time:

$$R = v_{ix} \cdot t = 43.3 \cdot 5.10 = \mathbf{221\ m}$$

Step 6 — check. With the direct formula: $R = v_i^2 \sin 2\theta / g = 50.0^2 \cdot \sin 60° / 9.8 = 2500 \cdot 0.866 / 9.8 = 221$ m. Same value.

Meaning: the stone hits the ground with the same speed as at launch (50.0 m/s), directed 30° downwards; the mass does not appear in any step.